



MADmap for multivariate discrete distribution



**goal:** approximate *multivariate* discrete posterior  $p(x)$ ,  $x \in \mathbb{N}^M$

augmented target:  $\tilde{p}(x, u) = p(x) \cdot 1_{[0,1]} \mathcal{M}(u)$

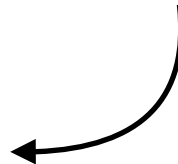
**idea: sequentially apply univariate methods**

conditional of mth entry given the rest


$$\tilde{p}_m(x_m, u_m) = \tilde{p}(x_m, u_m \mid x_{-m}, u_{-m})$$

$$= p(x_m \mid x_{-m}) \cdot 1_{[0,1]}(u_m)$$

only need full conditional of posterior!  
 (which  $\propto$  joint)

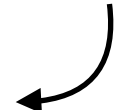




start with  $(x, u)$

update  $(x'_1, z'_1)$  with MAD map targeting  $\tilde{p}_1$

update  $(x'_2, u'_2)$  with MAD map targeting  $\tilde{p}_2$

conditioning on  $(x'_1, x_3, \dots, x_M)$  

•  
•  
•

update  $(x'_M, u'_M)$  with MAD map targeting  $\tilde{p}_M$

conditioning on  $x'_{-M}$  

end with  $(x', u') \equiv: T_{\text{MAD}}(x, u)$

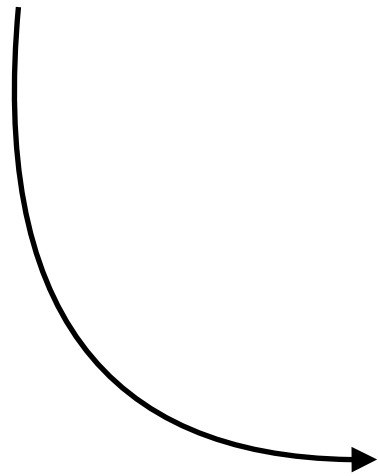
does it work?

does it work? yes!  $T_{\text{MAD}}$ :



does it work? yes!  $T_{\text{MAD}}$ :

- is invertible



each univariate pass is invertible  
inverse evaluated by evaluating  
with negative shift:  $-\xi$

$T_{\text{MAD}}$  invertible by evaluating in  
inverse order, i.e.,  $M$  to 1

does it work? yes!  $T_{\text{MAD}}$ :

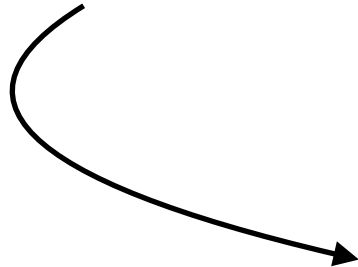
- is invertible
- has tractable density

if  $(x, u) \sim q_0$  then

$$p(T_{\text{MAD}}(x, u)) = \frac{q_0(T_{\text{MAD}}^{-1}(x, u))}{J_c(T_{\text{MAD}}^{-1}(x, u))}$$



"continuous restriction"  
of Jacobian



$$= \prod_{m=1}^M \frac{p_m(x_m)}{p_m(x'_m)}$$

does it work? yes!  $T_{\text{MAD}}$ :

- is invertible
- has tractable density
- is measure-preserving for augmented posterior



see paper for proof!

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**goal:** approximate *multivariate* discrete posterior  $p(x)$ ,  $x \in \mathbb{N}^M$


augmented target:  $\tilde{p}(x, u) = p(x) \cdot 1_{[0,1]^M}(u)$

**idea:** sequentially apply univariate map to *full conditionals*

start with  $(x, u)$

update  $(x'_1, u'_1)$  with MAD map targeting  $\tilde{p}_1$

update  $(x'_2, u'_2)$  with MAD map targeting  $\tilde{p}_2$

• conditioning on  $(x'_1, x'_3, \dots, x'_M)$    
•  
•

update  $(x'_M, u'_M)$  with MAD map targeting  $\tilde{p}_M$

conditioning on  $x'_{-M}$  

end with  $(x', u') =: T_{\text{MAD}}(x, u)$

does it work? yes!  $T_{\text{MAD}}$ :

- is invertible
- has tractable density
- is measure-preserving for augmented posterior

 see paper for proof!



# MAD Mix: flow family for discrete distributions

**goal:** approximate *multivariate* discrete posterior  $p(x)$ ,  $x \in \mathbb{N}^M$