Report

BUSINESS CASE IN REINSURANCE

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Summary

This report is related to our quotations. It shows our pricing process as a reinsurer, which was carried out for required reinsurance structures with a set of data.

After treating the given data properly, we will try to find models for each kind of losses by two approaches: probabilistic and exposure rating. Then we apply reinsurance structures for each model to obtain the pure premium and compare the results between two methods. From that and upon choosing a target ROE, we will determine our commercial prices.

In the last part of the report is our feedback post-placement.

Data treatment

1. As If

Our first step is to re-evaluate the historical data (Premium Income and Claims), which is the "As If" calculation, to obtain a comparable data with the year of coverage (2020). In order to do this, for each data, we use a different index:

- For Premium Income: *Inflation Rate*
- For Claims: *Cost Construction Index*

In the process of pricing, we will use these As If data instead of given historical data.

2. CLA split

We use the Claim As If and split them into CAT, Large and Attritional losses:

- To take out CAT, we count the number of claims in each day. Those claims in days with large number of small claims are considered as CAT.

Date	Nb Claim	Claim	Mean
10/3/2002	1514	48,720,470	32,180
7/31/2001	1480	8,026,820	5,424
3/15/2014	1378	15,941,326	11,568
3/12/2012	1243	16,966,539	13,650
9/24/2010	1231	2,443,186	1,985
9/25/2001	1120	2,736,102	2,443
4/13/2019	827	1,622,380	1,962
4/3/2001	453	2,806,734	6,196

However, there is a possibility that there exist some Large losses in those claims. After checking, we identify only one Large loss on 15/3/2014.

- For the rest of the data, we choose a threshold (T) to divide the Large and Attritional losses. We consider the approximation of 75% of the excess point in the first layer of XL option (750,000). The claims above $T = 550{,}000$ would be treated as Large losses.

Modelling and Simulations

1. CAT Losses

Our hypotheses are:

- Number of CAT follow Poisson distribution, with

  mean = sum of occurrence probability of all events
- Severity of CAT is Log-Normal distribution with expectation, standard deviation and max value given in the Event Loss Table (ELT).

First, we calculate:

$$\textbf{Probability of a CAT event to be event } i = \frac{\textbf{occurence probability of event } i}{\textbf{sum of occurence probability of all events}}$$

We then simulate CAT losses for 10,000 years:

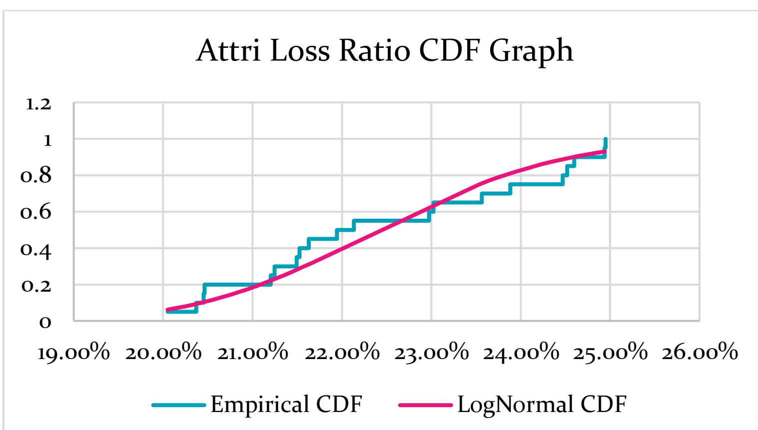
- For each year, we random the number of CAT losses (N) following the law of Poisson.
- Generate N claims, each is Log-Normally distributed with the max value is the given Max loss in ELT.

2. Large and Attritional Losses

2.1. Experience Method

a. Attritional losses

Assume that the loss ratio of Attritional losses follows Log-Normal distribution, we fit the historical data and get the result: LR Attri ~ $LN(\mu,\sigma^2)$ where $\mu = -1.49552$ and σ^2=0.00523.

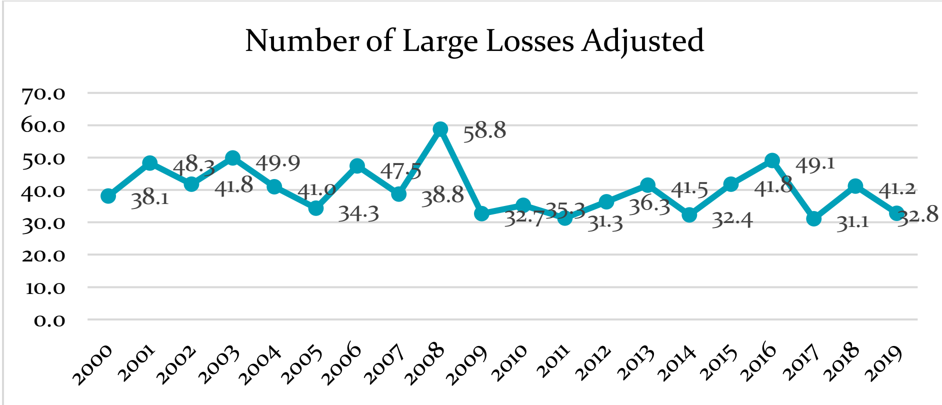


b. Large loss

Frequency

We count the number of Large losses for each year and then re-adjust them:

$$\textbf{Adjusted Number of Large Losses } (i) = \frac{\textbf{Number of Large Losses } (i) \textbf{ x Premium 2020}}{\textbf{Premium As If } (i)}$$

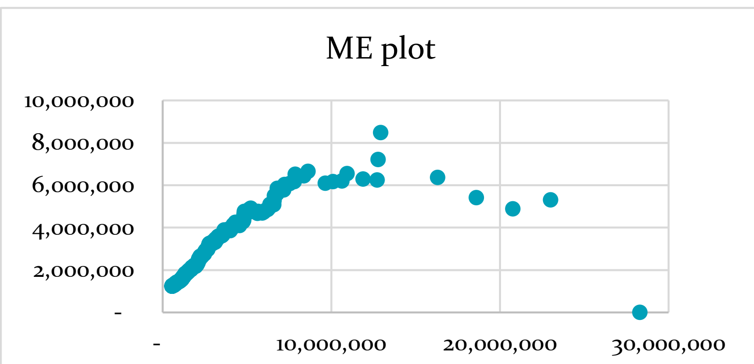


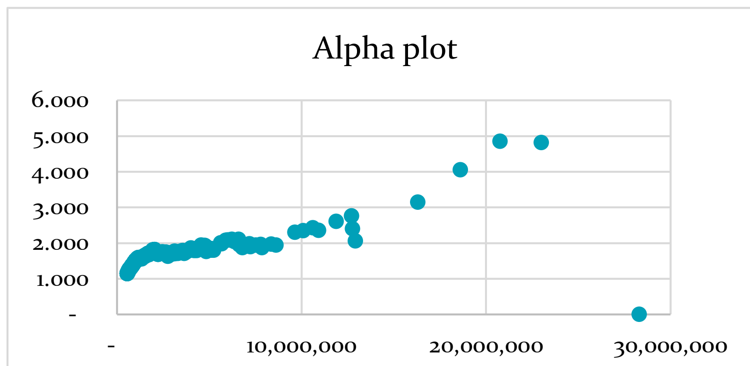
We obtain the average of 40.2 and variance of 55.8 for the adjusted number of large losses.

For the frequency model, we suppose the number of large losses follows either Poisson or Negative Binomial distribution. Since the variance we have here is greater than the mean of our sample, we decided to use the Negative Binomial, *NB (n, p)* with $n = 103$ and $p = 0.27996$.

Severity

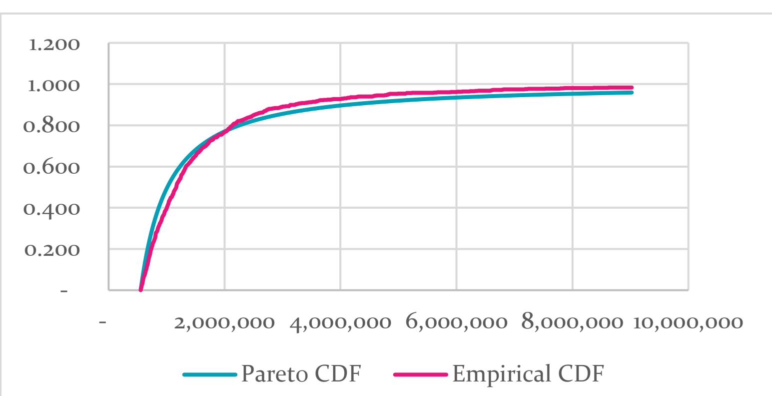
To obtain the model for severity of the Large losses, we first perform a mean-excess plot and α-plot:

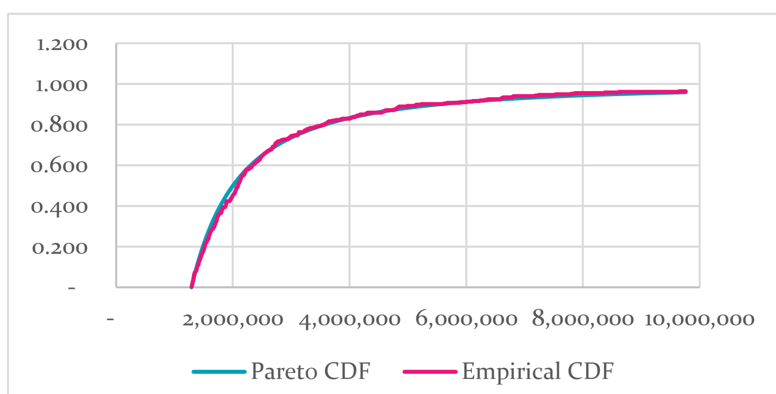




We notice that generally the ME plot is flat and increasing, which seems correspond to the ME plot of a Pareto distribution, *Pareto (A, α)*.

The following graphs show CDF of Pareto for different *A* and *α* in comparison with the empirical CDF:





In the graph on the left, we take A = 553,319.574 which is the lowest value of large loss claims, and α = 1.145. We can see that the two CDFs don't fit. So, we continue to increase A. As A = 1,291,448.029, α = 1.581, we have the graph on the right, where the Pareto CDF finally fit to the empirical one.

For the Large losses that are below A, we observe the empirical CDF and decide to use the Conditional Exponential distribution with mean λ equals to 870,048.39:

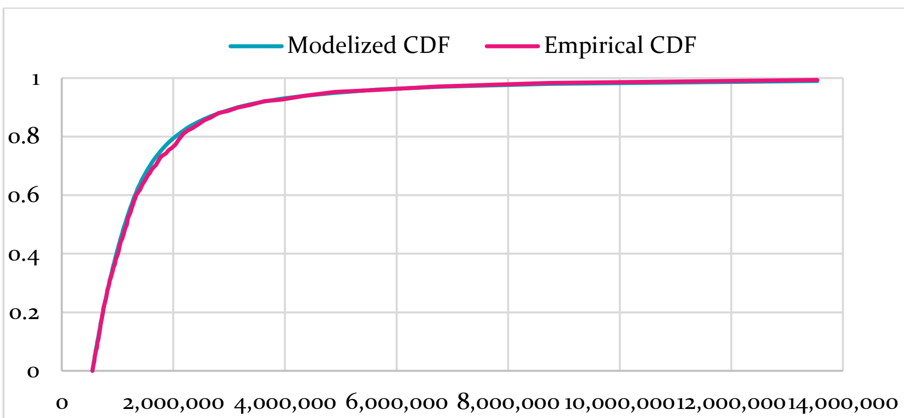
$$F_X(x \mid T < X < A) = \frac{e^{-T/\lambda} - e^{-x/\lambda}}{e^{-T/\lambda} - e^{-A/\lambda}}$$

Then we use the mixture distribution to modelizer the distribution of Large losses:

$$F(x) = \begin{cases} rF_1(x) & \text{for } T < x < A \\ (1-r)F_2(x) & \text{for } x > A \end{cases}$$

where $F_1(x) = \dfrac{e^{-T/\lambda} - e^{-x/\lambda}}{e^{-T/\lambda} - e^{-A/\lambda}}$; $F_2(x) = 1 - \left(\dfrac{A}{x}\right)^{\alpha}$ and $r = \dfrac{F'_2(A)}{F'_2(A) + F'_1(A)}$.

Hence, we have the following graph:



c. Simulation process

We simulate 10,000 years for each type of loss.

- *Attritional Loss*: Generate 10,000 LR Attri following the law of Log-Normal. Multiply with EPI to have the total attritional claim.
- *Large Loss*: For each simulation, generate number of claim (N) following Negative Binomial distribution. Then generate N claims following the distribution for severity we have found above.

2.2. Exposure rating

For this part we will use the given exposure curve:

$$G_{p,\alpha}(x) = \frac{x - \frac{1-p}{1+\alpha} x^{1+\alpha}}{1 - \frac{1-p}{1+\alpha}} \text{ with } \alpha = 2.96\% \text{ and } p = 0.135\%$$

From this we can deduce

- *Claim Severity model for each band*: Suppose that all have Sum Insured equal to the average sum insured of the band. The severity for losses in band *i* is

$$F_{X_i}(x) = F\left(\frac{x}{SI_i}\right) \text{where } F(d) = \begin{cases} 1 - \dfrac{G'(d)}{G'(0)} & 0 \le d < 1 \\ 1 & d = 1 \end{cases}.$$

  Then the average individual loss amount for band *i* is $E(X_i)$.
- *Claim Frequency model for each band*: we use the expected loss for each band in 2019 $E(S_i)$. Then, the expected number of loss band *i* is $E(N_i) = E(S_i)/E(X_i)$. Assume that the number of claims in a band follows a Poisson distribution $Poi(\lambda_i)$, with $\lambda_i = E(N_i)$.

Note that CAT losses are not included in these models.

From the two above models, we have the aggregate loss model across all bands:

- Number of simulations: 10,000
- For each simulation, random the number of losses for band *i* (N_i),
- Generate N_i loss amount following distribution F_{X_i},
- The total loss across all bands for one simulation is $S = \sum_i \sum_{k=1}^{N_i} X_{i,k}$.

Application of Reinsurance Structures

Option		Deductible	Cover	# Recs	1st Rec	Other Rec	AAD	AAL	Experience Method		Exposure Method	
									Without Clauses	With clauses	Without Clauses	With clauses
QS 60%									79,586,168		66,976,130	
SL	SL1	75%	125%						9,801,543		4,799,562	
	SL2	80%	120%						8,363,597		4,206,140	
XL Risk	XL1a	750,000	1,750,000	5	100%	200%	-	NA	24,293,534	1,049,609	29,518,093	1,050,000
	XL2a	2,500,000	27,500,000	3	100%	200%	-	NA	19,227,845	10,570,133	20,231,095	11,118,337
XL CAT	XL1b	1,500,000	2,500,000	NA	NA		2,500,000	7,500,000	691,153	85,062		
	XL2b	4,000,000	31,000,000	NA	NA		-	62,000,000	3,595,678	3,583,920		
XL Risk+CAT	XL1c	2,000,000	3,000,000	3	100%	100%	-	NA	12,438,567	2,687,382		
	XL2c	5,000,000	35,000,000	3	100%	100%	-	NA	15,285,419	10,640,222		

- Proportional reinsurance programs: From the simulation process, we have the aggregate loss distribution and loss ratio to price proportional reinsurance program. With the sample of 10,000 years, we get the average pure premium and its volatility.

- Non-proportional programs: From simulation by two methods we get large losses in each year for applying the XL structures. We can also calculate the price with clauses.

Optimal Reinsurance

After discussion, we have decided the profile of our company to be A. From the target ROE of a company rating A, we will estimate our profitability criteria then calculate the commercial price of each reinsurance option.

In order to compute ROE for QS option, we need to deduct Profit Ceded from Combined Operating Result. With the target ROE, we use Solver to obtain Profit Ceded percentage.

For SL and XL options, we use Solver to get the Loss Ratio of Reinsurer. From that, we can calculate the commercial prices.

Results and Conclusion

QS Option

QS **Cession** percentage	60%
Commission	30%
Profit ceded %	55.61%
Average LR CAT	5.05%
Average LR LL + Att	59.08%
Average LR Global	64.13%
RoE/RoRaC	6.00%

SL Option

	Deductible	Limit	Technical price (rate)	Commercial Price	Combined Ratio	RoE/RoRaC
SL1	75%	125%	4.77%	6.46%	76.36%	7.00%
SL2	80%	120%	4.07%	5.52%	76.36%	7.00%

XL Risk Option

	Deductible	Limit	# Recs	1st Rec	Other re	AAD	AAL	Technical price without clause	Rec Factor	Technical price (rate)	Commercial Price	Combined Ratio	RoE/RoRaC
XL1a	750,000	2,500,000	5	100%	200%	-	NA	24,293,534	4.32%	0.51%	0.59%	74.25%	7.00%
XL2a	2,500,000	30,000,000	3	100%	200%	-	NA	19,227,845	54.97%	5.15%	5.94%	74.25%	7.00%

XL Cat Option

	Deductible	Limit	# Recs	1st Rec	Other re	AAD	AAL	Technical price without clause	Rec Factor	Technical price (rate)	Commercial Price	Combined Ratio	RoE/RoRaC
XL1b	1,500,000	4,000,000	NA	NA		2,500,000	7,500,000	691,153	12.31%	0.04%	0.05%	74.25%	7.00%
XL2b	4,000,000	35,000,000	NA	NA		-	62,000,000	3,595,678	99.67%	1.75%	2.01%	74.25%	7.00%

XL Risk + Cat Option

	Deductible	Limit	# Recs	1st Rec	Other re	AAD	AAL	Technical price without clause	Rec Factor	Technical price (rate)	Commercial Price	Combined Ratio	RoE/RoRaC
XL1c	2,000,000	5,000,000	3	100%	100%	-	NA	12,438,567	21.61%	1.31%	1.51%	74.25%	7.00%
XL2c	5,000,000	40,000,000	3	100%	100%	-	NA	15,285,419	69.61%	5.18%	5.97%	74.25%	7.00%

Quotations

For the quotations, after comparing two methods, we decided to use experience method for pricing. Because from the results of exposure curve simulation, we have an average 43 large losses each year, which is relatively higher than the average 40.2 of historical data.

In conclusion, we would prefer QS and SL options since the volatility of Loss Ratio is relatively small. XL risk option is also in our preference choices since its volatility is not large either. XL CAT has a quite large volatility, so when writing this structure, we will need to pay attention to the security loading.

Feedback Post-Placement

We have some difficulties in finding insurers due to the difference in prices. Often, our prices are quite higher than them.

With prices more advantageous for the insurers, in the end we still signed some contracts. However, this leads to our loss for the next years (simulation). Therefore, it might be possible that our original prices are not too high compare to the real ones.

Annex

VBA functions used for the simulation process:

`poissoninv(x, lambda)`	*Inverse Poisson function* *Generate random count number given probability x and parameter lambda of Poisson distribution*
`NBinv(x, n, p)`	*Inverse Negative-Binomial (n, p)*
`lognormalinv(x, mu, sigma)`	*Inverse Log-Normal function with mu and sigma is mean and standard deviation of variable (ETL)*
`CDFLargeLossInv(x, T, A, alpha, lambda, r)`	*Inverse function of Large loss distribution function:* *Conditional Exponential (lambda)* *Pareto (A, alpha)* *T: Large Loss threshold* *r: parameter in mixture distribution*
`severity(x, alpha, p)`	*Severity function deduce from the given exposure curve*