

21.

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CS 271 - HW 5

1) Use Lagrange multiplier to solve the problem.

$$L = 16 - (x^2 + y^2) + \lambda(2x - y + 4) \quad (1)$$

take partial derivative and set it to 0.

$$\frac{\partial L}{\partial x} = -2x + \lambda(2) = 0 \Rightarrow x = \lambda$$

$$\frac{\partial L}{\partial y} = -2y + \lambda(-1) = 0 \Rightarrow y = -\frac{\lambda}{2}$$

Substituting while using the x, y value

$$L(\lambda) = 16 - \left(\lambda^2 + \left(-\frac{\lambda}{2}\right)^2\right) + \lambda\left(2\lambda - \frac{\lambda}{2} + 4\right)$$

$$L(\lambda) = 16 - \lambda^2 - \frac{\lambda^2}{4} + 2\lambda^2 - \frac{\lambda^2}{2} + 4\lambda$$

$$L = 16 + \frac{5\lambda^2}{4} + 4\lambda \Rightarrow \lambda =$$

$$L = \frac{64 + 5\lambda^2 + 16\lambda}{4}$$

$$\frac{\partial L}{\partial \lambda} = 10\lambda + 16 = 0 \Rightarrow \lambda = -\frac{16}{10} = -\frac{8}{5}$$

$$\Rightarrow x = -\frac{8}{5}$$

$$y = \frac{4}{5}$$

$$\lambda = -\frac{8}{5}$$

$$\text{To validate: } g(x) = 2x - y + 4 = 2\left(-\frac{8}{5}\right) - \left(\frac{4}{5}\right) + 4 = 0 \quad \checkmark$$

$$(3) \quad p_i = 2^{-(\lambda + 1/n)} = \frac{1}{e} 2^{-\lambda} \text{ for } i=1, 2, \dots, n. \quad (5.10)$$

$$\text{and } \sum_{i=1}^n p_i = 1 \quad (5.11)$$

$$\text{Show that } p_1 = p_2 = \dots = p_n = \frac{1}{n}.$$

We have n values.

$$\sum_{i=1}^n p_i = 1 = n \cdot \frac{1}{e} 2^{-\lambda}. \quad \text{Note: } \left(\frac{1}{n} \cdot n = 1 \right)$$

take log both side

$$\log(1) = \log\left(n \cdot \frac{1}{e} 2^{-\lambda}\right)$$

$$\Leftrightarrow 0 = \log \frac{n}{e} + \log 2^{-\lambda}.$$

$$\Leftrightarrow 0 = \log \frac{n}{e} + (-\lambda) \log 2.$$

$$\Leftrightarrow 0 = \log \frac{n}{e} - \log 2 \cdot \lambda \Leftrightarrow \lambda = \frac{\log \frac{n}{e}}{\log 2}$$

$$\Leftrightarrow \lambda = \log_2 \left(\frac{n}{e} \right) \quad \left[\log_a N = \frac{\log N}{\log a} \right]$$

$$\text{We have } p_i = \frac{1}{e} 2^{-\lambda} = \frac{1}{e} 2^{-\log_2 \left(\frac{n}{e} \right)} = \frac{1}{e \cdot 2^{\log_2 \frac{n}{e}}}$$

$$\Leftrightarrow \frac{1}{e \cdot n/e} = \frac{1}{n}. \quad \square$$

$$(4). L(\lambda) = \sum_{i=1}^n \lambda_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \lambda_i \lambda_j z_i z_j (X_i \cdot X_j) \quad (5.20)$$

where $X_i \cdot X_j = \underbrace{(x_i, y_i)^T (x_j, y_j)^T}_{(\text{dot product})} = x_i x_j + y_i y_j$

$$L(w_1, w_2, b, \lambda) = \frac{w_1^2 + w_2^2}{2} + \sum_{i=1}^n \lambda_i (1 - z_i (w_1 x_i + w_2 y_i + b))$$

$$\frac{\partial L}{\partial w_1} = \frac{2w_1}{2} - \sum_{i=1}^n \lambda_i z_i x_i = 0 \Rightarrow w_1 = \sum_{i=1}^n \lambda_i z_i x_i$$

$$\frac{\partial L}{\partial w_2} = \frac{2w_2}{2} - \sum_{i=1}^n \lambda_i z_i y_i = 0 \Rightarrow w_2 = \sum_{i=1}^n \lambda_i z_i y_i$$

$$\frac{\partial L}{\partial b} = \sum_{i=1}^n \lambda_i z_i = 0$$

$$\frac{\partial L}{\partial \lambda_i} = 1 - z_i (w_1 x_i + w_2 y_i + b) = 0 \text{ for all } i = 1, 2, \dots, n$$

$$L(x) = L(w_1, w_2, b, \lambda) = \frac{\left(\sum_{i=1}^n \lambda_i z_i x_i \right)^2}{2} + \frac{\left(\sum_{i=1}^n \lambda_i z_i y_i \right)^2}{2} + \sum_{i=1}^n \lambda_i - \sum_{i=1}^n \lambda_i z_i (w_1 x_i + w_2 y_i + b)$$

we have $\sum \lambda_i z_i = 0$

$$\Rightarrow L(w_1, w_2, b, \lambda) = \frac{\left(\sum_{i=1}^n \lambda_i z_i x_i \right)^2}{2} + \frac{\left(\sum_{i=1}^n \lambda_i z_i y_i \right)^2}{2} + \sum_{i=1}^n \lambda_i (1 - 0)$$

$$\Rightarrow L(\lambda) = \sum_{i=1}^n \lambda_i - \frac{1}{2} \left(\sum_{i=1}^n \sum_{j=1}^n (\lambda_i z_i x_i) (\lambda_j z_j x_j) \right) + \left(\sum_{i=1}^n \sum_{j=1}^n (\lambda_i z_i y_i) (\lambda_j z_j y_j) \right)$$

$$= L(\lambda) = \sum_{i=1}^n \lambda_i - \frac{1}{2} \left(\sum_{i=1}^n \sum_{j=1}^n (\lambda_i z_i \lambda_j z_j) (x_i x_j + y_i y_j) \right)$$

$$L(\lambda) = \sum_{i=1}^n \lambda_i - \frac{1}{2} \left(\sum_{i=1}^n \sum_{j=1}^n \lambda_i \lambda_j z_i z_j (X_i \cdot X_j) \right) \text{ where } (X_i \cdot X_j) = x_i x_j + y_i y_j \quad \checkmark$$

15. If MIPS instruction format would have a max number of instructions be there for MIPS processor?

$$g) f(x) = \sum_{i=1}^n \lambda_i z_i (x_i x) + b. \quad (5.21)$$

a) determine the weight associated with each component of a scored vector.
We have (5.21) is rewrite of 5.20.

$$\text{which is } f(x) = w_1 x + w_2 y + b = w \cdot x + b.$$

$$\text{Consider (5.21) } f(x) = \sum_{i=1}^n \lambda_i z_i (x_i x) + b. \quad \text{as } x_i x = x_i x + y_i y$$

$$= \sum_{i=1}^n \lambda_i z_i (x_i x + y_i y) + b$$

$$\Rightarrow \sum_{i=1}^n \lambda_i z_i x = (z, y) \Leftrightarrow \sum_{i=1}^n \lambda_i z_i (x_i + y_i) = w.$$

b) When we train the SVM for determine the weights of each feature, The weight states the more importance of combination of the features—each feature and hence more weight, more important. Apply this we can eliminate the lower weight of each features when compared to others and can achieve dimensionality reduction.

c) PCA: we are reduced noises and determine the important in terms of Principal Component which uses the linear combination of features. We will not get the direction values of importance (weight) of the features and so it is difficult to say which features are important. Because of that, it is difficult to archive dimensionality reduction in terms of features. The eigenvectors and eigenvalues can be determined from calculated the covariance matrix (higher the values, higher the variances, more important feature) and then there will no required for training.

SVM: the weights of the model are determined after performing the training.
Pros: when we get the weight of each features, so we can eliminate the features based on its importance weight (the more weight, the more important).

1.b.

When we train the SVM for determine the weight of each feature. The weight states the more important of each features and hence the more it weights, the more it is important. Apply in this, we can eliminate the lower weight of each features when compared to others and can achieve the dimensionality reduction.

1.c.

PCA:

While working with PCA, we are reduced noises and determine the important in terms of Principal Component which uses the linear combination of features. We will not get the direct values of importance weight of the features and so it will be hard to say which features are important. Because of that, it will be difficult to archive dimensionality reduction in terms of features.

The eigenvectors and eigenvalues can be calculated from Covariance matrix (higher the value, higher the variance, more important features) and then there will no requires for training.

SVM:

Cons:

The weights of the model are determined after performing the training.

Pros:

When we get the weight of each features then we can eliminate the feature based on its weights.

0.a.

$\gamma = 1.0, p = 2$

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
1 0]

0.9743589743589743

0.b.

$\gamma = 3.0, p = 2$

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
1 0]

0.9743589743589743

0.c.

$\gamma = 1.0, p = 4$

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 1 1 1 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
1 0]

0.9230769230769231

0.d.

$\gamma = 3.0, p = 4$

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 1 1 1 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0
1 0]

0.9230769230769231

11./

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 0 1 0 1 0
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0.9487179487179487

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0.9743589743589743

0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 0 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0

12./

1.9487179487179487

[0.19688024 -0.53494025 -0.70487142]]

Accuracy after removing the HMM feature ->

[-2.44489465 -2.24186897]]

1.9487179487179487

[-3.87596913]]

0. 0. 2.5 0. 1.25 1.25]

$$\tau[0] = 1.4999999999999982$$
$$\tau[1] = 2.7499999999999982$$
$$\tau[2] = 0.25000000000000018$$
$$\tau[3] = -3.4999999999999999$$

$\gamma[4] = -1.0$

$$[4] = -1$$

$\tau[5] = -1.0$

[5] = -1

[15.b

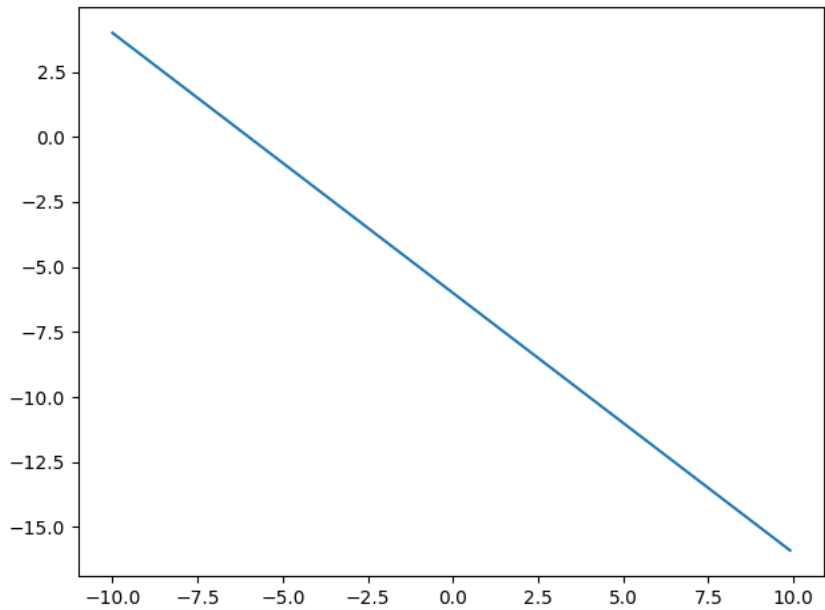
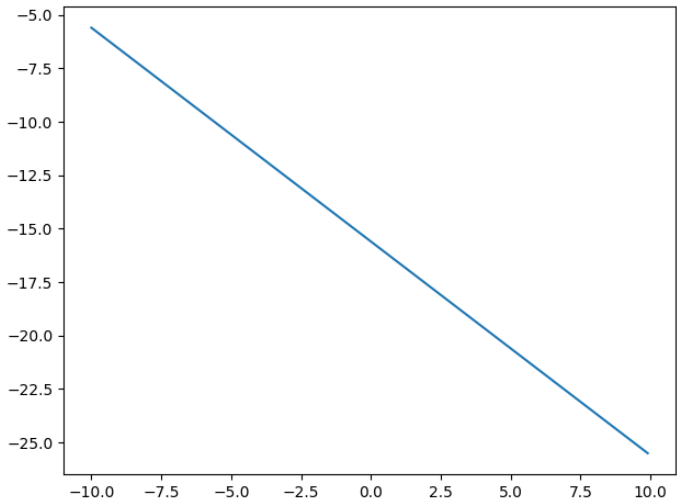
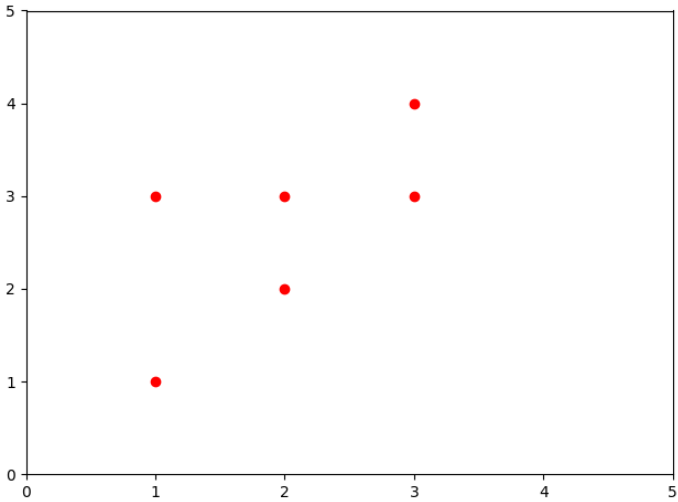
1.32173539 0. 3.09731961 0. 1.91905501 2.5]

15.60197599369006

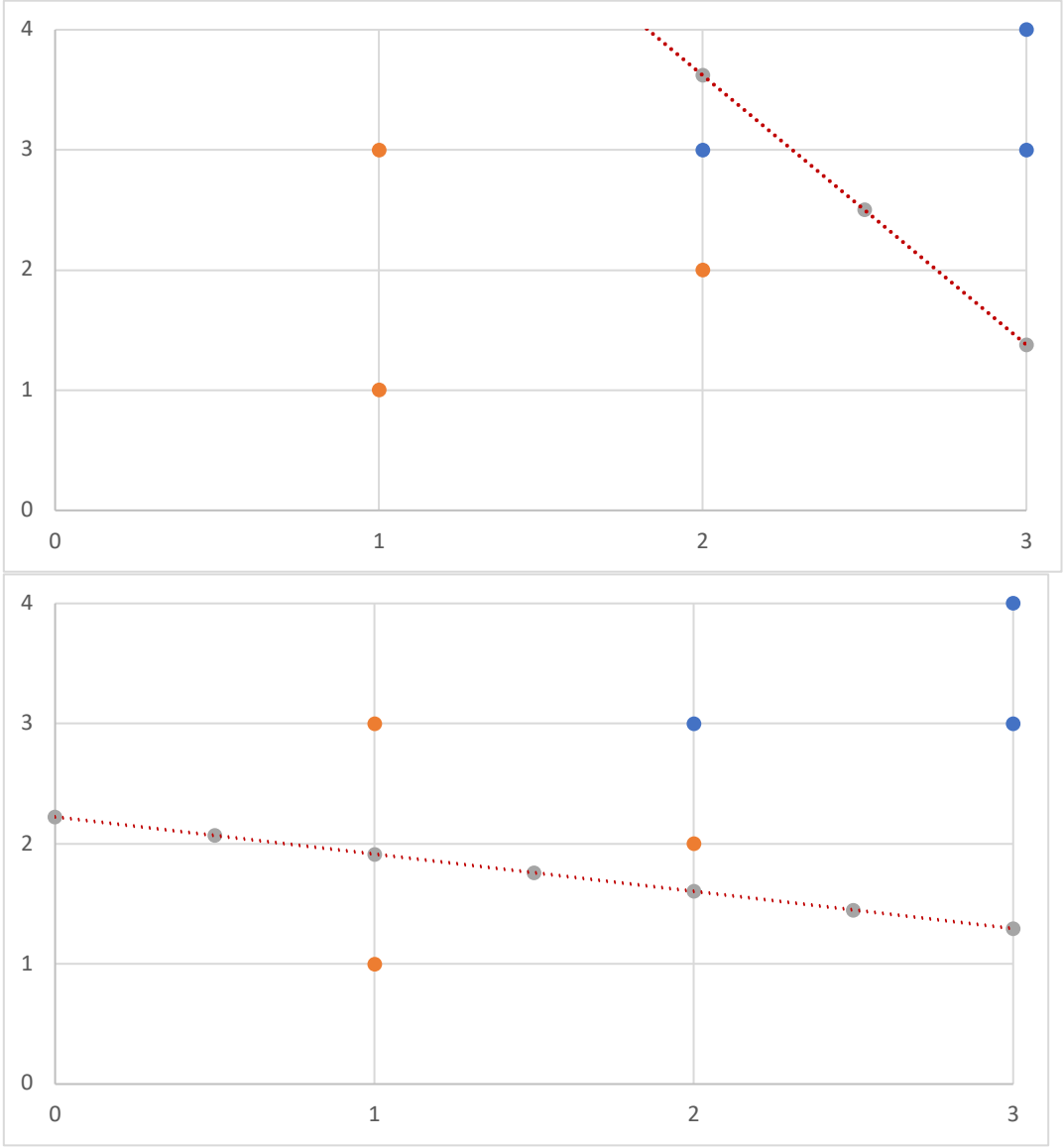
5.c.

'art a, we have b = -5.999999999999998 so the eqs for hyperplane is $x + y = -5.999999999999998$

'art b, we have b = -15.60197599369006 so the eqs for hyperplane is $x + y = -15.60197599369006$



group	x	y
A	3	3
A	3	4
A	2	3
B	1	1
B	1	3
B	2	2



REFERENCES

- 1. M. Stamp, “Introduction to Machine Learning with Applications in Information Security.”
- 2. Steven A.Cohen and Matthew W.Granade, “Models will run the world”