MAD map for multivariate discrete distributions

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goal: approximate *multivariate* discrete posterior $p(x),\ x \in \mathbb{N}^M$

augmented target: $\tilde{p}(x, u) = p(x) \cdot 1_{[0,1]^M}(u)$

idea: sequentially apply univariate map to *full conditionals* conditional of mth entry given the rest

$$\tilde{p}_m(x_m, u_m) = \tilde{p}(x_m, u_m \mid x_{-m}, u_{-m})$$

$$= p(x_m \mid x_{-m}) \cdot 1_{[0,1]}(u_m)$$

only need full conditional of posterior!

(which $\propto$ joint)

start with (x, u)

update (x'_1, u'_1) with MAD map targeting $\tilde{p}_1$

update (x'_2, u'_2) with MAD map targeting $\tilde{p}_2$

conditioning on $(x'_1, x_3, \cdots, x_M)$

⋮

update (x'_M, u'_M) with MAD map targeting $\tilde{p}_M$

conditioning on x'_{-M}

end with $(x', u') =: T_{\mathrm{MAD}}(x, u)$

does it work?

does it work? yes! T_{MAD}:

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- is invertible

each univariate pass is invertible

inverse evaluated by evaluating with negative shift: $-\xi$

T_{MAD} invertible by evaluating in inverse order, i.e., M to 1

does it work? yes! T_{MAD}:

- is invertible
- has tractable density

if $(x, u) \sim q_0$ then

$$p(T_{\mathrm{MAD}}(x, u)) = \frac{q_0(T_{\mathrm{MAD}}^{-1}(x, u))}{J_{\mathrm{c}}(T_{\mathrm{MAD}}^{-1}(x, u))}$$

"continuous restriction"
of Jacobian

$$= \prod_{m=1}^{M} \frac{p_m(x_m)}{p_m(x'_m)}$$

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- is measure-preserving for augmented posterior

see paper for proof!

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conditioning on $(x_1', x_3, \cdots, x_M)$

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MAD Mix: flow family for discrete distributions

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