Time Series and Financial Time Series

Project Work

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Abstract

The aim of this study is to investigate several energy commodities and evaluate their Value at Risk forecast for daily returns using a set of Generalized Autoregressive Conditional Heteroskedasticity models with different distribution specifications, in order to capture the typical features of financial data. The distributions considered for these models are: the Gaussian, the Student-t, the Generalized error distribution and their asymmetric counterparts. We also explored some mixed data sampling models, such as: GARCH MIDAS and Double Asymmetric GARCH MIDAS, which allowed us to asses the impact of COVID-19 pandemic on these commodities. For these latter models the low frequency component is the number of Covid deaths in the United States, while the distribution specification taken into consideration are the Gaussian and the Student-t.
In order to forecast the Value at Risk, a rolling window approach has been adopted. All the models have been evaluated in order to check their prediction ability, by using 2 methods: the backtest procedure and furthermore the MCS procedure by Hansen et al. (2011)[1] which provides a superior set of models.

1. Introduction and Literature review

This paper studies the effect of COVID-19 pandemic on the energy market by using Mixing Data Sampling (MIDAS) methods and comparing them with the well known Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models, bringing a new perspective in order to estimate the Value at Risk for the selected commodities.
Energy commodities have been subject of study of several research, especially in the field of risk management since energy risk has always been one of the main risk factors for many companies involved in key industrial sectors. The negative effects of the pandemic have spread worldwide, these effects include disruptions in global supply and demand chains and in the supply of goods. Commodity prices reacted strongly to the COVID-19 crisis, showing significant daily and weekly declines during the first lockdown and volatility has also increased.

In the literature, several models have been proposed to include macroeconomic variables as additional volatility determinants because of their influence, through expectations and announcements, on volatility Conrad et al. (2014)[2], Schwert (1989)[3] and Officer (1973)[4], among others. Many of these specifications extend the GARCH class of models with the idea that volatility dynamics is driven by two components, a short (high frequency) and a long (low frequency) one. In the spirit of the "mixing data sampling" (MIDAS) regression (Ghysels et al. (2007)[5], Ghysels et al. (2005)), the GARCH-MIDAS model (Engle et al. 2013)[6] addresses the issue of mixed frequency by letting the short-run component vary with the same frequency of the dependent variable while the long-run component filters the lower frequency MV observations.

In this section, we recall some of the most relevant contributions focusing on two papers: Laporta, Merlo, Petrella, 2018 "Selection of Value at Risk Models for Energy Commodities"; and Andreani, Candila, Morelli and Petrella, 2021 "Multivariate

[1] Peter R. Hansen, Asger Lunde and James M. Nason, "The Model Confidence Set"
[2] Conrad, "On the macroeconomic determinants of long-term volatilities and correlations in U.S. stock and crude oil markets"
[3] G. William Schwert, "Why Does Stock Market Volatility Change Over Time?"
[4] R. R. Officer, "The Variability of the Market Factor of the New York Stock Exchange"
[5] Eric Ghysels , Arthur Sinko Rossen Valkanov, "MIDAS Regressions: Further Results and New Directions"
[6] Robert Engle, Eric Ghysels and Bumjean Sohn, "Stock Market volatility and macroeconomic fundamentals"

Analysis of Energy Commodities during the COVID-19 Pandemic: Evidence from a Mixed-Frequency Approach".
Petrella et al. (2018)[7] investigate the performance of different GARCH specification in the univariate framework for modelling Value at Risk of seven energy commodities. The models considered are selected employing the Model Confidence Set procedure of Hansen et al. (2011) and the results show that the quantile approach outperform all the others, but in some specific case a combination of the other models (VaR aggregation) yields better results.

The second reference paper, Candila et al. (2021)[8], investigate six energy commodities in a multivariate framework by employing DCC-GARCH models, a distinctive feature of this class of models is the possibility to split the estimation phase into two steps: one for the univariate volatilities and another for the correlations (which we will not discuss). For the univariate specification, the GARCH-MIDAS and Double Asymmetric GARCH-MIDAS (DAGM) models have been employed, these models include a low-frequency variable (long run component). This allowed the authors to consider the information deriving from the long run component, "weekly deaths in the US related to COVID-19 infection", in order to estimate daily volatilities and the empirical results showed that correlation models based on the DAGM model deliver better results than those that do not account for the MIDAS component.

2. Methodology

2.1 Statistical Inference
In order to investigate the so called *stylized facts* of financial time series, it follows a brief description of the test used through the analysis, starting with the Augmented Dickey–Fuller test (ADF), which tests the null hypothesis of presence of a unit root versus the alternative hypothesis of absence of unit root (stationarity). The augmented Dickey–Fuller (ADF) statistic is a negative number, the more negative it is, the stronger the rejection of the hypothesis that there is a unit root at some level of confidence.

$$DF_\tau = \frac{\hat{\gamma}}{SE(\hat{\gamma})}$$

$H_0 : \gamma = 0$ or the series is a Random Walk
$H_1 : \gamma < 0$ or the process is stationary

In order to test normality the Jarque-Bera test has been employed, which allows to verify simultaneously if the skewness S and the kurtosis K are coherent with those under the Gaussian distribution. The null hypothesis is a joint hypothesis of the skewness and the *excess* of kurtosis being zero. If the series is normally distributed one should expect a skewness of 0 and an expected excess kurtosis of 0. As the definition of JB shows, any deviation from this increases the JB statistic.

$$JB = \frac{n}{6}\left(\hat{S}^2 + \frac{(\hat{K}-3)^2}{4}\right) \overset{H_0}{\sim} \chi^2_2$$

In order to account for autocorrelation, several tools can be employed: the autocorrelation function (ACF) and partial autocorrelation function (PACF) are useful qualitative tools to assess the presence of autocorrelation at individual lags; while the Ljung-Box Q-test assume, under the null hypothesis, that the first m autocorrelations are jointly zero.

A time series exhibiting conditional heteroscedasticity, or autocorrelation in the squared series, is said to have autoregressive conditional heteroscedastic (ARCH) effects. Engle's ARCH test is a Lagrange multiplier test to assess the significance of ARCH effects. Consider the time series y_t, the test uses the following regression:

$$y_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + ... + \alpha_p y_{t-q}^2$$

$H_0 = \alpha_1 = ... = \alpha_q = 0$
The ARCH-LM test statistic is TR^2, where R^2 comes from the regression, and under the null: $TR^2 \sim \chi^2_q$.

[7] Alessandro G. Laporta, Luca Merlo, Lea Petrella, "Selection of Value at Risk Models for Energy Commodities"
[8] Mila Andreani, Vincenzo Candila, Giacomo Morellia and Lea Petrella, "Multivariate Analysis of Energy Commodities during the COVID-19 Pandemic: Evidence from a Mixed-Frequency Approach"

2.2 Distribution specification

To model the empirical distributions of the log-returns of our commodities we considered 6 different distributions: Gaussian, Students-t, Generalized error distribution and their asymmetric counterparts.

The Gaussian distribution is a symmetric probability distribution described mainly by its first two moments: the mean and the variance. The general form of its probability density function is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{z-\mu}{\sigma})^2}$$

The Gaussian distribution has kurtosis equal to three and zero skewness, since it is symmetric in the mean.

A Student's t distribution has heavier tails than the Normal one, so it is able to capture leptokurtosis, a phenomenon often found in returns distributions. It depends on its parameter υ which indicates its degrees of freedom; the closer it gets to 0, the heavier the tails. Furthermore as $\upsilon \to \infty$ the distribution converges to the Normal one. The density function of a Student's t distribution with $\upsilon > 2$ is defined as:

$$f(x) = \frac{\Gamma(\frac{\upsilon+1}{2})}{\sqrt{\pi(\upsilon-2)}\Gamma\frac{\upsilon}{2}} \left(1 + \left(\frac{x^2}{(\upsilon-2)}\right)\right)^{-\frac{\upsilon+1}{2}}$$

The Student distribution has zero skewness and kurtosis equal to $6/(\upsilon-4)$ for $\upsilon > 4$

The Generalized Error distribution is a symmetrical unimodal member of the exponential family. The density function of a GED depends on three parameters: one of position (μ) for the mean; a scale one (σ) which defines the dispersion of the distribution; and κ which defines the form. It's density function is expressed as:

$$f(x) = \frac{\kappa \varepsilon^{-0.5|\frac{x-\mu}{\sigma}|\kappa}}{2^{1+\kappa^{-1}}\sigma\Gamma(\kappa^{-1})}$$

This family allows for tails that are either heavier than normal (when $\kappa < 2$) or lighter than normal (when $\kappa > 2$).

2.3 Models Specification

Financial data presents some features, known as stylized facts, such as: heavy tails, skewness and pronounced excess of kurtosis. Moreover, usually, there is no clear discernible pattern of behavior in the log returns, but usually there is some persistence in the squared returns which represent a proxy of the volatility of the returns. Another stylized fact is know as volatility clustering: low values of volatility followed by low values and high values of volatility followed by high values.[9] GARCH class of models aims at capturing some of these features and, in this section, we will consider a variety of models, such as: sGARCH, eGARCH, gjrGARCH, GARCH MIDAS, DAGM.

General Autoregressive Conditional Heteroskedasticity

Bollerslev proposed the General Autoregressive Conditional Heteroskedasticity model in 1986, which is an extension of Engle's (ARCH) model with a more flexible lag structure. In these models, the key concept is the conditional variance, that is, the variance conditional on the past. The GCH (general conditional heteroskedastic) model is defined as follows:

$$r_t = \mu_t + \sigma_t z_t$$
$$\mu_t = E(r_t|\mathscr{F}_{t-1})$$
$$\sigma_t^2 = Var(r_t|\mathscr{F}_{t-1})$$

[9]Rama Cont (2001), Empirical properties of asset returns: stylized facts and statistical issues

According to the empirical evidence, the mean structure of the returns is weak or equivalent to zero. So, for the sake of simplicity, we'll consider:

$$\mu_t = E(r_t|\mathscr{F}_{t-1}) = 0$$
$$r_t = \sigma_t z_t$$

where z_t is a sequence iid r.v. with zero mean and unit variance (independent of $\mathscr{F}_{t-1}$). So a GARCH(p,q) process can be expressed as following:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2$$

$$\text{with:} \quad \varepsilon_t = r_t \quad \text{where:} \quad \varepsilon_t|\mathscr{F}_{t-1} \sim N(0, \sigma_t^2)$$

the GARCH structure allows the magnitude of the noise ε_t to be a function of its past values. Thus, periods with high-volatility level (corresponding to large values of ε_{t-i}^2) will be followed by periods where the fluctuations have a smaller amplitude.[10] However, σ_t^2 does not depend only on the squared return plus an additive parameter ω, but also on σ_{t-1}^2, this formulation gives to the model a more flexible structure. To have a stable process with a positive variance, we must define some conditions. We put $\omega > 0$ and $\sum_{i=1}^{p} \alpha + \sum_{i=1}^{q} \beta < 1$ as stationarity conditions for the first purpose. We add additional constraint, $\alpha \geq 0$ and $\beta \geq 0$ in order to obtain a positive conditional variance.

The Exponential GARCH

In the basic GARCH model, above, since only squared residuals ε_{t-i}^2 enter the conditional variance equation, the signs of the residuals or shocks have no effect on conditional volatility. However, a stylized fact of financial time series is that bad news (negative shocks) tends to have a larger impact on volatility than good news (positive shocks). In order to account for this asymmetric effect (leverage effect), we introduce the E-GARCH(1,1) model:

$$log(\sigma_t^2) = \omega + \alpha g(\varepsilon_{t-1}) + \beta log(\sigma_{t-1}^2)$$

Where $g(\varepsilon_{t-1}) = \theta \varepsilon_{t-1} + \gamma(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|))$, $\theta \varepsilon_{t-1}$ determines the sign of the effect on the variance, while the second term $\gamma(|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|))$ determines the size of the effect. The process $g(\varepsilon_{t-1})$ allows the conditional variance to respond asymetrically to positive and negative news. Moreover, the conditional variance σ_t^2 is guaranteed to be positive, because the logarithm.

The Glosten-Jagannathan-Runkle GARCH

Beyond the E-GARCH model, the GJR (Glosten et al. 1993) is also able to account for the leverage effect by using an indicator function, which is equal to 1 when $r_{t-i} < 0$ and 0 otherwise. The dynamic equation for the conditional variance is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{1} \alpha_i r_{t-i}^2 + \sum_{j=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{i=1}^{q} \gamma_i r_{t-i}^2 I(r_{t-i} < 0)$$

The model is flexible, allowing the lags i of the past returns to display different asymmetries and, as special cases, when the model display no asymmetry ($\gamma_i = 0, \forall i$), we obtain the standard GARCH model. Sufficient condition for the positivity of σ^2 are $\omega > 0, \alpha \geq 0, \beta \geq 0, (\alpha + \gamma) \geq 0$. Under the assumption that z_t has asymmetric distribution the model is covariance stationary if and only if $\frac{\alpha+\beta+\gamma}{2} < 1$.

[10] C. Francq and M.J. Zakoian - GARCH models, 2^{nd} edition

GARCH MIDAS

Since macroeconomic variables (MVs) influence volatility as a result of expectations/announcements, a subset of GARCH models exists in which macroeconomic variables are directly specified in the long-term component. Because of the advantages of the MIDAS weighting system, the fact that the macroeconomic series are collected at different frequencies is not an issue. The GARCH-MIDAS (Engle et al., 2013) model is defined as:

$$r_{i,t} = \sqrt{\tau_t g_{i,t}} z_{i,t} \quad \text{with} \quad i = 1, ..., N_t$$

where:
$r_{i,t}$ log-return for day i of the period t;
N_t is the number of days for period t;
$z_{i,t}|\mathscr{F}_{t-1,t} \sim N(0,1)$ where $\mathscr{F}_{t-1,t}$ denotes the information set up to day $i-1$ of period t;
$g_{i,t}$ follows a unit-mean reverting GARCH(1,1) process (short-run component);
τ_t provides the slow moving average level of volatility (long-run component).

The short run component $g_{i,t}$ and the long-run component τ_t are defined as follows:

$$g_{i,t} = (1-\alpha-\beta) + \alpha \frac{(r_{i-1,t})^2}{\tau_t} + \beta g_{i-1,t} \quad and \quad \tau_t = exp\left(m - \theta \sum_{k=1}^{K} \delta_k(\omega) X_{t-k}\right)$$

where: m plays the role of an intercept; θ represents the coefficient of interest; $delta_k(\omega)$ is a proper function weighing the past K realizations of X_t. The model has two flaws: first, the short-run component lacks elements associated to negative returns (”bad news” increases volatility, known as the leverage effect); and second, positive and negative MV changes may have differing effects on the long-run component.

Double Asymmetric Garch MIDAS (DAGM)

The DAGM (Amendola et al., 2019) solve the first flaw of the previous model and accounts for the asymmetric effect of news. The model defines the short run component $g_{i,t}$ and the long-run component τ_t as follows:

$$g_{i,t} = (1-\alpha-\beta-\frac{\gamma}{2}) + (\alpha + \gamma \cdot I_{(r_{i-1,t}<0)}) \frac{(r_{i-1,t})^2}{\tau_t} + \beta g_{i-1,t}$$

$$\tau_t = exp\left(m - \theta^+ \sum_{k=1}^{K} \delta_k(\omega)^+ X_{t-k} \cdot I_{(X_{t-k}\geq 0)} + \theta^- \sum_{k=1}^{K} \delta_k(\omega)^- X_{t-k} \cdot I_{(X_{t-k}<0)}\right)$$

where: γ associated to negative daily returns; θ^+ and θ^- associated to positive and negative MV values, respectively; $\delta_k(\omega)^+$ and $\delta_k(\omega)^-$ represent the different system of weights for positive and negative MV realizations.
The parameter space contains four rather than two parameters related to τ_t, that is: $\Phi = \{\alpha, \beta, \gamma, m, \theta^+, \theta^-, \omega_2^+, \omega_2^-\}$ with no restrictions on θ^+ , θ^-.

2.4 Value at Risk (VaR)

Value at Risk is a widely used risk measure in finance. Let M be the set of all risks and $L \in M$ a financial loss, a risk measure is a map $\rho : M \to \mathbb{R}$ such that $\rho(L)$ identifies the amount of capital necessary to back a financial position with loss L. Hence,it summarizes the amount of capital necessary to compensate loss occurrences for as risky position, so that the risk is measured in monetary terms.

With $\alpha \in (0,1)$ being the confidence interval and $(1-\alpha)$ the significance level, the VaR of an asset, at a confidence interval α is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1-\alpha)$.

$$VaR_\alpha = inf\{l \in \mathbb{R} : P(L > l) \leq 1-\alpha\} = inf\{l \in \mathbb{R} : F_L(l) > \alpha\}$$

In probabilistic terms, VaR is thus simply a quantile of the loss distribution, so we get:

$$VaR_\alpha(L) = q_\alpha(L)$$

To overcome the problem of financial data inadequacy, the rolling window method can be employed, with the aim of constructing "new" observations using samples of consecutive existing observations.

2.5 Backtest

The backtesting technique relies on quantitative tests which scrutinize the model performances in terms of accuracy and efficiency with respect to a defined criterion.[11] The majority of backtesting techniques are based on hypothesis testing and, in the case in which the null hypothesis is rejected, the VaR forecasts do not present the characteristics required by the Backtest model, and therefore the underlying VaR model is considered to be inaccurate. If the null hypothesis is not rejected, the model is acceptably accurate. However, we should underline that the power of the test increases when the sample size gets larger. Thus, when a considerable amount of data is at our disposal, we are able to reject an inaccurate model without much difficulty.[12] Otherwise, the probability of classifying an inaccurate model as acceptably accurate could be high. The considered test are: "Unconditional Coverage Test (Kupiec test)", "Conditional Coverage Test (Christoffersen Test)" and the Dynamic Quantile test. The Engle and Manganelli's Dynamic Quantile test (2004), hereafter DQ, consists in testing some linear restrictions in a linear model that links the violations to a set of explanatory variables. The DQ test uses a regression in which the VaR violations are the dependent variables and the past violations, or any other information available, could be used as explanatory variable. The null hypothesis is:

$$H_0 : P(I_{t+1}|X_t) = p \Longleftrightarrow E[I_{t+1} - p|X_t] = 0$$

The unconditional coverage test examines if the percentage of violations is statistically similar to the VaR probability threshold p using a likelihood ratio test statistic. The Proportion Of Failure test, introduced by Kupiec, is the most used probability ratio test. The UC null hypothesis is:

$$H_0 : p = \hat{p} = \frac{x}{T}$$

The goal is to see if there is a significant difference between the observed failure rate, $\hat{p}$, and the hypothesized failure rate, p. The Likelihood Ratio is

$$LR_{UC} = -2ln\frac{(1-p)^{t-x}p^x}{[1-\frac{x}{T}]^{t-x}(\frac{x}{T})^x}$$

If the statistic is higher than the critical value, the model is rejected. The Conditional Coverage test incorporate the UC test and the independence test of the VaR violations:

$$LR_{CC} = LR_{UC} + LR_{ind}$$

$$LR_{ind} = -2ln\frac{(1-\pi)^{T_{00}+T_{10}}\pi^{T_{00}+T_{11}}}{(1-\pi_{01})^{T_{00}}\pi_{01}^{\pi01}(1-\pi_{11})^{T_{10}}\pi_{11}{}^{T_{11}}}$$

[11] Laporta, Merlo and Petrella - Selection of Value at Risk Models for Energy Commodities 2018

[12] Simona Roccioletti - Backtesting Value at Risk and Expected Shortfall, pag. 49

Where $T_{i,j}; i,j=0$, is the number of observation with a j is the number of observation with a j succeeding an i and $\pi = T_{01}+T_{11}+T_{00}+T_{10}+T_{11}$. The independence's null hypothesis is: $H_0 : \pi_{01} = \pi_{11}$, with π_{01} representing the probability of having a hit if in the last observation there was not a hit, and π_{11} is the probability of having a hit if in the last observation there was a hit.
The difference between the Unconditional Coverage test and the Conditional Coverage test is that the UC indicates that the violations are not significantly different from the significance level across days. The CC that the model gives VaR's violations with the right probability every day. It is a more complete analysis because it considers the problem of independence between the exceptions must be serial independent of each other. The numerator will be the maximum probability of hypothesis test under the assumption of being independent, the denominator indicates the maximum likelihood for the observed data.

2.6 MCS Procedure

The Model Confidence Set (MCS) procedure introduced by Hansen et al. has the objective of selecting the set of models, $M\star$, that consists of the best model(s) from a collection of models, M_0, where best is defined in terms of a criterion that is user-specified"[13]. The procedure is based on a collection of models M_0 and a criterion to evaluate these models empirically. The main features of the MCS procedure are an equivalence test, denoted with δM, and an elimination rule, denoted with eM. The first is applied to the set $M = M_0$ and if the test is rejected it means that the elements in M are not "good" in the same way. For the test is important to use the same significance level $1-\alpha$, while the latter it's used to eliminate an element which perform a poor sample performance from M. We start from the consideration of M_0, which contains a finite number of elements indexed by $i = 1,\dots,m_0$ that represent the loss function. A general notation for the loss function of the object i is $L_{i,t}$ with $t = 1,\dots,n$. The following steps is to write down the relative performance variable

$$d_{i,j} \equiv L_{i,t} - L_{j,t} \forall i,j \in M_0$$

Following the paper by Hansen we will consider the expected value of $d_{i,j,t} =_{i,j}$; which is finite and doesn't depend on the time t. Whenever $\mu_{i,j} < 0$ it means that the alternative i is preferred to the alternative j. We are now able to define $M\star$, the set of superior objectives as

$$\mathscr{M}\star \equiv i \in M_0 : \mu_{i,j} < 0 \; for \; all \; j \in M_0$$

Through the equivalence test we are able, as we already pointed out, to eliminate the objects which are significantly inferior to other elements of M_0. The hypothesis of the test are:

$$H_0, M : \mu_{i,j} = 0$$

$$H_1, M : \mu_{i,j} \neq 0$$

2.7 VaR Aggregation

The MCS procedure may deliver a set of several models with the same VaR predictive ability instead of a single superior one or, in the worst scenario, $M*_\alpha = M^0$ i.e. when no model is eliminated. In these circumstance it is not possible to be sure about the best model selection and the VaR aggregation addresses this question creating a single optimal model adopting the $weighted\ average\ framework$. This procedure consists of combining VaR forecasts which belongs to $\hat{M}*_\alpha$. By doing so it's possible to compare the performances of combined VaR with those of single models. Given the collection of superior set of models $\hat{M}*_\alpha$. For a set of weights ω_t such that $\sum_{t=1}^{m} *\omega_t = 1$, the weighted VaR is obtained as follows:

$$V\hat{a}R^{\alpha}_{AVE,t} = \sum_{t=1}^{m} *\omega_t V\hat{a}R^{\alpha}_{i,t}, *$$

The most used technique assigns equal weights for ω_t, that is $\omega_t = \frac{1}{m*}$ such that the resulting $V\hat{a}R^{\alpha}_{AVE,t}$ is simply the average of each $V\hat{a}R^{\alpha}_{i,t}, *$.

[13]Hansen et. al. 2011, "Model Confidence Set" p.2

3. Empirical Application

The analysis has been carried out on six energy commodities traded in the markets: West Texas Intermediate (WTI) crude oil, Europe Brent, Heating oil, Propane, Gasoline and Kerosene-Type Jet Fuel. The observation period starts the 28^{th} of february 2020 and ends the 04^{th} of June 2021. Missing data have been removed, thus the sample size consists of 317 daily spot prices. For the low frequency variable the number of observation is 68 weekly Covid-19 deaths.

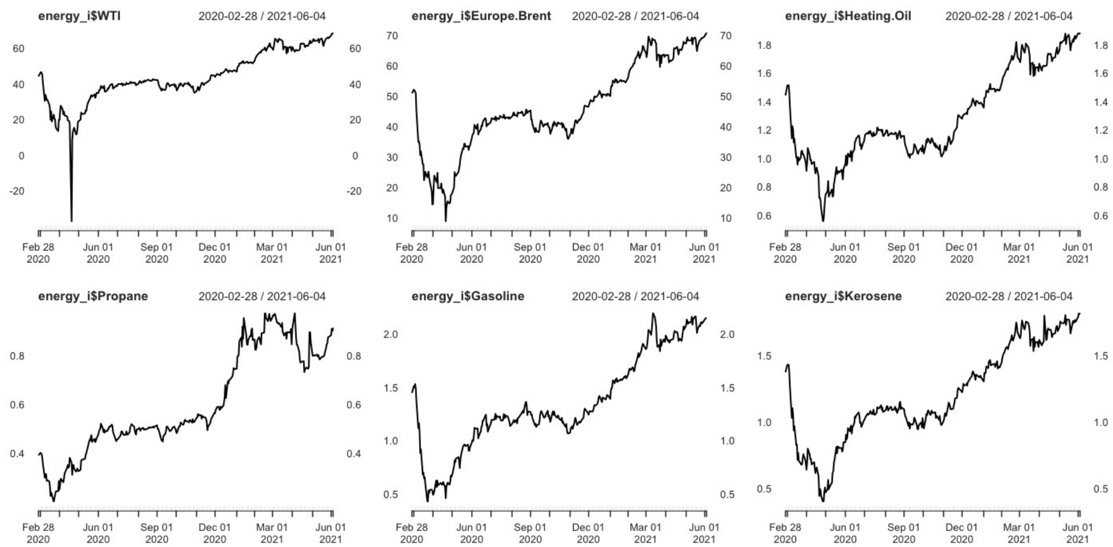


Figure 1. Daily prices for energy commodities: 1) WTI, 2) Brent 3) Heating Oil 4) Propane 5) Gasoline 6) Kerosene

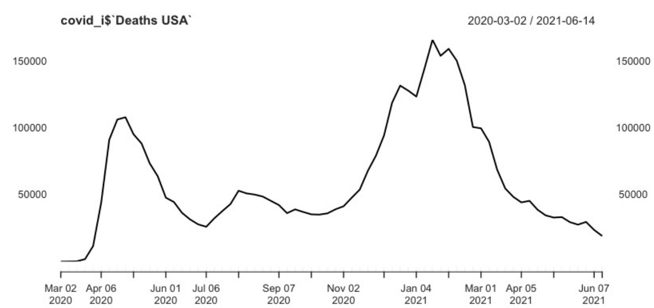


Figure 2. Weekly Covid-19 deaths U.S.

From a graphical analysis of figure 1 and 2, the series seem to follow a random walk (not stationarity). However, empirical evidence of non-stationarity is verified by means of the Augmented Dickey-Fuller test.

Table 1. Augmented Dickey Fuller test on commodities prices and Covid deaths

	WTI	Brent	Heating Oil	Propane	Gasoline	Kerosene	Covid Deaths
statistic	-3.698	-2.980	-2.995	-2.051	-3.637	-3.702	-2.709
p.value	0.024	0.163	0.157	0.555	0.030	0.024	0.287

Results presented in Table 1 are in contrast with the graphical analysis. The null hypothesis of non-stationarity is rejected in three series: WTI, Gasoline and Kerosene. However, these results could be due a structural break in these series, led by the massive decrease of commodity prices in the first period of the pandemic crisis. It's called a "structural break" when a time series abruptly changes at a point in time, and by applying conventional unit root tests, the result might be biased. In the paper of Candila et. al. (2018)[14] evidence of structural breaks have been found.

In financial applications, the focus rather than on prices is on returns, which give information on profits and losses. Log-returns are the first-order differences of the logarithm of the prices, where the logarithm are used in order to smooth out the volatility.

$$r_t = log\left(\frac{P_t}{P_{t-1}}\right)$$

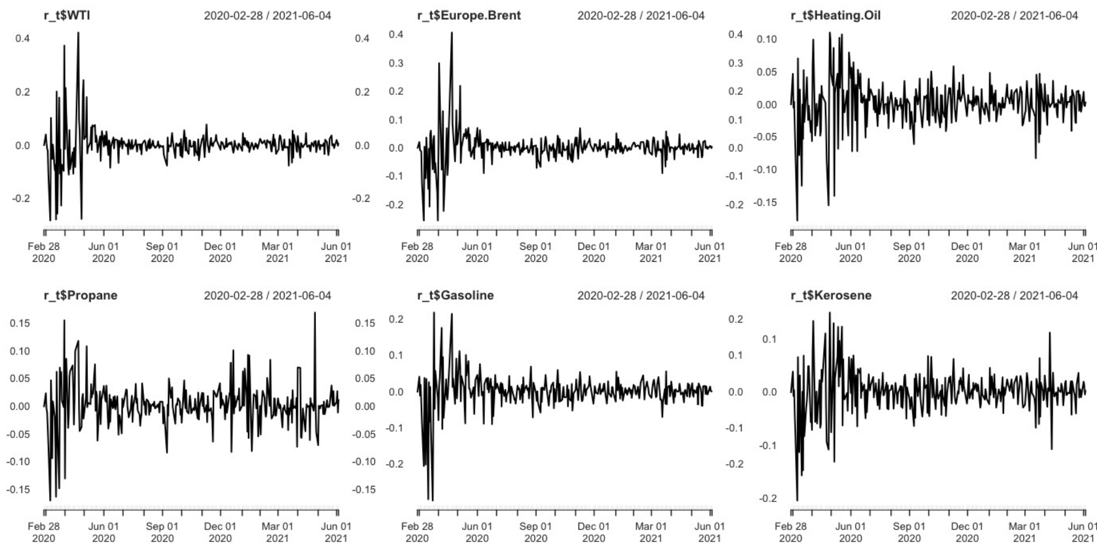


Figure 3. plot of returns

Table 2. Summary of log-returns and log differences of Covid deaths

Series	N	Mean	St. Dev.	Min	Max	Skewness	Kurtosis
WTI	315	0.004	0.063	−0.281	0.426	0.973	15.448
Europe.Brent	315	0.003	0.054	−0.256	0.412	0.992	18.058
Heating.Oil	315	0.002	0.034	−0.177	0.112	-0.800	4.959
Propane	315	0.003	0.039	−0.170	0.172	-0.131	4.759
Gasoline	315	0.002	0.047	−0.300	0.222	-1.399	14.420
Kerosene	315	0.002	0.041	−0.205	0.153	-0.406	4.235
Deaths USA	68	0.135	0.613	−0.289	3.761	4.071	18.491

All returns' series have zero mean and their distribution displays fat tails, high kurtosis and serial correlation (table 2). As already mentioned above, it is crucial to analyze graphically the distributions, figure 4 reports the QQplot, which is a graphical method for comparing two probability distributions by plotting their quantiles against each other. Also, a statistical test such as the Jarque Bera test has been employed, and, in this case, it strongly rejects the normal behavior of the data.
These empirical evidences has led us to consider other distributions to better fit the data other than the normal one, and expectations are that other errors distributions, which account for fat tails, might actually outperform the simpler Gaussian one.

[14]Mila Andreani, Vincenzo Candila, Giacomo Morellia and Lea Petrella, "Multivariate Analysis of Energy Commodities during the COVID-19 Pandemic: Evidence from a Mixed-Frequency Approach, pag. 9

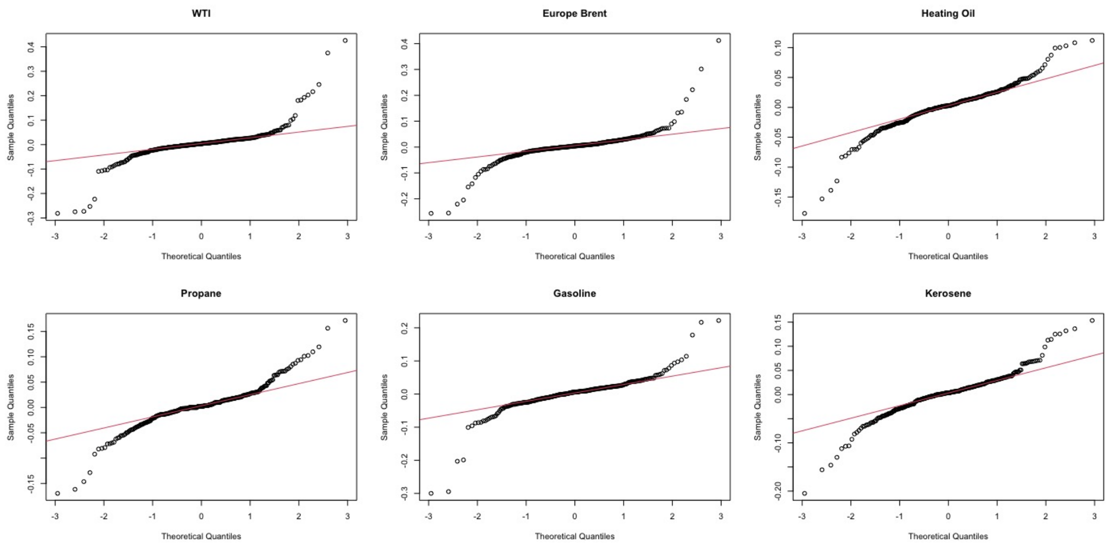


Figure 4. QQplot returns

As above mentioned, non-normality of the distribution can be seen also graphically, by the Q-Q plots. These plots show the leptokurtosis of all 6 commodities, since the left end pattern is below the line corresponding to normal quantiles and the right end is above it, showing heavier tails.

Table 3. Augmented Dickey Fuller, Jarque Bera, Ljung-Box, Box-Pierce and LM test on log differences

Series	ADF		JB		Ljung–Box		Box-Pierce		ARCH LM-test	
	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value	statistic	p-value
WTI	-6.264	0.010	3182.113	0.000	64.019	0.000	61.786	0.000	88.907	0.000
Brent	-5.768	0.010	4332.079	0.000	72.642	0.000	69.832	0.000	70.309	0.000
Heating Oil	-7.619	0.010	356.503	0.000	28.832	0.091	27.719	0.116	76.809	0.000
Propane	-6.229	0.010	298.176	0.000	33.514	0.029	32.099	0.042	65.425	0.000
Gasoline	-5.044	0.010	2832.344	0.000	40.773	0.003	39.305	0.006	128.499	0.000
Kerosene	-5.507	0.010	244.093	0.000	27.422	0.123	26.370	0.153	103.474	0.000
Covid Deaths	-2.709	0.010	9.163	0.010	57.299	0.000	53.556	0.000	52.980	0.000

Unit-root test for empirical evidence of stationarity have been performed, results in Table 3, and as expected while prices are usually non-stationarity, their first difference is always stationary, displaying the same features of a White Noise, as shown also in the ACF and PACF plots of the log-returns below (Figure 5-6).

Other tests performed are the Box–Pierce and the Ljung–Box, which test the null hypothesis of independence in a given time series, so whether the autocorrelations in the data are different from zero. The Box–Pierce test statistic is a simplified version of the Ljung–Box test statistic, which has shown poor performance in subsequent simulation studies. Both the tests were applied on the first 20 lags and the output is the following: two series, Heating Oil and Kerosene, do not reject H_0, so the returns are not correlated; while in the other series some correlation is present. This contradicts the stylized fact that returns are not correlated. However, the ACF plot (figure 5) show that in the first 20 lags, there are some lags exceeding the confidence intervals (blue bands).

Finally, the ARCH-LM test: an uncorrelated time series can still be serially dependent due to a dynamic conditional variance process. A time series exhibiting conditional heteroscedasticity or autocorrelation in the squared series is said to have autoregressive conditional heteroscedastic (ARCH) effects. ARCH-LM test check for the presence of Arch effect, means the presence of Heteroskedasticity. Under the null hypothesis the process have no arch effect and the results lead to rejection of

the null. It is a Lagrange multiplier test to assess the significance of ARCH effects. The test is set with 15 lags, considered necessary after a look to the ACF which is slowly decaying. Otherwise 10 lags should be enough.
This finding is important for our goals, because the null hypothesis of no ARCH effect is rejected. It is now possible to apply the Conditional Heteroskedasticity model family. Specifically GARCH(1,1)[15].

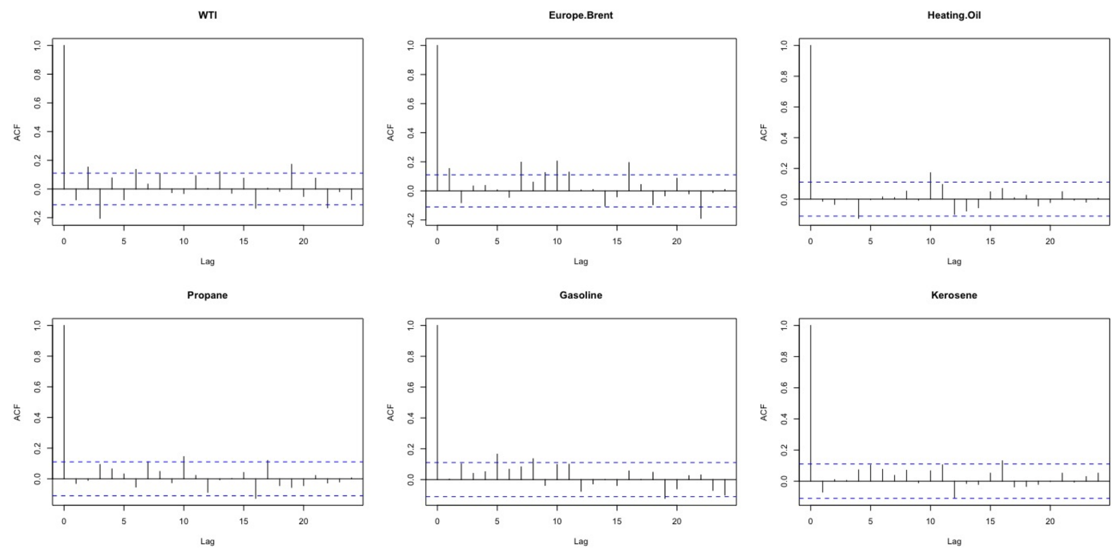


Figure 5. ACF of returns

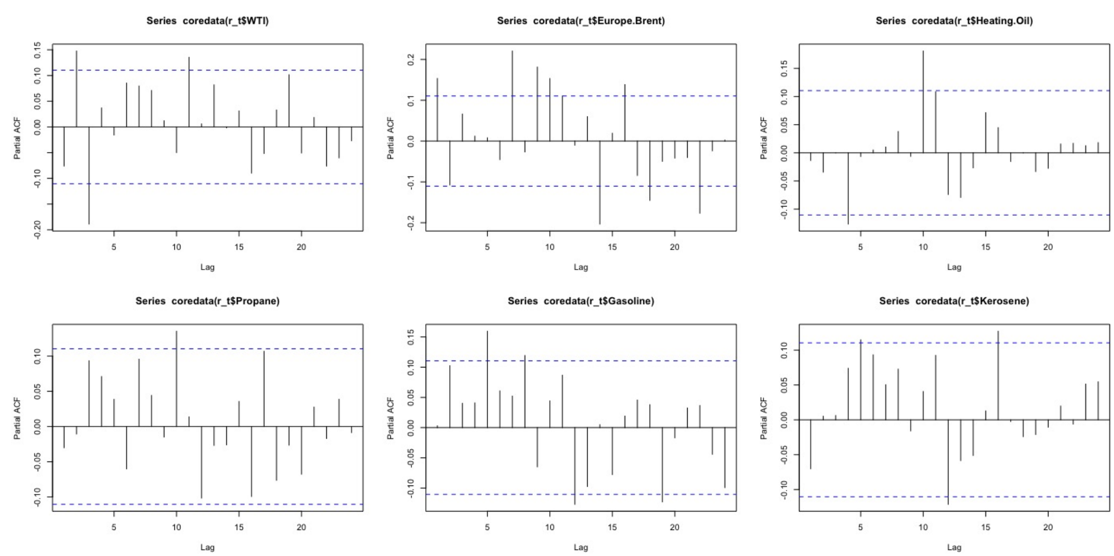


Figure 6. PACF of returns

[15]A forecast comparison of volatility models: does anything beat GARCH(1,1)?, Peter R.Hansen and Asger Lunde

One of the stylized fact of financial time series is that returns are uncorrelated and follow a White Noise process, so the ACF plot lies within the confidence bands (blue lines). This mean that it's not possible to use ARMA models and analyse the residuals. At this point, we could check other type of dependencies (e.g. quadratic), so we proceeded analyzing squared returns as they are a proxy for the volatility.

Table 4. Box-Pierce and Ljung-Box tests on squared returns

Series	Ljung–Box		Box-Pierce	
	statistic	p-value	statistic	p-value
WTI	289.39	0.000	278.41	0.000
Brent	177.62	0.000	170.73	0.000
Heating Oil	175.20	0.000	179.29	0.000
Propane	157.96	0.000	152.38	0.000
Gasoline	228.87	0.000	221.98	0.000
Kerosene	270.83	0.000	262.14	0.000

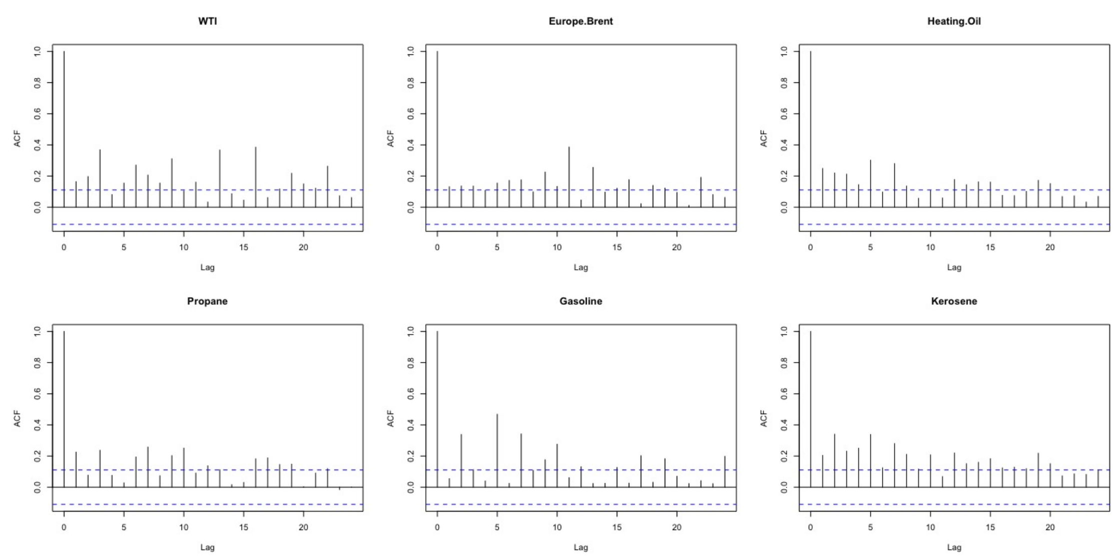
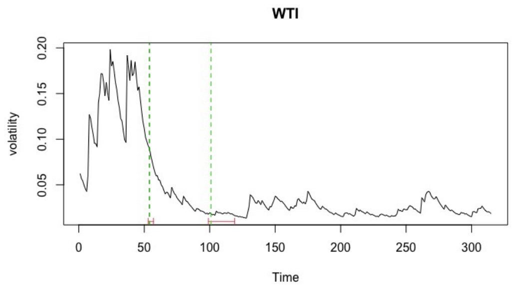
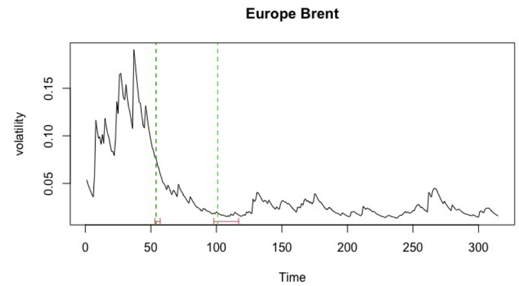


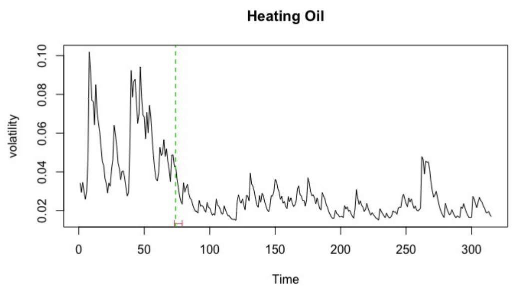
Figure 7. ACF of squared returns

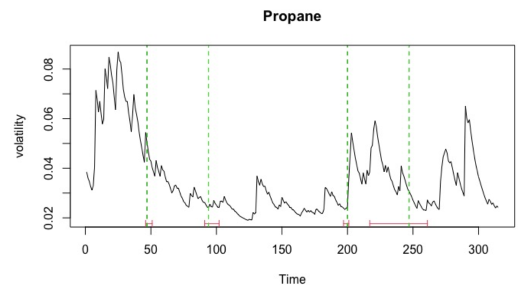
In figure 7, as expected, squared returns display some correlation between the lags.

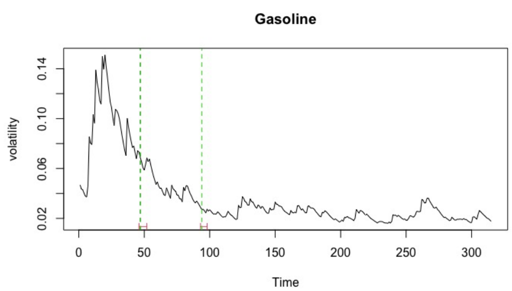
Finally, we can observe also another stylized fact: volality clustering, since in the first few months, until June 2020, there have been some huge spikes in the returns (figure 3). So, we can conclude that the evidences are in line with the expectations on the returns given by the stylized facts: fat tails, skewness, volatility clustering, abesence of autocorrelation in the returns, correlation in the squared returns. At this point, after performing all the tests, we can use the class of GARCH models including some different assumptions for the errors' distributions (Normal, Student-t, GED, Skew-Normal, Skew-Student-t, Skew-GED), in order to model the volatility.

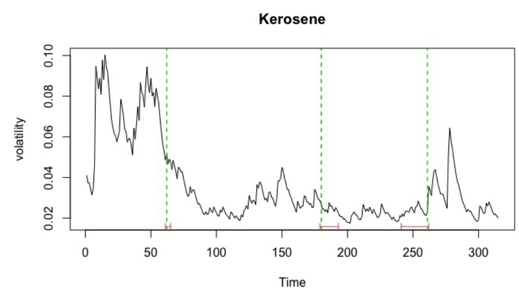












Following Candila et. al. (2020) a structural break research has been performed and reported in this section. We used a Chow test for parameter stability. There's evidence of structural break presence in each series and most of them are observed in similar time periods. WTI shows two structural break at $t = 54$ and $t = 101$, where $t = 0$ is 28^{th} of February and $t = 315$ is 4^{th} of June. The first structural break correspond to 19^{th} of May 2020 period of maximum alert of world governments due to the fast spread of covid-19 and also the maximum level of restrictions, imposing lockdown and stop to the supply chain. The second structural break correspond to uncertainty about the ongoing health situation during the summertime and the alert on the possibility of a spread of new variants of the virus. The same is replicated in Brent; while Gasoline $t = 47$ and $t = 94$; in Heating Oil is located in $t = 74$ and $t = 100$; Propane presents more breaks than any other series in $t = 47, t = 94, t = 200, t = 247$; these last two dates coincide with the new lockdown. Lastly Kerosene in $t = 62, t = 180, t = 261$. This results justifies the inclusion of a macroeconomic variable such as Covid-19 deaths and the further development of GARCH-MIDAS models according to Candila (2020).

The estimated coefficients are reported in the **Appendix** (Tables 14-22)

By looking at estimates of the models, we can have an early vision of their goodness. For each sGARCH and gjrGARCH model the ω parameter (intercept of the variance) is always 0, while eGARCH has only negative estimates for ω.
For α the situation is slighty different, as we can observe a similar behavior in all the series: sGARCH has always $\alpha > 0$ statistically significant, while eGARCH $\alpha < 0$ and gjrGARCH very close to zero. The only one which is different is gjrGARCH Skew Student t for Propane, where $\alpha = 0.165$. Regarding β, it's very high in every series and model, and it is even higher for those models with a lower associated α, this is coherent with the assumption of stationarity presented in the previous pages where $\alpha + \beta < 1$ since it is never violated.
Lastly, γ, the term that estimates the size of the effect of conditional variance and which is not estimated for sGARCH, generally results in higher eGARCH with respect to gjrGARCH. It is possible to notice such a scheme especially in Brent, Propane and Gasoline.

The focus can be shifted to the MIDAS models at this point. We set $K = 24$ in the fitting phase following the paper of Candila (2020). Our interest goes mainly on θ parameter which enters the τ_t long run component equation, recall section 2.3 "models specification". The results show that it is always statistically different from 0 and quite always has a large negative value. The only exception is Gasoline "GM-Skew-Normal" (0.19) and negative θ in the case of Heating Oil DAGM-Skew-Normal with an estimate of "0.02", which is actually the closer to 0 obtained so far. This means that, at least theoretically, the information brought by the macro-variable should increase the estimation of commodities' volatility.
Regarding the parameters ω_1 and ω_2 related to the weighting function, remind that the rumidas package in R sets $\omega_1 = 1$ for every model they are strongly significant.

In the following section, we will test these early conclusions with a series of procedures involved in verifying the accuracy of the forecasts generated by these models (Backtesting procedure and MCS).

First of all a brief description of the procedure performed for Value at Risk estimation is presented. In order to calculate VaR we divided returns in two sample: the first 235 observations will be the "in-sample" which is used to create our forecast, by using a rolling window approach, of the last 80 which will be the "out-sample", used to test the quality of the forecast. Note, that the forecast length is not randomly chosen, as a forecast length greater than 80 observations would return problems of convergence. Unfortunately, setting a different solver in R, did not provide better results.
The "ugarchroll" method allows us to perform a rolling estimation of our model. It returns the distributional forecast parameters necessary to calculate any required measure of the forecast density. We set the "out-sample" of observation as test and perform a rolling moving of one step ahead forecast of the conditional standard deviation; and re-estimate parameters every 1 observations. From there, we obtained the VaR, at 95% and 99% confidence level, using the roll's forecast of our returns and assigning the properties (shape, skew) of each distribution (norm, snorm, std, sstd, ged).

3.1 Backtesting results

Table 5. WTI crude oil Backtesting results at 99% and 95% confidence level

confidence level	99%			95%		
	LR.uc *p*	LR.cc *p*	DQ *p*	LR.uc *p*	LR.cc *p*	DQ *p*
sGARCH-norm	0.829	0.964	0.991	0.338	0.386	0.266
sGARCH-std	0.829	0.964	0.996	0.621	0.631	0.217
sGARCH-snorm	0.829	0.964	0.980	0.621	0.631	0.238
sGARCH-sstd	0.829	0.964	0.994	1	0.808	0.096
sGARCH-ged	0.829	0.964	0.994	0.338	0.386	0.248
sGARCH-sged	0.829	0.964	0.993	1	0.808	0.097
eGARCH-norm	0.010*	0.014*	0.000*	0.162	0.333	0.004*
eGARCH-std	0.257	0.500	0.397	0.069	0.187	0.008*
eGARCH-snorm	0.058	0.147	0.006*	0.338	0.468	0.073
eGARCH-sstd	0.829	0.964	0.997	0.162	0.333	0.099
eGARCH-ged	0.058	0.147	0.005*	0.069	0.187	0.011*
eGARCH-sged	0.829	0.964	0.997	0.621	0.498	0.129
gjrGARCH-norm	0.829	0.964	0.994	0.621	0.631	0.314
gjrGARCH-std	0.829	0.964	0.995	0.338	0.386	0.282
gjrGARCH-snorm	0.829	0.964	0.992	1	0.808	0.744
gjrGARCH-sstd	0.829	0.964	0.995	0.592	0.770	0.962
gjrGARCH-ged	0.829	0.964	0.995	0.338	0.386	0.304
gjrGARCH-sged	0.829	0.964	0.995	0.592	0.770	0.968
GM-Skew-norm	0.257	0.500	0.000*	0.592	0.770	0.239
GM-No Skew-norm	0.257	0.500	0.000*	0.592	0.770	0.241
GM-Skew-std	0.257	0.500	0.991	0.621	0.631	0.805
GM-No Skew-std	0.829	0.964	0.000*	0.258	0.501	0.230
DAGM-Skew-norm	0.829	0.964	0.000*	0.621	0.631	0.031*
DAGM-No Skew-norm	0.257	0.500	0.954	0.592	0.770	0.087
DAGM-Skew-std	0.829	0.964	0.984	1	0.808	0.601
DAGM-No Skew-std	0.257	0.500	0.000*	0.258	0.501	0.084

* Denotes that the null hypothesis is rejected at the 5% significance level.
Note: p = p-value

The initial backtest results on WTI VaR at 99% and 95% confidence levels are shown in table 5. The table shows the p-value of the test statistic associated to each test: Unconditional Coverage, Conditional Coverage and Dynamic Quantile test.
Regarding the performance of WTI VaR forecasts, there is evidence of good results in with standard GARCH considering all the distribution, all the models passed the test at both confidence levels. The highest values are obtained assuming Skew-Student t and Skew-GED errors' distributions at 99% but marginally significant at 10% for the 95%. These results concurs with the preliminary analysis performed on this series. In contrast, eGARCH obtained the worst results. Four models did not passed the Dynamic Quantile test at different confidence interval: Normal, Skew-Normal and GED did not passed the test at 99% confidence level; while Normal, Student t and GED failed in the 95%. Even in this case the best distributions are Skew-Student t and Skew-GED. Lastly, the gjrGARCH performs as good as standard one; the results are pretty similar: each model passed all the tests. The difference is the best model, which in this case is the Skew-Normal.

Passing through the MIDAS model an unexpected behavior appears. Three models did not passed the Dynamic Quantile test at 99% confidence level: Skew-Normal, Normal and Student t; but a very good performance is can be observed for the Skew-Student t considering both intervals. This again is a confirm of the previous tests, but also indicates that the inclusion of the macroeconomic variable computing the VaR forecast increase the performance of the standard GARCH models for $\alpha = 5\%$, but this performance decreases considering $\alpha = 1\%$; and the same happens when considering the Double asymmetric models. This fact can evidence a problem of the MIDAS models, that is: the increase of second type errors due to the reduction of the sample necessary to obtain its estimations, since the returns series should start $K+1$ periods after the macro-variable. We should evidence that the power of the test increases when the sample size gets larger. Thus it should be kept in mind the risk of uncorreclty do no rejet a null hypothesis.

Table 6. Brent Backtesting results at 99% and 95% confidence level

confidence level	99%			95%		
	LR.uc p	LR.cc p	DQ p	LR.uc p	LR.cc p	DQ p
sGARCH-norm	0.828	0.964	0.740	1	0.807	0.200
sGARCH-std	0.828	0.964	0.998	0.620	0.631	0.631
sGARCH-snorm	0.828	0.964	0.995	0.592	0.769	0.476
sGARCH-sstd	0.828	0.964	0.997	1	0.807	0.177
sGARCH-ged	0.828	0.964	0.998	1	0.807	0.194
sGARCH-sged	0.828	0.964	0.997	0.592	0.769	0.467
eGARCH-norm	0.058	0.033*	0.000*	0.337	0.468	0.187
eGARCH-std	0.828	0.964	0.903	0.162	0.333	0.155
eGARCH-snorm	0.058	0.033*	0.000*	0.620	0.497	0.255
eGARCH-sstd	0.828	0.964	0.997	0.337	0.468	0.141
eGARCH-ged	0.828	0.964	0.996	0.162	0.333	0.183
eGARCH-sged	0.828	0.964	0.997	0.337	0.468	0.170
gjrGARCH-norm	0.829	0.964	0.998	1	0.807	0.518
gjrGARCH-std	0.829	0.964	0.998	0.337	0.468	0.179
gjrGARCH-snorm	0.828	0.964	0.997	0.257	0.500	0.987
gjrGARCH-sstd	0.828	0.964	0.998	0.592	0.769	0.481
gjrGARCH-ged	0.828	0.964	0.998	0.337	0.468	0.197
gjrGARCH-sged	0.829	0.964	0.998	0.257	0.500	0.987
GM-Skew-norm	0.828	0.964	0.972	0.257	0.500	0.094
GM-No Skew-norm	0.828	0.964	0.980	0.257	0.500	0.105
GM-Skew-std	0.828	0.964	0.966	0.257	0.500	0.094
GM-No Skew-std	0.829	0.964	0.963	0.258	0.501	0.101
DAGM-Skew-norm	0.829	0.964	0.996	0.257	0.500	0.981
DAGM-No Skew-norm	0.828	0.964	0.996	0.257	0.500	0.106
DAGM-Skew-std	0.829	0.964	0.991	0.592	0.769	0.480
DAGM-No Skew-std	0.828	0.964	0.997	0.592	0.769	0.442

* Denotes that the null hypothesis is rejected at the 5% significance level.

Regarding Brent's VaR performances, it's possible to see a different behavior with respect to WTI. The sGARCH model results seem to be better for both confidence level, even though at 95% confidence level these results are even better, with none of the statistics being rejected. The similarity with the previous series is present only in terms of bad results of eGARCH models, but in this case, only if we consider an $\alpha = 1\%$. Normal and Skew-Normal did not pass Conditional Coverage and Dynamic Quantile tests.Again gjrGARCH obtain consistent performance just like the standard.

The GM and DAGM models have excellent result on this series: all the models pass the backtests and with very good results.

Table 7. Heating oil Backtesting results at 99% and 95% confidence level

confidence level	99%			95%		
	LR.uc *p*	LR.cc *p*	DQ *p*	LR.uc *p*	LR.cc *p*	DQ *p*
sGARCH-norm	0.257	0.499	0.735	0.620	0.631	0.805
sGARCH-std	0.257	0.499	0.723	0.620	0.631	0.798
sGARCH-snorm	0.257	0.499	0.700	1	0.807	0.856
sGARCH-sstd	0.828	0.964	0.999	1	0.807	0.850
sGARCH-ged	0.257	0.499	0.716	0.620	0.631	0.804
sGARCH-sged	0.828	0.964	0.999	1	0.807	0.865
eGARCH-norm	0.058	0.147	0.056	1	0.807	0.851
eGARCH-std	0.058	0.147	0.057	0.337	0.468	0.283
eGARCH-snorm	0.058	0.147	0.057	1	0.807	0.857
eGARCH-sstd	0.257	0.499	0.867	1	0.807	0.857
eGARCH-ged	0.058	0.147	0.057	1	0.807	0.849
eGARCH-sged	0.257	0.499	0.866	1	0.807	0.863
gjrGARCH-norm	0.058	0.147	0.041*	1	0.807	0.851
gjrGARCH-std	0.058	0.147	0.031*	1	0.807	0.827
gjrGARCH-snorm	0.257	0.499	0.821	1	0.807	0.868
gjrGARCH-sstd	0.828	0.964	0.999	1	0.807	0.856
gjrGARCH-ged	0.058	0.147	0.035*	1	0.807	0.840
gjrGARCH-sged	0.829	0.964	0.999	1	0.807	0.869
GM-Skew-norm	0.828	0.964	0.981	0.592	0.769	0.428
GM-No Skew-norm	0.828	0.964	0.999	0.257	0.500	0.098
GM-Skew-std	0.828	0.964	0.978	0.257	0.500	0.085
GM-No Skew-std	0.828	0.964	0.995	0.257	0.501	0.094
DAGM-Skew-norm	0.828	0.964	0.986	0.592	0.769	0.439
DAGM-No Skew-norm	0.828	0.964	0.996	0.592	0.769	0.443
DAGM-Skew-std	0.828	0.964	0.959	0.257	0.500	0.080
DAGM-No Skew-std	0.828	0.964	0.996	0.592	0.769	0.038*

* Denotes that the null hypothesis is rejected at the 5% significance level.

Heating Oil results are very different with the previous series. Standard GARCH still obtain good results, but in this specific case eGARCH for the first time follows sGARCH in goodness of forecasts: at 95% all the test are passed, while at 99% all test are passed at 5% significance level but all distributions except Skew-Student t and Skew-GED, are marginally rejected. This is the first series found that obtain acceptable results with this kind of model. In contrast with previous results, gjrGARCH did not pass Dynamic Quantile test for Normal, Skew-Student t and GED, but it is still very good for 95%. Meanwhile, the last two models display bad results for DAGM Skew-Student t (marginally rejected with 0.08) and DAGM-Student t (rejected).

Table 8. Propane Backtesting results at 99% and 95% confidence level

confidence level	99%			95%		
	LR.uc p	LR.cc p	DQ p	LR.uc p	LR.cc p	DQ p
sGARCH-norm	0.257	0.500	0.342	1	0.808	0.374
sGARCH-std	0.829	0.964	0.990	1	0.808	0.618
sGARCH-snorm	0.257	0.500	0.398	1	0.808	0.418
sGARCH-sstd	0.829	0.964	0.988	1	0.808	0.604
sGARCH-ged	0.257	0.500	0.446	1	0.808	0.524
sGARCH-sged	0.257	0.500	0.485	1	0.808	0.583
eGARCH-norm	0.257	0.500	0.269	1	0.808	0.351
eGARCH-std	0.257	0.500	0.546	1	0.808	0.656
eGARCH-snorm	0.058	0.147	0.000*	0.621	0.631	0.731
eGARCH-sstd	0.257	0.500	0.537	1	0.808	0.618
eGARCH-ged	0.257	0.500	0.393	1	0.808	0.499
eGARCH-sged	0.257	0.500	0.371	1	0.808	0.440
gjrGARCH-norm	0.257	0.500	0.448	1	0.808	0.539
gjrGARCH-std	0.257	0.500	0.568	1	0.808	0.699
gjrGARCH-snorm	0.257	0.500	0.469	1	0.808	0.541
gjrGARCH-sstd	0.257	0.500	0.574	1	0.808	0.694
gjrGARCH-ged	0.257	0.500	0.471	1	0.808	0.579
gjrGARCH-sged	0.257	0.500	0.461	1	0.808	0.567
GM-Skew-norm	0.010*	0.030*	0.000*	0.069	0.078	0.070
GM-No Skew-norm	0.058	0.147	0.011*	1	0.808	0.677
GM-Skew-std	0.257	0.500	0.994	0.592	0.770	0.752
GM-No Skew-std	0.001*	0.005*	0.996	0.621	0.676	0.762
DAGM-Skew-norm	0.829	0.964	0.545	1	0.808	0.974
DAGM-No Skew-norm	0.829	0.964	0.469	1	0.808	0.960
DAGM-Skew-std	0.257	0.499	0.489	0.592	0.770	0.976
DAGM-No Skew-std	0.257	0.499	0.529	0.258	0.501	0.972

* Denotes that the null hypothesis is rejected at the 5% significance level.

In all the tests, considering all the models, only four were rejected at 99% (eGARCH Skew-Normal, GM-Skew-Normal, GM-Normal and GM t), which clearly indicates that the normal distribution is incapable of capturing the features of this series. This results is confirmed looking at 95% confidence level, in fact the only test marginally rejected is GM-Skew-Normal only for the LR unconditional coverage. The best model looking at $\alpha = 0.05$ are gjrGARCH with very good results of Uncoditional and Conditional Coverage tests, but also the GM family have good performance, their Dynamic Quantile p-value are better, especially the DAGM one. In this case the GM and DAGM models prefer the Normal distribution. This is also confirmed at 99% confidence level for DAGM.

Table 9. Gasoline Backtesting results at 99% and 95% confidence level

confidence level	99%			95%		
	LR.uc *p*	LR.cc *p*	DQ *p*	LR.uc *p*	LR.cc *p*	DQ *p*
sGARCH-norm	0.829	0.964	0.999	1	0.374	0.209
sGARCH-std	0.829	0.964	0.998	1	0.374	0.217
sGARCH-snorm	0.829	0.964	0.999	1	0.374	0.158
sGARCH-sstd	0.829	0.964	0.999	1	0.374	0.189
sGARCH-ged	0.829	0.964	0.998	1	0.374	0.210
sGARCH-sged	0.829	0.964	0.999	1	0.374	0.169
eGARCH-norm	0.829	0.964	0.956	0.162	0.307	0.556
eGARCH-std	0.829	0.964	0.962	0.162	0.307	0.529
eGARCH-snorm	0.829	0.964	0.996	1	0.808	0.703
eGARCH-sstd	0.829	0.964	0.987	0.592	0.770	0.665
eGARCH-ged	0.829	0.964	0.937	0.162	0.307	0.480
eGARCH-sged	0.829	0.964	0.989	0.592	0.770	0.683
gjrGARCH-norm	0.829	0.964	0.993	0.621	0.498	0.232
gjrGARCH-std	0.829	0.964	0.985	1	0.374	0.224
gjrGARCH-snorm	0.829	0.964	0.999	0.621	0.498	0.188
gjrGARCH-sstd	0.829	0.964	0.993	1	0.374	0.222
gjrGARCH-ged	0.829	0.964	0.985	1	0.374	0.227
gjrGARCH-sged	0.829	0.964	0.993	1	0.374	0.222
GM-Skew-norm	0.829	0.964	0.999	1	0.374	0.071
GM-No Skew-norm	0.204	0.447	0.997	0.067	0.185	0.943
GM-Skew-std	0.828	0.964	0.998	0.592	0.769	0.218
GM-No Skew-std	0.204	0.447	0.997	0.067	0.185	0.942
DAGM-Skew-norm	0.828	0.964	0.999	0.067	0.185	0.946
DAGM-No Skew-norm	0.828	0.964	0.997	0.067	0.185	0.939
DAGM-Skew-std	0.828	0.964	0.999	1	0.373	0.067
DAGM-No Skew-std	0.204	0.447	0.997	0.067	0.185	0.939

* Denotes that the null hypothesis is rejected at the 5% significance level.

For gasoline, there is no rejection at both 99% and 95% confidence levels. Gasoline is the first series that passes every tests, it proves good prediction ability for all the models. The LR_{uc} at 95% is very close to the rejection region for DAGM models, except for DAGM skew student t. Same for GM for normal distributions and for no skew student t.

DQ test performed well, but for GM Skew Norm at 95% seems to be not the optimal one for this commodity, since it is close to the rejection region.

Table 10. Kerosene Backtesting results at 99% and 95% confidence level

confidence level	99%			95%		
	LR.uc p	LR.cc p	DQ p	LR.uc p	LR.cc p	DQ p
sGARCH-norm	0.257	0.500	0.000*	0.621	0.631	0.018*
sGARCH-std	0.829	0.964	0.990	0.621	0.631	0.022*
sGARCH-snorm	0.257	0.500	0.000*	0.621	0.631	0.017*
sGARCH-sstd	0.829	0.964	0.978	0.621	0.631	0.024*
sGARCH-ged	0.829	0.964	0.990	0.621	0.631	0.024*
sGARCH-sged	0.829	0.964	0.983	1	0.808	0.011*
eGARCH-norm	0.058	0.147	0.000*	0.162	0.190	0.000*
eGARCH-std	0.058	0.147	0.000*	0.338	0.386	0.006*
eGARCH-snorm	0.058	0.147	0.000*	0.338	0.386	0.001*
eGARCH-sstd	0.257	0.500	0.000*	0.338	0.386	0.007*
eGARCH-ged	0.058	0.147	0.000*	0.162	0.190	0.001*
eGARCH-sged	0.257	0.500	0.000*	0.338	0.386	0.002*
gjrGARCH-norm	0.257	0.500	0.000*	0.162	0.190	0.002*
gjrGARCH-std	0.257	0.500	0.000*	0.162	0.190	0.005*
gjrGARCH-snorm	0.257	0.500	0.000*	0.162	0.190	0.002*
gjrGARCH-sstd	0.257	0.500	0.000*	0.338	0.386	0.009*
gjrGARCH-ged	0.257	0.500	0.000*	0.162	0.190	0.004*
gjrGARCH-sged	0.257	0.500	0.000*	0.338	0.386	0.008*
GM-Skew-norm	0.829	0.964	0.000*	0.258	0.501	0.025*
GM-No Skew-norm	0.829	0.964	0.000*	0.258	0.501	0.026*
GM-Skew-std	0.829	0.964	0.000*	0.592	0.770	0.024*
GM-No Skew-std	0.829	0.964	0.000*	0.592	0.770	0.025*
DAGM-Skew-norm	0.829	0.964	0.000*	0.258	0.501	0.009*
DAGM-No Skew-norm	0.829	0.964	0.000*	0.258	0.501	0.009*
DAGM-Skew-std	0.257	0.500	0.000*	0.592	0.770	0.003*
DAGM-No Skew-std	0.829	0.964	0.000*	0.592	0.770	0.009*

* Denotes that the null hypothesis is rejected at the 5% significance level.

For Kerosene, the situation gets worse for the DQ test. At 99% confidence interval all models are rejected except sGARCH student t, sGARCH skew student t, sGARCH ged and sGARCH skew-ged, whereas at 95% are all rejected. LR conditional and unconditional coverage performed well, There was no rejection in this series at either confidence level, but at 99%, unconditional coverage of eGARCH models is very close to the rejection region, except for skew student t and skew ged.

3.2 MCS

After the backtesting which allowed us to find out the best models, we introduce the MCS procedure in order to obtain the Superior Set of Models (SSM). In doing so, we have estimated the losses from the VaRs and then constructed the ”Superior Set of Models” by using the R package ”MCS”.

In the following section we present a table for each commodity at both 99% and 95% confidence interval.

At 99% WTI superior set of models indicates that eGARCH models are the best ones, except for the one with a normal distribution. One of the common feature is that normal distributions have not great results. DAGM presents very heterogeneous results, gjrGARCH seems to be an homogeneous medium-low group and student-t is the most efficient distribution.
At 95% situation dramatically changed for eGARCH. gjr GARCH performed well and only DAGM skew norm is discarded.
At 99% Brent superior set of models shows, as we’ve seen before for WTI, that eGARCH models are the best one and DAGM models are heterogeneous. At 95% gjr GARCH models have the best outcomes and they present, like the other ones, homogeneous outcomes. GM and DAGM are not significantly adequate to forecast Brent.
At 99% among the first in the ranking are eGARCH skew student t skew ged and skew norm. Heating oil is the first series to give good outocomes for the DAGM models. At 95% sGARCH are the best ones, followed by eGARCH and gjr GARCH.
At 99% Propane has particular outcomes, GM models with std distribution performed well, but at the same time, GM normal and skew normal have one of the worst gjr GARCH overall has demonstrated pretty fine outcomes, unlike sGARCH and eGARCH.

Gasoline presents similar results at both confidence intervals, eGARCH and gjr GARCH take the first positions. At 95% MCS has removed skew student t distribution for GM and DAGM.
Looking at our last table, we end up with a very particular case. This is the only other case together with Heating Oil for which the standard GARCH outperforms all others models. It’s quite strange finding all the first six position in the first class of models (99%) , and even stranger that the best one is the Normal one given the fact that this was rejected by the Dynamic Quantile test in the previous section. The worst model with this confidence interval is DAGM-Skew-Student t. Moving to the 95% confidence interval the situation seems in line with some others results, highlighting good results of gjrGARCH which takes the first position using the Skew-Student t distribution, and the second with Skew-GED. The MIDAS class of models did not performed very well in this case, even if neither a model has been eliminated but still takes the last eight positions.

Table 11. Superior Set of Models

Models	WTI	Brent	Heating Oil	Propane	Gasoline	Kerosene
Panel A: Confidence level 99%						
sGARCH-norm	13	17	22	21	4	1
sGARCH-std	6	7	17	8	9	2
sGARCH-snorm	16	23	19	19	15	4
sGARCH-sstd	14	18	18	11	16	6
sGARCH-ged	7	14	16	9	6	3
sGARCH-sged	21	24	20	5	17	5
eGARCH-norm	19	12	25	23	2	22
eGARCH-std	1	1	15	18	12	18
eGARCH-snorm	8	15	4	22	1	24
eGARCH-sstd	5	3	1	20	18	14
eGARCH-ged	3	2	12	12	11	21
eGARCH-sged	4	4	2	13	21	20
gjrGARCH-norm	15	13	26	17	3	23
gjrGARCH-std	11	6	23	7	7	10
gjrGARCH-snorm	17	21	13	15	8	25
gjrGARCH-sstd	22	11	10	10	13	7
gjrGARCH-ged	12	10	21	4	5	13
gjrGARCH-sged	24	22	6	2	14	11
GM-Skew-norm	18	20	24	24	10	15
GM-No Skew-norm	23	9	8	25	20	8
GM-Skew-std	9	19	14	1	25	12
GM-No Skew-std	25	8	11	3	24	9
DAGM-Skew-norm	26	26	5	16	19	19
DAGM-No Skew-norm	10	16	3	26	22	17
DAGM-Skew-std	2	25	7	14	26	26
DAGM-No Skew-std	20	5	9	6	23	16
Panel B: Confidence level 95%						
sGARCH-norm	10	8	1	19	8	11
sGARCH-std	13	16	6	7	6	8
sGARCH-snorm	7	6	4	13	17	10
sGARCH-sstd	8	13	3	22	11	7
sGARCH-ged	12	11	2	21	9	9
sGARCH-sged	6	7	5	11	15	6
eGARCH-norm	15	12	9	15	14	17
eGARCH-std	20	18	8	17	13	12
eGARCH-snorm	16	10	14	14	2	18
eGARCH-sstd	14	17	13	24	7	3
eGARCH-ged	17	15	7	18	16	16
eGARCH-sged	19	14	15	20	18	13
gjrGARCH-norm	3	1	11	8	5	15
gjrGARCH-std	11	9	10	5	1	4
gjrGARCH-snorm	2	2	16	6	12	14
gjrGARCH-sstd	4	5	17	16	3	1
gjrGARCH-ged	5	4	12	2	10	5
gjrGARCH-sged	9	3	18	3	4	2
GM-Skew-norm	21	20	22	23	21	23
GM-No Skew-norm	23	23	24	-	20	22
GM-Skew-std	1	26	21	12	-	24
GM-No Skew-std	25	22	25	10	24	26
DAGM-Skew-norm	-	24	20	4	19	21
DAGM-No Skew-norm	22	25	19	-	23	19
DAGM-Skew-std	18	21	23	9	-	25
DAGM-No Skew-std	24	19	26	1	22	20

3.3 VaR Aggregation

Most of the time MCS procedure doesn't yield a single superior model but a set of several models with the same VaR predictive ability. For this reason we use the weighted average framework that consists in combining VaR models which belong to $M*_a$ in order to compare the performance of combined VaR with those of single models. The average weights we use are uniform and exponential. The first methodology is the Uniform one and it consists in giving the same weight to all the models. In accordance with this, for each commodity we attribute weight $\frac{1}{M*}$, where $M*$ corresponds to the number of models selected within the MCS procedure. The second method is the Exponential one and it consists of assigning a different weights to each model through the t-statistics resulting from the MCS procedure. The Table 12 and 13 show the new backtesting by using the VaR aggregation with uniform and exponential weights at confidence level 95% and 99%.

Table 12. VaR Aggregation exponential weights Backtesting results

confidence level	99%				95%			
	LR_{uc} p	LR_{cc} p	DQ p	AE	LR_{uc} p	LR_{cc} p	DQ p	AE
WTI	0.828	0.964	0.959	1.25	0.592	0.769	0.884	0.75
Brent	0.828	0.964	0.961	1.25	0.592	0.769	0.784	0.75
Heating Oil	0.828	0.964	0.997	1.25	0.592	0.769	0.819	0.75
Propane	0.257	0.499	0.174	2.5	1	0.807	0.356	1
Gasoline	0.828	0.964	0.944	1.25	1	0.373	0.350	1
Kerosene	0.257	0.499	0.000*	2.5	1	0.807	0.011*	1

* Denotes that the null hypothesis is rejected at the 5% significance level.

Table 13. VaR Aggregation uniform weights Backtesting results

confidence level	99%				95%			
	LR_{uc} p	LR_{cc} p	DQ p	AE	LR_{uc} p	LR_{cc} p	DQ p	AE
WTI	0.828	0.964	0.931	1.25	0.592	0.769	0.869	0.75
Brent	0.828	0.964	0.954	1.25	0.592	0.769	0.867	0.75
Heating Oil	0.828	0.964	0.995	1.25	0.592	0.769	0.779	0.75
Propane	0.257	0.499	0.158	2.5	1	0.807	0.304	1
Gasoline	0.828	0.964	0.969	1.25	1	0.373	0.344	1
Kerosene	0.257	0.499	0.000*	2.5	0.592	0.769	0.008*	0.75

* Denotes that the null hypothesis is rejected at the 5% significance level.

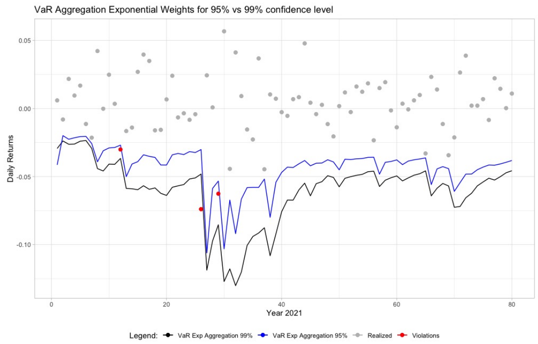


Figure 8. WTI - VaR Aggregation violations

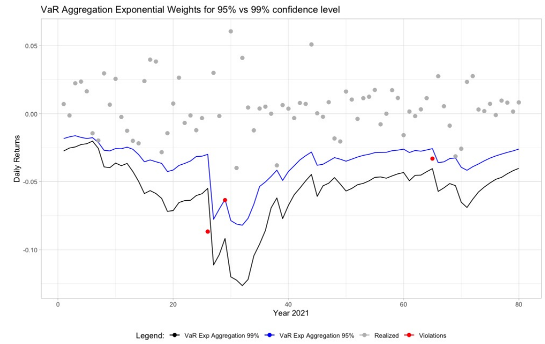


Figure 9. Brent - VaR Aggregation violations

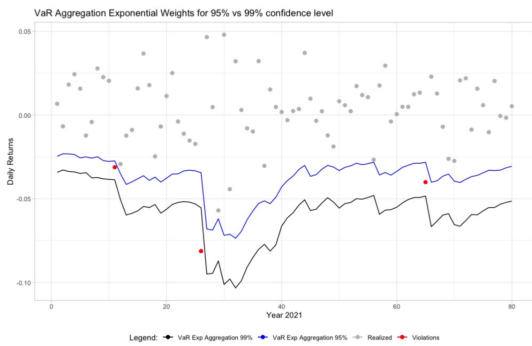


Figure 10. Heating Oil - VaR Aggregation violations

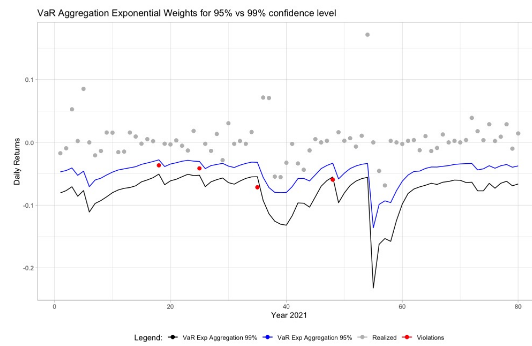


Figure 11. Propane - VaR Aggregation violations

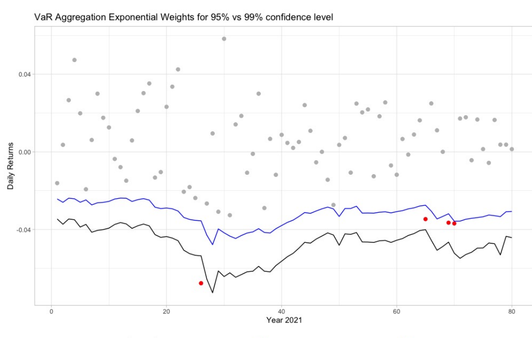


Figure 12. Gasoline - VaR Aggregation violations

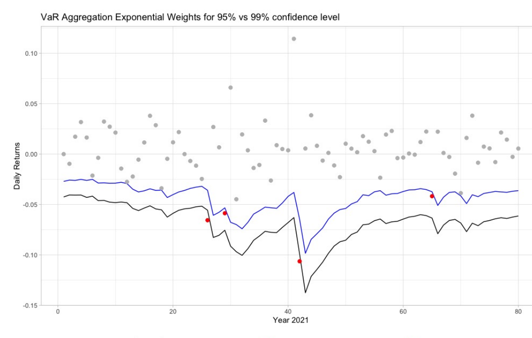


Figure 13. Kerosene - VaR Aggregation violations

4. Conclusions

The goal of this study is to find the most accurate model for predicting VaR of six commodities: WTI, Brent, Heating Oil, Propane, Gasoline and Kerosene. Both approaches, backtesting and MCS procedure, were employed to assess their forecasting capacity. Moreover, various models have been used to enrich the analysis, such as Mixed Data Sampling method including Covid-19 weekly U.S. deaths as macroeconomic variable. After estimating the VaR at both 99% and 95% confidence levels, which are the most used by practitioners, the results have been compared. The MCS procedure by Hansen, Lunde and Nason (2011) highlights set of superior models ”SSM” and, since different models belong to the SSM, we concluded the analysis aggregating the VaR estimates provided by surviving models.

Starting with WTI, the best model according to the Backtest procedure are: standard GARCH skew-Student-t and Skew-GED at 99% confidence level and gjrGARCH Skew-Norm and DAGM Skew-student-t for 95%, while the MCS procedure classifies as first eGARCH Student-t at 99% and GM Skew Student-t at 95%. The first result is contradictory. Even if the backtest did not reject the eGARCH Student-t, because of the poor performance of this model, in general, it should not be considered the best model. VaR aggregation results seems to be very good for both confidence intervals, since no test has rejected the model, either with exponential or uniform weights, but it might underestimate the losses with an AE ratio equal to 1.25 for 99% and overestimate for 95%, AE = 0.75.

Brent Crude Oil backtest seems to be quite similar to WTI and, as already specified, the performance of standard GARCH are quite good for all confidence levels and for every distribution, but with some preference in Normal, Skew-Student t and GED, while GM and DAGM obtained results way better than WTI. According to the MCS, the best mode is the eGARCH-Student-t for 99% and gjr-Normal for 95% confidence level, while the DAGM-No Skew-Student-t displays good backtesting results and also obtained a relatively high position at 99% confidence level. When combining all the models, we could draw the same conclusions as for WTI: VaR aggregation pass all the tests with exponential and uniform weights, with some results slightly better in the Dynamic Quantile test and the same underestimation of the losses at 99% (AE = 1.25) and overestimation at 95%.

Heating Oil's backtest indicates us that the best models for 99% confidence level are standard GARCH, eGARCH and gjr-GARCH with Skew-Student-t distribution and GM and DAGM Normal and Studen-t. Regarding 95% confidence level, gjr-GARCH and eGARCH outperforms all the others without exclusions for any distributions. The MCS places first the eGARCH Skew-Student-t and then also all the DAGMs performs well at 99%; while sGARCHs are better at 95%. Also in this case, VaR aggregation performed well and passed all the test, displaying the same characteristics mentioned before.

When comes to backtesting, Propane performs better with simpler GARCH models rather then GARCH MIDAS, especially at 99% confidence level. The only model which did not pass the backtest is eGARCH Skew-Student-t at 99% confidence level. For MIDAS models, GMs performances are bad at 99% confidence level, while acceptable at 95%, especially the GM-Normal. DAGM obtained very good results at both confidence levels, but the most notable are the DAGM-Normal and the DAGM-Skew-Normal. According to the MCS, GM Student-t and Skew Student-t are among the first in the ranking at 99% confidence level, but immediately after we have the gjr class of models, among which the gjr skew-GED is the best. At 95% gjrGARCH and DAGM models can be declared the best, considering both backtest and MCS. VaR aggregation, despite the good results of the backetest for exponential and uniform, seems to extremely underestimate risk at 99% with AE ratio of 2.5, while it's perfect at 95% scoring 1.

Gasoline is the best series in terms of backtesting results; not a single model has been rejected by any test. In this case, it's difficult to say which is the best. According to the MCS the best is the eGARCH at 99% and gjrGARCH-std at 95%. The superior set of models at this confidence level doesn't include GM-Skew-Student t and DAGM-Skew-Student-t. VaR aggregation's backtest doesn't reject the null for any test.

Lastly, Kerosene which is the most problematic series. Only few models pass the Dynamic Quantile test (sGARCH Student-t, Skew Student-t, GED and Skew GED), but for the rest we obtained all bad results even if some statistic of Conditional Coverage and Uncoditional Coverage were not rejected. MCS places sGARCH Normal and Student-t in the first two position at 99% confidence level and gjrGARCH Skew-Student-t and Skew GED at 95%, while the lower ranks are occupied by DAGM Skew Student-t and GM Student-t. For VaR aggregation we have the first case of rejection of some tests: Dynamic Quantile reject the null with 1% significance at both confidence level, plus once again the aggregation overestimates the losses at 99% c.i. and underestimates them at 95%.

In conclusion, we can state that the best model is quite often a gjrGARCH, usually skewed and with a Student-t distribution; while for Mixed Frequency models the best one is always the DAGM Student-t. By looking at all the test's results, it could be stated that a more conservative approach seems better. Even with the inclusion of such an important low frequency variable, within the MIDAS models, the GARCH forecasts seems to be more accurate for the simpler class of GARCH models and in general they are preferred to the mixed frequency approach. This results could be due to the exclusion, in our study, of the multivariate correlation specification, which has been used by Candila (2020) and that could improve the global estimation; or, as specified above, to the possible increase of second type errors when performing the tests, as the power of the test relies on the sample size[16].

[16] Roccioletti, "Backtesting Value at Risk and Expected Shortfall", pag. 49

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Appendix

Table 14. Matrix of coefficient for WTI GARCH models

	Model	μ	ω	α	β	γ	skew	shape
1	sGARCH-norm	0.0033	0.0000	0.1627	0.8269	-	-	-
2	sGARCH-std	0.0040	0.0000	0.2006	0.7845	-	-	3.9919
3	sGARCH-snorm	0.0030	0.0000	0.1613	0.8309	-	0.8418	-
4	sGARCH-sstd	0.0026	0.0000	0.2153	0.7765	-	0.8011	4.1993
5	sGARCH-ged	0.0038	0.0000	0.1699	0.8087	-	-	1.1617
6	sGARCH-sged	0.0028	0.0000	0.1732	0.8098	-	0.8698	1.1895
7	eGARCH-norm	0.0025	-0.0732	-0.1809	0.9896	0.2322	-	-
8	eGARCH-std	0.0032	-0.1125	-0.1629	0.9858	0.2497	-	4.9484
9	eGARCH-snorm	0.0022	-0.0546	-0.1861	0.9921	0.2481	0.7965	-
10	eGARCH-sstd	0.0020	-0.1069	-0.1680	0.9852	0.2951	0.7814	5.1571
11	eGARCH-ged	0.0033	-0.0932	-0.1669	0.9891	0.2326	-	1.2717
12	eGARCH-sged	0.0024	-0.0787	-0.1742	0.9900	0.2622	0.8075	1.3616
13	gjrGARCH-norm	0.0017	0.0000	0.0178	0.8402	0.2656	-	-
14	gjrGARCH-std	0.0030	0.0000	0.0270	0.8319	0.2413	-	4.9491
15	gjrGARCH-snorm	0.0013	0.0000	0.0131	0.8405	0.2907	0.8135	-
16	gjrGARCH-sstd	0.0016	0.0000	0.0363	0.8197	0.2802	0.7714	5.0715
17	gjrGARCH-ged	0.0029	0.0000	0.0236	0.8360	0.2391	-	1.2633
18	gjrGARCH-sged	0.0018	0.0000	0.0262	0.8295	0.2679	0.8262	1.3264

Table 15. Matrix of coefficient for Europe Brent GARCH models

	Model	μ	ω	α	β	γ	skew	shape
1	sGARCH-norm	0.0035	0.0000	0.1602	0.8267	-	-	-
2	sGARCH-std	0.0040	0.0000	0.1974	0.8015	-	-	3.3997
3	sGARCH-snorm	0.0030	0.0000	0.1553	0.8296	-	0.8473	-
4	sGARCH-sstd	0.0027	0.0000	0.1966	0.8019	-	0.8649	3.4324
5	sGARCH-ged	0.0040	0.0000	0.1632	0.8152	-	-	1.0617
6	sGARCH-sged	0.0028	0.0000	0.1557	0.8208	-	0.8615	1.0981
7	eGARCH-norm	0.0029	-0.0736	-0.1251	0.9888	0.2325	-	-
8	eGARCH-std	0.0036	-0.1227	-0.1092	0.9836	0.2796	-	3.7428
9	eGARCH-snorm	0.0027	-0.0599	-0.1173	0.9907	0.2330	0.8618	-
10	eGARCH-sstd	0.0025	-0.1278	-0.1031	0.9822	0.2836	0.8753	3.7467
11	eGARCH-ged	0.0038	-0.1021	-0.1088	0.9875	0.2431	-	1.1132
12	eGARCH-sged	0.0027	-0.0925	-0.1018	0.9881	0.2452	0.8687	1.1448
13	gjrGARCH-norm	0.0025	0.0000	0.0258	0.8620	0.1837	-	-
14	gjrGARCH-std	0.0035	0.0000	0.0465	0.8471	0.1766	-	3.8944
15	gjrGARCH-snorm	0.0020	0.0000	0.0308	0.8630	0.1756	0.8542	-
16	gjrGARCH-sstd	0.0024	0.0000	0.0551	0.8443	0.1714	0.8695	3.9117
17	gjrGARCH-ged	0.0037	0.0000	0.0357	0.8551	0.1621	-	1.1238
18	gjrGARCH-sged	0.0024	0.0000	0.0432	0.8538	0.1570	0.8656	1.1558

Table 16. GARCH MIDAS Coefficient Matrix of WTI and Brent

	Estimate	Std. Error	t value	Pr(>|t|)	Estimate	Std. Error	t value	Pr(>|t|)
	WTI GM - Skew - norm				Brent GM - Skew - norm			
α	0.0001	0.0479	0.0021	0.9983	0.0001	0.0824	0.0012	0.9990
γ	0.7238	0.3042	2.3795	0.0173	0.4798	0.3529	1.3595	0.1740
β	0.0139	0.0644	0.2160	0.8290	0.1549	0.2176	0.7120	0.4765
m	-7.1268	0.2562	-27.8183	0.0000	-7.1794	0.2759	-26.0201	0.0000
θ	-15.0453	1.7934	-8.3892	0.0000	-13.1300	2.4748	-5.3055	0.0000
ω_2	1.7113	0.1491	11.4765	0.0000	2.2784	0.4109	5.5450	0.0000
	WTI GM - No Skew - norm				Brent GM - No Skew - norm			
α	0.3499	0.2294	1.5253	0.1272	0.0010	0.0068	0.1477	0.8825
β	0.0010	0.3450	0.0029	0.9977	0.9980	0.0872	11.4471	0.0000
m	-7.2193	0.6203	-11.6390	0.0000	-7.1927	0.3325	-21.6334	0.0000
θ	-14.7500	19.7871	-0.7454	0.4560	-10.9439	14.5786	-0.7507	0.4528
ω_2	1.7642	0.3817	4.6223	0.0000	2.3019	0.5176	4.4469	0.0000
	WTI GM - Skew - std				Brent GM - Skew - std			
α	0.0001	1.0407	0.0001	0.9999	0.0001	0.0614	0.0017	0.9987
γ	0.2667	0.6909	0.3860	0.6995	0.4938	0.3841	1.2855	0.1986
β	0.7682	2.1444	0.3582	0.7202	0.1750	0.2274	0.7697	0.4415
m	-7.6092	1.0762	-7.0707	0.0000	-7.1159	0.3756	-18.9473	0.0000
θ	-1.0794	2.8011	-0.3854	0.7000	-13.8749	8.5404	-1.6246	0.1042
ω_2	1.0291	0.7075	1.4546	0.1458	2.1304	0.4009	5.3145	0.0000
shape	7.1960	6.0998	1.1797	0.2381	13.5861	3.0115	4.5113	0.0000
	WTI GM - No Skew - std				Brent GM - No Skew - std			
α	0.3970	0.1930	2.0574	0.0396	0.0878	0.1446	0.6071	0.5438
β	0.0010	0.3448	0.0029	0.9977	0.3331	0.6866	0.4852	0.6275
m	-7.1770	0.2749	-26.1116	0.0000	-7.1663	0.1920	-37.3257	0.0000
θ	-13.1378	6.0178	-2.1832	0.0290	-15.5142	2.9987	-5.1736	0.0000
ω_2	1.6806	0.2250	7.4698	0.0000	2.1328	0.3270	6.5215	0.0000
shape	28.3542	0.0047	5985.0828	0.0000	8.5046	2.4147	3.5221	0.0004
	WTI DAGM - Skew - norm				Brent DAGM - Skew - norm			
α	0.0001	0.0965	0.0010	0.9992	0.0001	0.2350	0.0005	0.9996
γ	0.7168	0.3396	2.1104	0.0348	0.4553	0.3796	1.1993	0.2304
β	0.0083	0.1244	0.0670	0.9466	0.7712	0.0491	15.7030	0.0000
m	-6.5834	0.5701	-11.5476	0.0000	-2.6482	0.2122	-12.4787	0.0000
θ^+	-19.9593	3.7536	-5.3174	0.0000	-0.2070	0.1874	-1.1049	0.2692
ω_2^+	1.9707	0.8303	2.3736	0.0176	1.9959	0.3233	6.1737	0.0000
θ^-	-8.1632	35.0273	-0.2331	0.8157	0.1070	0.6590	0.1623	0.8711
ω_2^-	1.2287	6.0900	0.2018	0.8401	1.9994	1.2697	1.5748	0.1153
	WTI DAGM - No Skew - norm				Brent DAGM - No Skew - norm			
α	0.3351	0.3573	0.9377	0.3484	0.0523	0.8443	0.0619	0.9506
β	0.0010	0.2633	0.0039	0.9969	0.9467	0.9341	1.0135	0.3108
m	-5.9375	4.4102	-1.3463	0.1782	-6.7334	11.1253	-0.6052	0.5450
θ^+	-25.0562	58.7683	-0.4264	0.6698	-7.0649	217.7476	-0.0324	0.9741
ω_2^+	2.2666	0.9993	2.2682	0.0233	2.0941	21.8552	0.0958	0.9237
θ^-	1.4230	24.4579	0.0582	0.9536	-0.6377	38.1410	-0.0167	0.9867
ω_2^-	3.7808	13.2263	0.2859	0.7750	2.0176	67.1209	0.0301	0.9760
	WTI DAGM - Skew - std				Brent DAGM - Skew - std			
α	0.0029	0.0602	0.0475	0.9621	0.0001	0.1347	0.0007	0.9994
γ	0.7905	0.3870	2.0428	0.0411	0.1693	0.1851	0.9145	0.3605
β	0.0112	0.0696	0.1608	0.8723	0.8564	0.2691	3.1826	0.0015
m	-5.9830	0.5003	-11.9595	0.0000	-7.0671	0.4223	-16.7349	0.0000
θ^+	-22.6956	3.3142	-6.8479	0.0000	-7.5049	2.9232	-2.5674	0.0102
ω_2^+	2.0720	0.2132	9.7193	0.0000	2.2539	1.3244	1.7018	0.0888
θ^-	-2.1626	7.2840	-0.2969	0.7665	-1.3399	2.0495	-0.6538	0.5133
ω_2^-	14.9842	5.4909	2.7289	0.0064	1.2328	1.3307	0.9265	0.3542
shape	55.6913	0.1153	482.8045	0.0000	9.2247	3.8612	2.3891	0.0169
	WTI DAGM - No Skew - std				Brent DAGM - No Skew - std			
α	0.3780	0.1470	2.5717	0.0101	0.0482	0.0331	1.4591	0.1445
β	0.0010	0.2778	0.0037	0.9971	0.9508	0.0428	22.2223	0.0000
m	-4.9353	0.4682	-10.5406	0.0000	-6.6422	0.2769	-23.9862	0.0000
θ^+	-30.7098	0.8840	-34.7399	0.0000	-3.0924	1.8209	-1.6983	0.0894
ω_2^+	2.3927	0.2133	11.2176	0.0000	1.8448	1.3602	1.3563	0.1750
θ^-	11.6686	4.9700	2.3478	0.0189	1.3512	0.7533	1.7936	0.0729
ω_2^-	1.0017	1.0604	0.9447	0.3448	2.1808	0.9517	2.2916	0.0219
shape	54.9362	0.0515	1066.0767	0.0000	4.4969	1.9199	2.3422	0.0192

Table 17. Matrix of coefficient for Heating Oil GARCH models

	Model	μ	ω	α	β	γ	skew	shape
1	sGARCH-norm	0.0029	0.0001	0.2730	0.6807	-	-	-
2	sGARCH-std	0.0031	0.0000	0.1572	0.8156	-	-	6.8363
3	sGARCH-snorm	0.0025	0.0000	0.1900	0.7672	-	0.7849	-
4	sGARCH-sstd	0.0023	0.0000	0.1586	0.8083	-	0.8214	8.0208
5	sGARCH-ged	0.0029	0.0000	0.1927	0.7769	-	-	1.3671
6	sGARCH-sged	0.0023	0.0000	0.1917	0.7688	-	0.8080	1.4609
7	eGARCH-norm	0.0015	-0.4306	-0.1563	0.9399	0.3224	-	-
8	eGARCH-std	0.0024	-0.3133	-0.1129	0.9578	0.2750	-	8.4166
9	eGARCH-snorm	0.0013	-0.3963	-0.1458	0.9448	0.2925	0.7942	-
10	eGARCH-sstd	0.0015	-0.3716	-0.1266	0.9491	0.2867	0.8144	10.6245
11	eGARCH-ged	0.0023	-0.3720	-0.1311	0.9500	0.3057	-	1.4452
12	eGARCH-sged	0.0013	-0.4215	-0.1410	0.9421	0.3108	0.8065	1.5384
13	gjrGARCH-norm	0.0013	0.0001	0.0490	0.7343	0.3337	-	-
14	gjrGARCH-std	0.0022	0.0000	0.0440	0.8063	0.2073	-	8.9103
15	gjrGARCH-snorm	0.0011	0.0000	0.0357	0.7655	0.2937	0.7848	-
16	gjrGARCH-sstd	0.0013	0.0000	0.0431	0.7830	0.2477	0.7988	11.5272
17	gjrGARCH-ged	0.0022	0.0000	0.0536	0.7637	0.2659	-	1.4734
18	gjrGARCH-sged	0.0012	0.0000	0.0537	0.7439	0.2992	0.7941	1.5567

Table 18. Matrix of coefficient for Propane GARCH models

	Model	μ	ω	α	β	γ	skew	shape
1	sGARCH-norm	0.0035	0.0000	0.1229	0.8438	-	-	-
2	sGARCH-std	0.0027	0.0001	0.2826	0.7164	-	-	2.9287
3	sGARCH-snorm	0.0036	0.0001	0.1251	0.8405	-	1.0134	-
4	sGARCH-sstd	0.0022	0.0001	0.2762	0.7228	-	0.9637	2.9403
5	sGARCH-ged	0.0022	0.0001	0.1897	0.7787	-	-	0.8537
6	sGARCH-sged	0.0022	0.0001	0.1898	0.7787	-	0.9997	0.8522
7	eGARCH-norm	0.0013	-0.1747	-0.1004	0.9704	0.2463	-	-
8	eGARCH-std	0.0024	-0.5041	-0.0506	0.9203	0.5222	-	2.6590
9	eGARCH-snorm	0.0012	-0.1712	-0.1005	0.9709	0.2455	0.9927	-
10	eGARCH-sstd	0.0017	-0.4600	-0.0592	0.9267	0.5173	0.9488	2.6498
11	eGARCH-ged	0.0022	-0.2875	-0.0641	0.9582	0.3134	-	0.8584
12	eGARCH-sged	0.0019	-0.2741	-0.0664	0.9600	0.3107	0.9920	0.8582
13	gjrGARCH-norm	0.0016	0.0000	0.0443	0.8675	0.1572	-	-
14	gjrGARCH-std	0.0026	0.0001	0.1991	0.7519	0.0959	-	2.9802
15	gjrGARCH-snorm	0.0015	0.0000	0.0434	0.8693	0.1576	0.9883	-
16	gjrGARCH-sstd	0.0019	0.0001	0.1652	0.7788	0.1133	0.9533	3.0124
17	gjrGARCH-ged	0.0021	0.0000	0.0767	0.8432	0.1343	-	0.8754
18	gjrGARCH-sged	0.0018	0.0000	0.0718	0.8500	0.1357	0.9908	0.8744

Table 19. GM Coefficient Matrix of Heating Oil and Propane

	Estimate	Std. Error	t value	Pr(>|t|)	Estimate	Std. Error	t value	Pr(>|t|)
	Heating Oil GM - Skew - norm				Propane GM - Skew - norm			
α	0.0001	0.1420	0.0007	0.9994	0.0790	0.1094	0.7223	0.4701
γ	0.0724	0.2673	0.2709	0.7865	0.9679	1.2855	0.7529	0.4515
β	0.9627	0.0719	13.3921	0.0000	0.1256	0.5485	0.2291	0.8188
m	-7.4002	1.0024	-7.3822	0.0000	-7.0753	1.3547	-5.2229	0.0000
θ	-2.1469	23.4920	-0.0914	0.9272	10.4818	2.3386	4.4821	0.0000
ω_2	1.5665	5.3859	0.2909	0.7712	3.3491	0.3727	8.9868	0.0000
	Heating Oil GM - No Skew - norm				Propane GM - No Skew - norm			
α	0.0010	0.0010	1.0105	0.3123	0.3709	0.1534	2.4172	0.0156
β	0.9724	0.0449	21.6400	0.0000	0.6097	0.1604	3.8004	0.0001
m	-7.5084	0.1397	-53.7439	0.0000	-5.7077	1.1761	-4.8531	0.0000
θ	-8.6329	2.3529	-3.6690	0.0002	-15.1475	16.0007	-0.9467	0.3438
ω_2	2.8700	1.8696	1.5351	0.1248	1.0806	0.5218	2.0707	0.0384
	Heating Oil GM - Skew - std				Propane GM - Skew - std			
α	0.0001	0.0593	0.0017	0.9987	0.2706	0.1383	1.9563	0.0504
γ	0.0400	0.0579	0.6911	0.4895	0.0939	0.4371	0.2147	0.8300
β	0.9789	0.1102	8.8792	0.0000	0.5801	0.2009	2.8872	0.0039
m	-7.4018	0.3309	-22.3715	0.0000	-6.4973	0.7220	-8.9992	0.0000
θ	-5.7600	4.4613	-1.2911	0.1967	-5.6039	5.3375	-1.0499	0.2938
ω_2	1.9917	0.9140	2.1791	0.0293	1.0011	0.6233	1.6060	0.1083
shape	12.0065	14.0866	0.8523	0.3940	4.0111	1.2474	3.2156	0.0013
	Heating Oil GM - No Skew - std				Propane GM - No Skew - std			
α	0.0001	0.0010	0.1006	0.9199	0.2952	0.1318	2.2402	0.0251
β	0.5152	0.3727	1.3825	0.1668	0.5930	0.1404	4.2247	0.0000
m	-7.3763	0.1748	-42.1991	0.0000	-6.6081	0.6701	-9.8619	0.0000
θ	-11.1879	4.9704	-2.2509	0.0244	-5.2578	4.2409	-1.2398	0.2151
ω_2	2.2889	0.6089	3.7589	0.0002	1.0011	0.4134	2.4214	0.0155
shape	15.2618	0.4190	36.4235	0.0000	3.9498	1.2666	3.1185	0.0018
	Heating Oil DAGM - Skew - norm				Propane DAGM - Skew - norm			
α	0.0001	0.2287	0.0004	0.9997	0.1436	0.1012	1.4184	0.1561
γ	0.0451	0.3542	0.1273	0.8987	0.6874	0.6747	1.0188	0.3083
β	0.9763	0.3330	2.9323	0.0034	0.2783	0.2607	1.0677	0.2856
m	-7.1215	4.5971	-1.5491	0.1214	-3.5499	0.9056	-3.9198	0.0001
θ^+	-7.0620	32.8964	-0.2147	0.8300	-18.6413	3.8391	-4.8556	0.0000
ω_2^+	3.8522	10.9135	0.3530	0.7241	1.7619	0.2660	6.6241	0.0000
θ^-	0.0276	6.3201	0.0044	0.9965	37.3945	4.0121	9.3204	0.0000
ω_2^-	2.0143	10.4981	0.1919	0.8478	1.0010	0.3955	2.5312	0.0114
	Heating Oil DAGM - No Skew - norm				Propane DAGM - No Skew - norm			
α	0.0372	0.1362	0.2727	0.7851	0.5591	0.1588	3.5217	0.0004
β	0.9618	0.1561	6.1623	0.0000	0.4379	0.1389	3.1514	0.0016
m	-7.2739	1.5813	-4.5999	0.0000	3.9017	7.8574	0.4966	0.6195
θ^+	-3.1691	29.6405	-0.1069	0.9149	-54.6880	11.5982	-4.7152	0.0000
ω_2^+	3.2274	4.1624	0.7754	0.4381	1.6020	0.4388	3.6505	0.0003
θ^-	-0.5428	0.7237	-0.7500	0.4532	74.0549	16.7397	4.4239	0.0000
ω_2^-	1.8784	2.5293	0.7427	0.4577	1.0010	0.7460	1.3418	0.1797
	Heating Oil DAGM - Skew - std				Propane DAGM - Skew- std			
α	0.0001	0.0811	0.0012	0.9990	0.0929	0.3229	0.2876	0.7737
γ	0.0392	0.1049	0.3734	0.7088	0.4350	0.6570	0.6622	0.5079
β	0.9793	0.1343	7.2929	0.0000	0.4941	0.2430	2.0331	0.0420
m	-7.0775	0.6868	-10.3055	0.0000	-4.1875	3.6205	-1.1566	0.2474
θ^+	-7.9083	5.8538	-1.3510	0.1767	-11.7951	34.2220	-0.3447	0.7303
ω_2^+	2.6002	2.5023	1.0391	0.2987	1.4602	2.5319	0.5767	0.5641
θ^-	-1.3943	6.9140	-0.2017	0.8402	36.1233	40.8194	0.8850	0.3762
ω_2^-	1.0053	7.5351	0.1334	0.8939	1.0010	2.3199	0.4315	0.6661
*shape*e	12.4686	1.9375	6.4355	0.0000	5.5186	4.1524	1.3290	0.1839
	Heating Oil DAGM - No Skew - std				Propane DAGM - No Skew - std			
α	0.0001	0.0041	0.0243	0.9806	0.2161	0.1601	1.3493	0.1772
β	0.9986	0.4859	2.0549	0.0399	0.5372	0.1222	4.3949	0.0000
m	-6.8473	0.2531	-27.0566	0.0000	-4.5650	0.9580	-4.7652	0.0000
θ^+	-10.0110	2.3101	-4.3337	0.0000	-10.7601	6.1607	-1.7466	0.0807
ω_2^+	4.3596	1.1632	3.7479	0.0002	1.4870	0.7047	2.1100	0.0349
θ^-	3.4869	2.7481	1.2688	0.2045	33.8849	13.3079	2.5462	0.0109
ω_2^-	3.3002	3.3627	0.9814	0.3264	1.0011	0.9833	1.0181	0.3086
shape	10.8892	3.6943	2.9476	0.0032	4.2835	2.4905	1.7199	0.0854

Table 20. Matrix of coefficient for Gasoline GARCH models

	Model	μ	ω	α	β	γ	skew	shape
1	sGARCH-norm	0.0037	0.0000	0.1279	0.8566	-	-	-
2	sGARCH-std	0.0040	0.0000	0.0905	0.8796	-	-	5.1263
3	sGARCH-snorm	0.0034	0.0000	0.1023	0.8754	-	0.8768	-
4	sGARCH-sstd	0.0029	0.0000	0.0754	0.8946	-	0.8608	4.9535
5	sGARCH-ged	0.0049	0.0000	0.1061	0.8666	-	-	1.2540
6	sGARCH-sged	0.0032	0.0000	0.0833	0.8869	-	0.8650	1.2106
7	eGARCH-norm	0.0031	-0.0554	-0.0640	0.9918	0.2251	-	-
8	eGARCH-std	0.0038	-0.1103	-0.0384	0.9852	0.1933	-	5.1758
9	eGARCH-snorm	0.0028	-0.0577	-0.0605	0.9916	0.1919	0.8817	-
10	eGARCH-sstd	0.0028	-0.1010	-0.0352	0.9864	0.1730	0.8707	5.0434
11	eGARCH-ged	0.0046	-0.0938	-0.0463	0.9881	0.2087	-	1.2675
12	eGARCH-sged	0.0030	-0.0910	-0.0473	0.9882	0.1748	0.8764	1.2116
13	gjrGARCH-norm	0.0030	0.0000	0.0494	0.8792	0.0910	-	-
14	gjrGARCH-std	0.0037	0.0000	0.0427	0.8954	0.0558	-	5.4265
15	gjrGARCH-snorm	0.0026	0.0000	0.0232	0.9064	0.0801	0.8644	-
16	gjrGARCH-sstd	0.0026	0.0000	0.0297	0.9118	0.0525	0.8593	5.2525
17	gjrGARCH-ged	0.0046	0.0000	0.0490	0.8856	0.0634	-	1.2857
18	gjrGARCH-sged	0.0028	0.0000	0.0227	0.9120	0.0636	0.8623	1.2334

Table 21. Matrix of coefficient for Kerosene GARCH models

	Model	μ	ω	α	β	γ	skew	shape
1	sGARCH-norm	0.0035	0.0000	0.1672	0.8069	-	-	-
2	sGARCH-std	0.0034	0.0000	0.1660	0.8025	-	-	7.4601
3	sGARCH-snorm	0.0033	0.0000	0.1493	0.8269	-	0.9137	-
4	sGARCH-sstd	0.0028	0.0000	0.1578	0.8134	-	0.8867	7.2646
5	sGARCH-ged	0.0037	0.0000	0.1737	0.7969	-	-	1.3185
6	sGARCH-sged	0.0031	0.0000	0.1585	0.8130	-	0.9262	1.3402
7	eGARCH-norm	0.0020	-0.1372	-0.1156	0.9800	0.2010	-	-
8	eGARCH-std	0.0028	-0.1283	-0.0937	0.9824	0.1917	-	8.5748
9	eGARCH-snorm	0.0019	-0.1231	-0.1162	0.9820	0.1874	0.8803	-
10	eGARCH-sstd	0.0022	-0.1242	-0.0928	0.9827	0.1957	0.8717	8.5212
11	eGARCH-ged	0.0031	-0.1281	-0.1001	0.9829	0.1930	-	1.3797
12	eGARCH-sged	0.0021	-0.1253	-0.1045	0.9827	0.1880	0.8912	1.4103
13	gjrGARCH-norm	0.0023	0.0000	0.0563	0.8435	0.1566	-	-
14	gjrGARCH-std	0.0028	0.0000	0.0667	0.8461	0.1170	-	8.5360
15	gjrGARCH-snorm	0.0022	0.0000	0.0434	0.8603	0.1528	0.8990	-
16	gjrGARCH-sstd	0.0022	0.0000	0.0611	0.8528	0.1213	0.8804	8.2281
17	gjrGARCH-ged	0.0032	0.0000	0.0665	0.8422	0.1283	-	1.3762
18	gjrGARCH-sged	0.0023	0.0000	0.0522	0.8556	0.1367	0.9035	1.3977

Table 22. GM: Coefficient Matrix Gasoline and Kerosene

	Estimate	Std. Error	t value	Pr(>|t|)	Estimate	Std. Error	t value	Pr(>|t|)
	Gasoline GM - Skew - norm				Kerosene GM - Skew - norm			
α	0.0002	0.0617	0.0025	0.9980	0.0001	0.0102	0.0098	0.9922
γ	0.0925	0.3579	0.2585	0.7960	0.0205	0.0924	0.2222	0.8242
β	0.9526	0.2841	3.3531	0.0008	0.9290	0.0813	11.4242	0.0000
m	-6.8692	0.7017	-9.7898	0.0000	-7.1371	0.1633	-43.7097	0.0000
θ	0.1975	0.3010	0.6561	0.5117	-9.5884	4.1229	-2.3257	0.0200
ω_2	1.5740	2.1087	0.7464	0.4554	2.5869	1.1207	2.3082	0.0210
	Gasoline GM - No Skew - norm				Kerosene GM - No Skew - norm			
α	0.0384	0.3641	0.1055	0.9160	0.0010	0.0038	0.2638	0.7920
β	0.9606	0.3991	2.4066	0.0161	0.3946	0.7532	0.5240	0.6003
m	-7.0208	1.9109	-3.6741	0.0002	-7.1315	0.1747	-40.8216	0.0000
θ	-3.3796	21.8835	-0.1544	0.8773	-9.9485	7.2906	-1.3646	0.1724
ω_2	5.2213	447.9955	0.0117	0.9907	3.1543	5.7134	0.5521	0.5809
	Gasoline GM - Skew - std				Kerosene GM - Skew - std			
α	0.0001	0.3305	0.0003	0.9998	0.0001	0.1439	0.0007	0.9994
γ	0.0535	0.1094	0.4890	0.6248	0.3157	0.3197	0.9873	0.3235
β	0.9722	0.3337	2.9129	0.0036	0.0019	0.3424	0.0055	0.9956
m	-7.0850	1.5992	-4.4303	0.0000	-7.0288	0.1966	-35.7537	0.0000
θ	-3.1100	5.9437	-0.5232	0.6008	-10.1487	2.6270	-3.8632	0.0001
ω_2	2.9495	5.1799	0.5694	0.5691	3.0117	1.0002	3.0111	0.0026
$shape$	11.1078	3.7728	2.9442	0.0032	22.0455	0.1270	173.6320	0.0000
	Gasoline GM - No Skew - std				Kerosene GM - No Skew - std			
α	0.0001	0.1533	0.0007	0.9995	0.0001	0.0004	0.2438	0.8074
β	0.4772	7.3367	0.0650	0.9481	0.6181	0.3968	1.5574	0.1194
m	-7.2433	0.1701	-42.5741	0.0000	-7.0838	0.2197	-32.2441	0.0000
θ	-8.4942	2.6896	-3.1582	0.0016	-10.7376	6.4705	-1.6595	0.0970
ω_2	2.7264	0.5338	5.1074	0.0000	2.5886	0.8608	3.0073	0.0026
$shape$	12.5588	1.0316	12.1738	0.0000	20.8868	0.0462	452.4074	0.0000
	Gasoline DAGM - Skew - norm				Kerosene DAGM - Skew - norm			
α	0.0001	0.0203	0.0049	0.9961	0.0001	0.0380	0.0026	0.9979
γ	0.0496	0.0311	1.5927	0.1112	0.0234	0.1287	0.1815	0.8559
β	0.9741	0.0390	25.0038	0.0000	0.6974	0.2116	3.2963	0.0010
m	-7.4247	0.4303	-17.2566	0.0000	-7.3306	0.5084	-14.4191	0.0000
θ^+	-2.7023	2.8558	-0.9462	0.3440	-8.2520	4.6307	-1.7820	0.0747
ω_2^+	17.9071	9.4179	1.9014	0.0573	4.0706	1.7655	2.3056	0.0211
θ^-	-6.9475	8.9687	-0.7746	0.4386	-9.5816	2.9651	-3.2314	0.0012
ω_2^-	1.8274	0.7168	2.5492	0.0108	1.6428	0.9692	1.6950	0.0901
	Gasoline DAGM - No Skew - norm				Kerosene DAGM - No Skew - norm			
α	0.0001	0.1451	0.0007	0.9994	0.0001	0.0028	0.0361	0.9712
β	0.3111	2.0949	0.1485	0.8820	0.9989	0.0496	20.1266	0.0000
m	-7.7977	0.4573	-17.0521	0.0000	-6.9367	5.9467	-1.1665	0.2434
θ^+	-4.7771	3.3354	-1.4322	0.1521	-11.2127	59.0147	-0.1900	0.8493
ω_2^+	3.2084	1.0273	3.1230	0.0018	4.0110	30.1185	0.1332	0.8941
θ^-	-13.0737	5.0015	-2.6140	0.0089	-5.2417	33.1878	-0.1579	0.8745
ω_2^-	1.7762	0.5267	3.3723	0.0007	1.8790	12.7833	0.1470	0.8831
	Gasoline DAGM - Skew - std				Kerosene DAGM - Skew - std			
α	0.0001	0.2631	0.0004	0.9997	0.0001	0.1111	0.0009	0.9993
γ	0.0314	0.1550	0.2028	0.8393	0.1020	0.0783	1.3034	0.1924
β	0.9832	0.0786	12.5042	0.0000	0.9339	0.0957	9.7618	0.0000
m	-8.0778	3.3355	-2.4217	0.0154	-6.8587	0.5654	-12.1312	0.0000
θ^+	1.2154	28.9121	0.0420	0.9665	-3.2368	2.4175	-1.3389	0.1806
ω_2^+	1.3633	0.7660	1.7797	0.0751	1.7842	1.7050	1.0465	0.2954
θ^-	-11.9126	48.2760	-0.2468	0.8051	0.1682	1.3069	0.1287	0.8976
ω_2^-	1.4286	10.0333	0.1424	0.8868	2.1376	1.0860	1.9684	0.0490
$shape$	21.4945	1.6022	13.4155	0.0000	15.3576	0.2143	71.6601	0.0000
	Gasoline DAGM - No Skew - std				Kerosene DAGM - No Skew - std			
α	0.0187	0.2452	0.0763	0.9392	0.0001	0.0381	0.0026	0.9979
β	0.9803	0.2968	3.3024	0.0010	0.5597	0.8881	0.6303	0.5285
m	-1.5573	11.6702	-0.1334	0.8938	-7.0640	0.5016	-14.0833	0.0000
θ^+	-2.6419	53.2924	-0.0496	0.9605	-9.7718	19.4694	-0.5019	0.6157
ω_2^+	1.0020	23.6236	0.0424	0.9662	3.5280	12.9879	0.2716	0.7859
θ^-	0.5795	21.9608	0.0264	0.9789	-6.4096	12.3675	-0.5183	0.6043
ω_2^-	2.0164	62.9915	0.0320	0.9745	1.7845	4.2820	0.4167	0.6769
$shape$	2.0099	0.0424	47.4200	0.0000	17.6872	12.2056	1.4491	0.1473

Code

Here it is presented the R code used for the project, but for the sake of brevity, we will include only the code related to WTI crude oil since the other 5 commodities share the same commands and procedures.

```
setwd("/Users/gianlorenzo/Desktop/FINASS/Time Series/Project/")

graphics.off()
rm(list = ls())

# libraries
library(readxl)
library(xts)
library(highfrequency)
library(tseries)
library(ggplot2)
library(fBasics)
library(rugarch)
library(rumidas)
library(FinTS)
library(MCS)
library(psych)
library(car)
library(forecast)
library(stargazer)
library(broom)
library(xtable)
###

energy_data <- read_excel("energydata_covid.xlsx", sheet = 1, skip = 4)
covid_data <- read_excel("energydata_covid.xlsx", sheet = 2, skip = 4)

energy_data <- na.omit(energy_data)
covid_data <- na.omit(covid_data)

class(energy_data)
class(covid_data)

#################### time series transformation (transform data.frame into xts)
Date_energy<-strptime(energy_data$...1, "%Y-%m-%d",tz="GMT")
energy_i<-as.xts(energy_data[,2:ncol(energy_data)],Date_energy)

range(time(energy_i))

Date_covid<-strptime(covid_data$...1, "%Y-%m-%d",tz="GMT")
covid_i<-as.xts(covid_data[,2:ncol(covid_data)],Date_covid)

range(time(covid_i))

### plot
plot(energy_i$WTI, grid.col = NA)
plot(covid_i$`Deaths USA`, grid.col = NA)

### ADF      H_0 = no stationarity
adf.test(energy_i$WTI, alternative= "stationary")
```

```
adf.test(covid_i$`Deaths USA`,alternative= "stationary")

# ACF for each series
acf(coredata(energy_i$WTI))

# PACF for each series
pacf(coredata(energy_i$WTI))

# describe (summary statistics)
describe(energy_i$WTI)
describe(covid_i$`Deaths USA`)

### log-returns ###
r_t<-makeReturns(energy_i)
N<-length(r_t$WTI)

r_t<- na.omit(r_t)

diff_covid <-makeReturns(covid_i)
L<-length(diff_covid)

diff_covid <- na.omit(diff_covid)

### Squared returns
r_t_sq <- r_t^2

### plot first differences
plot(r_t$WTI, grid.col = NA)
plot(diff_covid)

# ACF for each energy series
acf(coredata(r_t$WTI))

# PACF for returns of the energy series
pacf(coredata(r_t$WTI))

# describe (summary statistics)
describe(r_t$WTI)
describe(covid_i$`Deaths USA`)

# ADF first differences
adf.test(r_t$WTI, alternative= "stationary")
adf.test(diff_covid$`Deaths USA`,alternative= "stationary")

# ACF for squared returns of the energy series
acf(coredata(r_t_sq$WTI))

###### QQNORM
qqnorm(r_t[,1], main = "WTI")
qqline(r_t[,1], col = 2, distribution= qnorm)

# Skewness
skewness(r_t[,1]) # WTI

# Excess of Kurtosis (so kurtosis -3)
```

```r
kurtosis(r_t[,1]) # WTI

# Jarque Bera test      H_0 : Normality. If p-value < 0.05 we reject H_0
jarque.bera.test(r_t[,1]) # WTI
jarque.bera.test(diff_covid$`Deaths USA`)

#test correlation (White Noise test) H_0 : No correlation
Box.test(r_t[,1],type="Ljung-Box",lag=20)
Box.test(r_t[,1],type="Box-Pierce",lag=20)

# squared returns
Box.test(r_t_sq[,1],type="Ljung-Box",lag=20) #p-value < 2.2e-16
Box.test(r_t_sq[,1],type="Box-Pierce",lag=20) #p-value < 2.2e-16

# Heteroscedasticity test H_0 : No heteroscedasticity # p-value < 0.05 we reject H_0
ArchTest(r_t[,1], lag = 30)

#### ARCH TESTS ####
ArchTest(r_t$WTI, lags = 12)
# We have an  ARCH effect, meaning that there is presence of heteroskedasticity

#significance of mean different from zero test. H_0 : true mean is equal zero
t.test(r_t[,1]) #p-value > 0.05 we do not reject H_O
########


###### GARCH #####
# GARCH models with different distribution errors:
# Normal, T-student, Skew Normal, Skew t, GED

## FITTING
# we define the variety of GARCH models we consider
spec.comp<-list()
models<- c("sGARCH","eGARCH","gjrGARCH")
distributions<-c("norm","std","snorm","sstd","ged","sged")
spec.comp<-list()
m<-c()
d<-c()
for(m in models) {
  for(d in distributions) {
    spec.comp[[paste(m,d,sep="-")]] <-
      ugarchspec(mean.model= list(armaOrder= c(0,0)),
                 variance.model=list(model=m, garchOrder= c(1,1)),
                 distribution.model=d)
  }
}

specifications<- names(spec.comp)
specifications

#Fit WTI data into the models
fitmod.WTI<- list()
for(s in specifications) {
  fitmod.WTI[[s]] <- ugarchfit(spec= spec.comp[[s]], r_t$WTI)
}
```

```r
fitmod.WTI

########## Strcutural Breaks
library(strucchange)

sigma.s = fitmod.WTI$`sGARCH-norm`@fit$sigma
sigma.s = as.ts(sigma.s, start = first(time(r_t)), end = last(time(r_t)), frequency = 1)

test2 <- Fstats(sigma.s~1) #Gets a sequence of fstatistics for all possible
# break points within the middle 70% of myts1
sigma.fs <- test2$Fstats #These are the fstats
bp.sigma <- breakpoints(sigma.s~1) #Gets the breakpoint based on the F-stats
plot(sigma.s) #plots the series myts1
lines(bp.sigma) #plots the break date implied by the sup F test
bd.sigma <- breakdates(bp.sigma) #Obtains the implied break data (2018.35,
# referring to day 128 (0.35*365 = day number))
sctest(test2) #Obtains a p-value for the implied breakpoint
ci.sigma <- confint(bp.sigma) #95% CI for the location break date
plot(sigma.s, main = "Kerosene", ylab = "volatility")
lines(ci.sigma) #This shows the interval around the estimated break date
lines(bp.sigma, col = "green")


# Perform the rolling forecasts

#### WTI ROLL ####
WTI.roll<-list()
for(s in specifications) {
  WTI.roll[[s]]<- ugarchroll(spec=spec.comp[[s]], data= r_t$WTI,
                             forecast.length= 80,
                             refit.every=1, refit.window= "moving")
}

WTI.roll
summary(WTI.roll)


# Extracting VaR from Rolling forecast #
####### VaR WTI #######
VaR.WTI=list()
for(s in specifications) {
  VaR.WTI[[s]]<-as.data.frame(WTI.roll[[s]], which="VaR")
}
VaR.WTI
str(VaR.WTI)



###### GARCH MIDAS #####
# Estimate GARCH models with an observed variable at weekly frequency (Covid deaths)

#### Time lapse between dates (for mv_into_mat)
# diff_time <- as.numeric(difftime(first(r_t_est), first(covid_i), units = "day"))
# (diff_time < 0 | ((diff_time - K * num_obs_k) < 0))
difftime(strptime("2020-08-25", format = "%Y-%m-%d"), strptime("2020-03-02", format = "%Y-%m-%d"), units="days")
```

```r

r_t_est <- r_t['2020-08-15/2021-06-04']                                          # K = 24
K<- 24 # number of lagged realizations entering the long-run equation
covid_mv<-mv_into_mat(r_t_est$WTI,diff_covid,K=K,"weekly")

dim(covid_mv)
length(r_t_est$WTI)

# out of sample
n_oos = 80

# Fit the data into the models                                                    # WTI
fit.gm_WTI <- c()
models<- c("GM","DAGM")
distributions<-c("norm","std")
skew<-c("YES", "NO")
m<-c()
d<-c()
s<-c()
for(m in models) {
  for(d in distributions) {
    for(s in skew) {
      fit.gm_WTI[[paste(m,s,d,sep="-")]] <- ugmfit(model=m, skew=s,distribution=d,
                                                   daily_ret=r_t_est$WTI,mv_m=covid_mv, K=K,
                                                   out_of_sample=n_oos)
    }
  }
}

specifications2<- names(fit.gm_WTI)
specifications2

fit.gm_oos$`GM-YES-norm`$est_vol_oos
summary.rumidas(fit.gm_oos$`GM-YES-norm`)


#############################
#      VaR Garch MIDAS     #
############################

###### Rolling windows for VaR Garch Midas
library(PerformanceAnalytics)

VaR.calc <- function(x){
  x * (qdist(distribution = "norm" , p = 0.01))
}

roll.cond_vol = c()
for(j in specifications2){
  roll.cond_vol[[j]] <- apply.rolling(fit.gm_WTI[[j]]$est_vol_oos, width = 1, trim = TRUE, gap = 1, by = 1, FUN = "VaR.calc")
}


# We recreate the VaR list
# VaR Garch Midas = conditional vol * quantile distribution
```

```r
# Creating VaR object list in R as that coming from the ugarchroll

gm_std = specifications2[c(3, 4, 7, 8)]
gm_norm = specifications2[- c(3, 4, 7, 8)]

#### VaR for GM models with normal dist ####                                                    # WTI
# VaR 1% GM norm
VaR.WTI.gm.99 = c()
for(j in gm_norm){
  VaR.WTI.gm.99[[j]] = as.data.frame(fit.gm_WTI[[j]]$est_vol_oos * (qdist(distribution = "norm" , p = 0.01)))
}
VaR.WTI.gm.99

# rename V1 into alpha(1%)
for(j in gm_norm){
  names(VaR.WTI.gm.99[[j]]) = "alpha(1%)"
}

# VaR 5% GM norm
VaR.WTI.gm.95 = c()
for(j in gm_norm){
  VaR.WTI.gm.95[[j]] = as.data.frame(fit.gm_WTI[[j]]$est_vol_oos * (qdist(distribution = "norm" , p = 0.05)))
}
VaR.WTI.gm.95

for(j in gm_norm){
  VaR.WTI.gm.99[[j]]$`alpha(5%)` <- VaR.WTI.gm.95[[j]]$V1
}

for(j in gm_norm){
  VaR.WTI.gm.99[[j]]$realized <- as.double(tail(r_t_est$WTI, n_oos))
}

# rename VaR.WTI.gm.99 into VaR.WTI.gm_norm
VaR.WTI.gm_norm = VaR.WTI.gm.99
rm(VaR.WTI.gm.99, VaR.WTI.gm.95)


#### VaR for GM models with std dist ####
# vector with shapes for std
shapes <- c(fit.gm_WTI[["GM-YES-std"]]$rob_coef_mat[7,1],
            fit.gm_WTI[["GM-NO-std"]]$rob_coef_mat[6,1],
            fit.gm_WTI[["DAGM-YES-std"]]$rob_coef_mat[9,1],
            fit.gm_WTI[["DAGM-NO-std"]]$rob_coef_mat[8,1])

# VaR 1% GM std
VaR.WTI.gm_std.99= c()
for(ss in shapes){
  for(j in gm_std){
    VaR.WTI.gm_std.99[[j]] = as.data.frame(fit.gm_WTI[[j]]$est_vol_oos * (qdist(distribution = "std", shape = ss, p = 0.01)))
  }
}
VaR.WTI.gm_std.99

# rename V1 into alpha(1%)
```

```
for(j in gm_std){
  names(VaR.WTI.gm_std.99[[j]]) = "alpha(1%)"
}

# VaR 5% GM std
VaR.WTI.gm_std.95 = c()
for(ss in shapes){
  for(j in gm_std){
    VaR.WTI.gm_std.95[[j]] = as.data.frame(fit.gm_WTI[[j]]$est_vol_oos * (qdist(distribution = "std", shape = ss, p = 0.05)))
  }
}
VaR.WTI.gm_std.95

for(j in gm_std){
  VaR.WTI.gm_std.99[[j]]$`alpha(5%)` <- VaR.WTI.gm_std.95[[j]]$V1
}

for(j in gm_std){
  VaR.WTI.gm_std.99[[j]]$realized <- as.double(tail(r_t_est$WTI, n_oos))
}

# rename VaR.WTI.gm_std.99 into VaR.WTI.gm
VaR.WTI.gm_std = VaR.WTI.gm_std.99
rm(VaR.WTI.gm_std.99, VaR.WTI.gm_std.95)

# Combine the 2 VaR gm list into 1
VaR.WTI.gm = c(VaR.WTI.gm_norm, VaR.WTI.gm_std)
rm(VaR.WTI.gm_norm, VaR.WTI.gm_std) # remove obejcts we don't need anymore

# Merging the 2 VaR obeject, rember to extract VaR from ugarchroll
VaR.WTI = c(VaR.WTI, VaR.WTI.gm)
rm(VaR.WTI.gm)
#####



##############################
### Backtesting VaR
#############################

spec = c(specifications, specifications2)

################## WTI KUPIEC, CHRISTOFFSEN, DQ PER WTI 5 E 1 CONF.  ################
# install.packages("segMGarch")
library(segMGarch)

# VaR Backtest 99%                                                                # WTI
VaR.WTI_backtest.99 = matrix(NA, length(spec), 2)
for(s in 1:length(spec)) {
  back = VaRTest(actual = VaR.WTI[[s]]$realized, VaR = VaR.WTI[[s]]$`alpha(1%)`, alpha = .01, conf.level=0.99)
  VaR.WTI_backtest.99[s,] = c(back$uc.LRp, back$cc.LRp)
}
rownames(VaR.WTI_backtest.99) = spec
colnames(VaR.WTI_backtest.99) = names(back)[c(6, 11)]
VaR.WTI_backtest.99
```

```

# VaR Backtest 95%
VaR.WTI_backtest.95 = matrix(NA, length(spec), 2)
for(s in 1:length(spec)) {
  back = VaRTest(actual = VaR.WTI[[s]]$realized, VaR = VaR.WTI[[s]]$`alpha(5%)`, alpha = .05, conf.level=0.95)
  VaR.WTI_backtest.95[s,] = c(back$uc.LRp, back$cc.LRp)
}
rownames(VaR.WTI_backtest.95) = spec
colnames(VaR.WTI_backtest.95) = names(back)[c(6, 11)]
VaR.WTI_backtest.95


VaR.WTI_back = matrix(data = NA, nrow = length(spec), ncol = 4)
rownames(VaR.WTI_back ) = spec
colnames(VaR.WTI_back) = names(back)[c(6, 11, 6, 11)]

VaR.WTI_back[,1] = VaR.WTI_backtest.99[,1]
VaR.WTI_back[,2] = VaR.WTI_backtest.99[,2]
VaR.WTI_back[,3] = VaR.WTI_backtest.95[,1]
VaR.WTI_back[,4] = VaR.WTI_backtest.95[,2]
stargazer(VaR.WTI_back, summary = F)
xtable(VaR.WTI_back)


###############
#Report given information about the VaR backtest with VaR 1%  ## SOLO PER CONFRONTO CON RUGARCH ##
rep.WTI<- list()
for(j in spec) {
  rep.WTI[[j]]<- as.data.frame(report(WTI.roll[[j]], type= "VaR", VaR.alpha= 0.01, conf.level=0.99))
}
###############

DQ.WTI.99 = list()
for (s in spec) {
  DQ.WTI.99[[s]]<- DQtest(r_t$WTI, VaR= VaR.WTI[[s]]$`alpha(1%)`, VaR_level= 0.99)
}

DQ.WTI.95 = list()
for (s in spec) {
  DQ.WTI.95[[s]]<- DQtest(r_t$WTI, VaR= VaR.WTI[[s]]$`alpha(1%)`, VaR_level= 0.95)
}

# alternative
library(GAS)
BacktestVaR(VaR.WTI$`sGARCH-norm`$realized, VaR.WTI$`sGARCH-norm`$`alpha(1%)`, 0.01, Lags = 4)



##############################
### LOSS & MCS
#############################

###### LOSS WTI #########                                                                # WTI
# Store di tutti gli alpha (Per LOSS, vedi sotto)
WTI.eval_01 <- list()
```

```
for(j in spec){
  WTI.eval_01[[j]] <- VaR.WTI[[j]]$`alpha(1%)`
}


WTI.eval_05 <- list()
for(j in spec){
  WTI.eval_05[[j]] <- VaR.WTI[[j]]$`alpha(5%)`
}

# Loss
WTI.Loss.01 <- do.call(cbind, lapply(spec,
                                     function(s) { LossVaR(tau= 0.01, realized= tail(r_t$WTI, n_oos), # /100
                                                           evaluated= WTI.eval_01[[s]])})) # /100
colnames(WTI.Loss.01) = spec
WTI.Loss.01

WTI.Loss.05<- do.call(cbind, lapply(spec,
                                    function(s) { LossVaR(tau= 0.05, realized= tail(r_t$WTI, n_oos), # /100
                                                          evaluated= WTI.eval_05[[s]])})) # /100
colnames(WTI.Loss.05)<- spec
WTI.Loss.05

##### MCS PROCEDURES ######
SSM.WTI<- MCSprocedure(Loss= WTI.Loss.01, alpha=0.01, B=5000,
                       statistic= "Tmax")
SSM.WTI

stargazer(SSM.WTI@show)
xtable(SSM.WTI@show)


SSM5.WTI<- MCSprocedure(Loss= WTI.Loss.05, alpha=0.05, B=5000,
                        statistic= "Tmax")
stargazer(SSM5.WTI@show)
xtable(SSM5.WTI@show, digits = 3)
######



##############################
### VaR aggregation
#############################

## VaR Aggregation 99% with Exponential weights
t.stat<-SSM.WTI@show[,2]
t.stat
t.stat<- as.numeric(t.stat)
t.stat

#Denominator
dif<- c()
delta_s<- c()
for(s in 1:length(t.stat)) {
  for(j in 1:length(t.stat)) {
```

```
    dif[j] <- (t.stat[j] - t.stat[s])
  }
  delta_s[s]<- min(dif)
}
exp.delta_s <- exp(delta_s)
denominator <- sum(exp.delta_s)


#Smallest t-statistic
i.1 <- which.min(t.stat)
i.1

#Numerator
numerator <- c()
for(i in 1:length(t.stat)) {
  numerator[i]<- (t.stat[i.1] - t.stat[i])
}
numerator <- exp(numerator)

weights <- numerator/denominator
weights

sum(weights)  #it should be equal one

#Apply weights to VaR 1%
exp.weight.VaR1<- VaR.WTI[[1]]$`alpha(1%)`*weights[1]+
  VaR.WTI[[2]]$`alpha(1%)`*weights[2]+
  VaR.WTI[[3]]$`alpha(1%)`*weights[3]+
  VaR.WTI[[4]]$`alpha(1%)`*weights[4]+
  VaR.WTI[[5]]$`alpha(1%)`*weights[5]+
  VaR.WTI[[6]]$`alpha(1%)`*weights[6]+
  VaR.WTI[[7]]$`alpha(1%)`*weights[7]+
  VaR.WTI[[8]]$`alpha(1%)`*weights[8]+
  VaR.WTI[[9]]$`alpha(1%)`*weights[9]+
  VaR.WTI[[10]]$`alpha(1%)`*weights[10]+
  VaR.WTI[[11]]$`alpha(1%)`*weights[11]+
  VaR.WTI[[12]]$`alpha(1%)`*weights[12]+
  VaR.WTI[[13]]$`alpha(1%)`*weights[13]+
  VaR.WTI[[14]]$`alpha(1%)`*weights[14]+
  VaR.WTI[[15]]$`alpha(1%)`*weights[15]+
  VaR.WTI[[16]]$`alpha(1%)`*weights[16]+
  VaR.WTI[[17]]$`alpha(1%)`*weights[17]+
  VaR.WTI[[18]]$`alpha(1%)`*weights[18]+
  VaR.WTI[[19]]$`alpha(1%)`*weights[19]+
  VaR.WTI[[20]]$`alpha(1%)`*weights[20]+
  VaR.WTI[[21]]$`alpha(1%)`*weights[21]+
  VaR.WTI[[22]]$`alpha(1%)`*weights[22]+
  VaR.WTI[[23]]$`alpha(1%)`*weights[23]+
  VaR.WTI[[24]]$`alpha(1%)`*weights[24]+
  VaR.WTI[[25]]$`alpha(1%)`*weights[25]+
  VaR.WTI[[26]]$`alpha(1%)`*weights[26]

exp.weight.VaR1
plot.ts(exp.weight.VaR1)
```

```
VaRTest(actual = tail(r_t$WTI, n_oos), VaR= exp.weight.VaR1, alpha = 0.01, conf.level = 0.99)
DQtest(tail(r_t$WTI, n_oos), VaR = exp.weight.VaR1, VaR_level = 0.99)
BacktestVaR(tail(r_t$WTI, n_oos), exp.weight.VaR1, 0.01, Lags = 4)


#Uniform weights 99%
weights = rep(1/length(t.stat), length(t.stat))

uniform.VaR1 <- VaR.WTI[[1]]$`alpha(1%)`*weights[1]+
  VaR.WTI[[2]]$`alpha(1%)`*weights[2]+
  VaR.WTI[[3]]$`alpha(1%)`*weights[3]+
  VaR.WTI[[4]]$`alpha(1%)`*weights[4]+
  VaR.WTI[[5]]$`alpha(1%)`*weights[5]+
  VaR.WTI[[6]]$`alpha(1%)`*weights[6]+
  VaR.WTI[[7]]$`alpha(1%)`*weights[7]+
  VaR.WTI[[8]]$`alpha(1%)`*weights[8]+
  VaR.WTI[[9]]$`alpha(1%)`*weights[9]+
  VaR.WTI[[10]]$`alpha(1%)`*weights[10]+
  VaR.WTI[[11]]$`alpha(1%)`*weights[11]+
  VaR.WTI[[12]]$`alpha(1%)`*weights[12]+
  VaR.WTI[[13]]$`alpha(1%)`*weights[13]+
  VaR.WTI[[14]]$`alpha(1%)`*weights[14]+
  VaR.WTI[[15]]$`alpha(1%)`*weights[15]+
  VaR.WTI[[16]]$`alpha(1%)`*weights[16]+
  VaR.WTI[[17]]$`alpha(1%)`*weights[17]+
  VaR.WTI[[18]]$`alpha(1%)`*weights[18]+
  VaR.WTI[[19]]$`alpha(1%)`*weights[19]+
  VaR.WTI[[20]]$`alpha(1%)`*weights[20]+
  VaR.WTI[[21]]$`alpha(1%)`*weights[21]+
  VaR.WTI[[22]]$`alpha(1%)`*weights[22]+
  VaR.WTI[[23]]$`alpha(1%)`*weights[23]+
  VaR.WTI[[24]]$`alpha(1%)`*weights[24]+
  VaR.WTI[[25]]$`alpha(1%)`*weights[25]+
  VaR.WTI[[26]]$`alpha(1%)`*weights[26]

uniform.VaR1
plot.ts(uniform.VaR1)

VaRTest(actual = tail(r_t$WTI, n_oos), VaR= uniform.VaR1, alpha = 0.01, conf.level = 0.99)
DQtest(tail(r_t$WTI, n_oos), VaR = uniform.VaR1, VaR_level = 0.99)
BacktestVaR(tail(r_t$WTI, n_oos), uniform.VaR1, 0.01, Lags = 4)


## VaR Aggregation 95%
#Exponential weights
t.stat<-SSM5.WTI@show[,2]
t.stat<- as.numeric(t.stat)

#Denominator
dif<- c()
delta_s<- c()
for(s in 1:length(t.stat)) {
  for(j in 1:length(t.stat)) {
    dif[j] <- (t.stat[j] - t.stat[s])
  }
```

```
  delta_s[s]<- min(dif)
}
exp.delta_s <- exp(delta_s)
denominator <- sum(exp.delta_s)


#Smallest t-statistic
i.5 <- which.min(t.stat)
i.5

#Numerator
numerator <- c()
for(i in 1:length(t.stat)) {
  numerator[i]<- (t.stat[i.5] - t.stat[i])
}
numerator <- exp(numerator)

weights <- numerator/denominator
weights

sum(weights)  #it should be equal one

#Apply weights to VaR 5%
exp.weight.VaR5<- VaR.WTI[[1]]$`alpha(1%)`*weights[1]+
  VaR.WTI[[2]]$`alpha(5%)`*weights[2]+
  VaR.WTI[[3]]$`alpha(5%)`*weights[3]+
  VaR.WTI[[4]]$`alpha(5%)`*weights[4]+
  VaR.WTI[[5]]$`alpha(5%)`*weights[5]+
  VaR.WTI[[6]]$`alpha(5%)`*weights[6]+
  VaR.WTI[[7]]$`alpha(5%)`*weights[7]+
  VaR.WTI[[8]]$`alpha(5%)`*weights[8]+
  VaR.WTI[[9]]$`alpha(5%)`*weights[9]+
  VaR.WTI[[10]]$`alpha(5%)`*weights[10]+
  VaR.WTI[[11]]$`alpha(5%)`*weights[11]+
  VaR.WTI[[12]]$`alpha(5%)`*weights[12]+
  VaR.WTI[[13]]$`alpha(5%)`*weights[13]+
  VaR.WTI[[14]]$`alpha(5%)`*weights[14]+
  VaR.WTI[[15]]$`alpha(5%)`*weights[15]+
  VaR.WTI[[16]]$`alpha(5%)`*weights[16]+
  VaR.WTI[[17]]$`alpha(5%)`*weights[17]+
  VaR.WTI[[18]]$`alpha(5%)`*weights[18]+
  VaR.WTI[[19]]$`alpha(5%)`*weights[19]+
  VaR.WTI[[20]]$`alpha(5%)`*weights[20]+
  VaR.WTI[[21]]$`alpha(5%)`*weights[21]+
  VaR.WTI[[22]]$`alpha(5%)`*weights[22]+
  VaR.WTI[[23]]$`alpha(5%)`*weights[23]+
  VaR.WTI[[24]]$`alpha(5%)`*weights[24]+
  VaR.WTI[[25]]$`alpha(5%)`*weights[25]

exp.weight.VaR5
plot.ts(exp.weight.VaR5)

VaRTest(actual = tail(r_t$WTI, n_oos), VaR= exp.weight.VaR5, alpha = 0.05, conf.level = 0.95)
DQtest(tail(r_t$WTI, n_oos), VaR = exp.weight.VaR5, VaR_level = 0.95)
BacktestVaR(tail(r_t$WTI, n_oos), exp.weight.VaR5, 0.05, Lags = 4)
```

```r

#Uniform weights 95%
weights = rep(1/length(t.stat), length(t.stat))

uniform.VaR5 <- VaR.WTI[[1]]$`alpha(1%)`*weights[1]+
  VaR.WTI[[2]]$`alpha(5%)`*weights[2]+
  VaR.WTI[[3]]$`alpha(5%)`*weights[3]+
  VaR.WTI[[4]]$`alpha(5%)`*weights[4]+
  VaR.WTI[[5]]$`alpha(5%)`*weights[5]+
  VaR.WTI[[6]]$`alpha(5%)`*weights[6]+
  VaR.WTI[[7]]$`alpha(5%)`*weights[7]+
  VaR.WTI[[8]]$`alpha(5%)`*weights[8]+
  VaR.WTI[[9]]$`alpha(5%)`*weights[9]+
  VaR.WTI[[10]]$`alpha(5%)`*weights[10]+
  VaR.WTI[[11]]$`alpha(5%)`*weights[11]+
  VaR.WTI[[12]]$`alpha(5%)`*weights[12]+
  VaR.WTI[[13]]$`alpha(5%)`*weights[13]+
  VaR.WTI[[14]]$`alpha(5%)`*weights[14]+
  VaR.WTI[[15]]$`alpha(5%)`*weights[15]+
  VaR.WTI[[16]]$`alpha(5%)`*weights[16]+
  VaR.WTI[[17]]$`alpha(5%)`*weights[17]+
  VaR.WTI[[18]]$`alpha(5%)`*weights[18]+
  VaR.WTI[[19]]$`alpha(5%)`*weights[19]+
  VaR.WTI[[20]]$`alpha(5%)`*weights[20]+
  VaR.WTI[[21]]$`alpha(5%)`*weights[21]+
  VaR.WTI[[22]]$`alpha(5%)`*weights[22]+
  VaR.WTI[[23]]$`alpha(5%)`*weights[23]+
  VaR.WTI[[24]]$`alpha(5%)`*weights[24]+
  VaR.WTI[[25]]$`alpha(5%)`*weights[25]

uniform.VaR5
plot.ts(uniform.VaR5)

VaRTest(actual = tail(r_t$WTI, n_oos), VaR= uniform.VaR5, alpha = 0.05, conf.level = 0.95)
DQtest(tail(r_t$WTI, n_oos), VaR = uniform.VaR5, VaR_level = 0.95)
BacktestVaR(tail(r_t$WTI, n_oos), uniform.VaR5, 0.05, Lags = 4)


###### VaR Violations Grafico
library(ggplot2)

realized = VaR.WTI$`sGARCH-norm`$realized

#### VaR Violation comparison
qplot(y = exp.weight.VaR1 , x = 1:80 , geom = 'line', color='black') +
  geom_point(aes(x = 1:80 , y = realized, color = as.factor(realized < exp.weight.VaR1)) , size = 2)+
  geom_line(aes(y=  exp.weight.VaR5, x=1:80, color='blue'))+
  geom_point(aes(x = 1:80 , y = realized, color = as.factor(realized < exp.weight.VaR5)) , size = 2)+
  scale_color_manual(values = c('black','blue', 'grey','red'),
                     labels=c("VaR Exp Aggregation 99%","VaR Exp Aggregation 95%","Realized" , "Violations")) +
  labs(y = 'Daily Returns' , x = 'Year 2021',
       title = 'VaR Aggregation Exponential Weights for 95% vs 99% confidence level') + theme_light() +
  theme(legend.position = "bottom")+
  labs(color = "Legend:")
```