

A note on a sports league scheduling problem

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Abstract

Sports league scheduling is a difficult task in the general case. In this short note, we report two improvements to an existing enumerative search algorithm for a NP-hard sports league scheduling problem known as “prob026” in CSPLib. These improvements are based on additional rules to constraint and accelerate the enumeration process. The proposed approach is able to find a solution (schedule) for all prob026 instances for a number T of teams ranging from 12 to 70, including several T values for which a solution is reported for the first time.

Keywords: sports league scheduling, prob026 in CSPLib, balanced tournament design, enumerative search, constraints

1. Introduction

The sports league scheduling problem studied in this note, called “prob026” in CSPLib [1] and also known as the “balanced tournament design” problem in combinatorial design theory [2, pages 238-241], is a NP-hard problem [3] that seems to be first introduced in [4]:

- There are $T = 2n$ teams (i.e., T even). The season lasts $W = T - 1$ weeks. Weeks are partitioned into $P = T/2$ slots called “periods” or “stadiums”. Each week, one match is scheduled in every period;
- c_H constraint: All teams play each other exactly once (Half competition);
- c_P constraint: No team plays more than twice in a Period. This constraint may be motivated by the equal distribution of stadiums to teams;
- c_W constraint: Every team plays exactly one game in every Week of the season, i.e., all teams are different in a week.

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The problem then is to schedule a tournament with respect to these definitions and constraints. A solution to prob026 is a complete assignment of $D = \{(t, t'), 1 \leq t < t' \leq T\}$ items (couples of *teams*) to variables of $X = \{x = \langle p, w \rangle, 1 \leq p \leq P, 1 \leq w \leq W\}$ (couples of *periods* and *weeks*) verifying the constraint set $C = \{c_{\mathcal{H}}, c_{\mathcal{P}}, c_{\mathcal{W}}\}$, $\langle p, w \rangle = (t, t')$ meaning that team t meets team t' in period p and week w . Thus, a solution can be conveniently represented by a $P \times W$ sized table, whose items are integer couples (t, t') , see Table 1 for an example of a valid schedule for $T = 8$. For $T = 70$ teams, this represents a problem with 2 415 variables and 2 415 values per variable. There are $T(T-1)/2$ matches to be scheduled. A valid schedule can be thought of as a particular permutation of these matches. So, for T teams, the search space size is $[T(T-1)/2]!$.

Table 1: A valid schedule for 8 teams.

Periods	Weeks						
	1	2	3	4	5	6	7
1	1,2	6,8	2,5	4,5	4,7	3,8	1,7
2	3,7	5,7	3,4	1,8	5,6	2,4	2,6
3	4,6	1,4	7,8	3,6	2,8	1,5	3,5
4	5,8	2,3	1,6	2,7	1,3	6,7	4,8

Direct construction methods exist when $(T-1) \bmod 3 \neq 0$ [5, 6] or $T/2$ is odd [7, 8]. However, finding a solution (schedule) in the general case for any arbitrary T remains a highly challenging task. Indeed, to our knowledge, the best performing search algorithm [9] can solve all the instances for T up to 50, but only some cases when $50 < T \leq 70$. Other representative solution approaches include integer programming [10] (limited to $T \leq 12$), transformation into the SAT problem [11] ($T \leq 20$), distributed approach ($T \leq 28$ according to [12]), constraint programming [13] and tabu search [14] ($T \leq 40$).

In this paper, we present two improvements to the **Enumerative Algorithm (EnASS)** proposed in [9]. With the proposed enhancements, **all** the instances for $12 \leq T \leq 70$ can now be solved.

We provide in the next section a brief recall of the original **EnASS** method. We show then in the following sections a new **EnASS** variant that solves **all** instances up to $T = 60$ (including the problematic $T \bmod 4 = 0$ cases) and another new variant that solves all the $12 \leq T \leq 70$ instances.

2. A brief recall of the EnASS algorithm

EnASS starts with a complete \bar{s} conflicting assignment. \bar{s} is built, in linear-time complexity, to satisfy the $c_{\mathcal{W}}$ and $c_{\mathcal{H}}$ constraints (thanks to patterned one-factorization [2, page 662]). At this stage, the remaining $c_{\mathcal{P}}$ constraint is not verified in \bar{s} , see Table 2 where team 8 appears more than twice in the 4th period.

Table 2: Initial conflicting \bar{s} schedule for 8 teams.

Periods	Weeks						
	1	2	3	4	5	6	7
1	1,2	2,3	3,4	4,5	5,6	6,7	1,7
2	3,7	1,4	2,5	3,6	4,7	1,5	2,6
3	4,6	5,7	1,6	2,7	1,3	2,4	3,5
4	5,8	6,8	7,8	1,8	2,8	3,8	4,8

Algorithm 1 EnASS: An overview.

Require: Two periods (p and \bar{p}) and a week (w)

- 1: **if** $p = P + 1$ **then** // A solution is obtained since all periods are filled and valid according to \mathcal{R}
 - 2: **return true**
 - 3: **end if**
 - 4: **if** $w = w_l + 1$ **then** // Period p is filled and valid according to \mathcal{R} , try to fill next period
 - 5: **return** EnASS($p + 1, w_f, 1$)
 - 6: **end if**
 - 7: **if** $\bar{p} = P + 1$ **then** // Backtrack since no match from week w in \bar{s} can be scheduled in period p of week w without violating \mathcal{R}
 - 8: **return false**
 - 9: **end if**
 - 10: **if** $\exists 1 \leq p' < p : \langle p', w \rangle = \bar{s}(\bar{p}, w)$ **then** // The $\bar{s}(\bar{p}, w)$ match is already scheduled, try next match
 - 11: **return** EnASS($p, w, \bar{p} + 1$)
 - 12: **end if**
 - 13: $\langle p, w \rangle \leftarrow \bar{s}(\bar{p}, w)$ // Schedule the $\bar{s}(\bar{p}, w)$ match in period p of week w
 - 14: **if** \mathcal{R} is locally verified and EnASS($p, w + 1, 1$) = **true** **then** // The previous assignment and next calls lead to a solution
 - 15: **return true**
 - 16: **end if**
 - 17: // From this point, \mathcal{R} is locally violated or next calls lead to a failure
 - 18: Undo step 13 // Backtrack
 - 19: **return** EnASS($p, w, \bar{p} + 1$) // Try next value for $\langle p, w \rangle$
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Roughly speaking, **EnASS** uses \bar{s} to search for a valid tournament by filling a $P \times W$ table (initially empty) row by row, see Algorithm 1 where w_f and w_l are the first and last weeks **EnASS** considers when filling any period p ($1 \leq w_f < w_l \leq W$), $\bar{s}\langle \bar{p}, w \rangle$ is the match in \bar{s} scheduled in period \bar{p} and week w , and \mathcal{R} is a set of properties (or “Requirements”) that (partial or full) solutions must verify. **EnASS** admits three integer parameters: p and w specify which $\langle p, w \rangle$ variable is currently considered, \bar{p} specifies the value assignment tried (see step 13). The function returns TRUE if a solution has been found or FALSE otherwise. Backtracks are sometimes performed in the latter case. **EnASS** is called first, after the \bar{s} initialization, with $p = 1, w = w_f$ and $\bar{p} = 1$ meaning that it tries to fill the slot in the first period of week w_f with the $\bar{s}\langle 1, w_f \rangle$ match.

The basic **EnASS** skeleton presented in Algorithm 1 solves prob026 only up to $T = 12$ when the \mathcal{R} set is restricted to $\{c_P\}$ while considering the first week as invariant with respect to \bar{s} (i.e., $\forall 1 \leq p \leq P, \langle p, 1 \rangle = \bar{s}\langle p, 1 \rangle$) with $w_f = 2$ (since the first week is invariant) and $w_l = W$. Note that making the first week invariant helps to avoid some evident symmetries mentioned in [9, see Sect. 4 and 5.3].

To tackle larger-size problems, several **EnASS** variants were considered in [9]. **EnASS**₀ solved prob026 up to $T = 32$, except the $T = 24$ case, including in \mathcal{R} an implicit property (called “ c_D ” in [9]) of all prob026 solutions: $\mathcal{R}_0 = \{c_P, c_D\}$. The c_D property was not originally mentioned in the seminal definition of the problem [4] and seems to be first introduced in [8]. **EnASS**₁, derived from **EnASS**₀ by further including an “implied” requirement (r_{\Rightarrow}), solved all instances up to $T = 50$: $\mathcal{R}_1 = \{c_P, c_D, r_{\Rightarrow}\}$. Finally, **EnASS**₂ solved some cases (when $T \bmod 4 \neq 0$) for T up to 70 with two additional invariants (r_I and r_V): $\mathcal{R}_2 = \{c_P, c_D, r_{\Rightarrow}, r_I, r_V\}$.

3. Solving all instances of prob026 up to $T = 60$

The rule r'_{\Rightarrow} used to solve **all** prob026 instances up to $T = 60$ resembles the original r_{\Rightarrow} requirement introduced in [9, Sect. 7]. Like r_{\Rightarrow} , r'_{\Rightarrow} fixes more than one variable (two exactly, to be more precise) when exploring a new branch in the search tree. The difference between r_{\Rightarrow} and the new r'_{\Rightarrow} rule is the weeks that are concerned: While r_{\Rightarrow} connects any week $w_f \leq w \leq P$ to week $T - w + 1$, the r'_{\Rightarrow} constraint links any week $1 \leq w \leq P - 1$ together with week $W - w + 1$. More formally, $\forall 1 \leq w \leq P - 1, r'_{\Rightarrow}(p, w) \Leftrightarrow \langle p, w \rangle = \bar{s}\langle \bar{p}, w \rangle \Rightarrow \langle p, W - w + 1 \rangle = \bar{s}\langle \bar{p}, W - w + 1 \rangle$.

This leads to **EnASS**₃ which comes from the **EnASS**₁ algorithm from [9] by replacing in \mathcal{R}_1 the r_{\Rightarrow} requirement with the new r'_{\Rightarrow} rule: $\mathcal{R}_3 = \{c_P, c_D, r'_{\Rightarrow}\}$. Like for **EnASS**₁, step 13 in the basic **EnASS** description (see Algorithm 1) may be adapted since one additional variable has now to be instantiated and w_l has to be set to $P - 1$ before running **EnASS**₃. Steps 4–6 have also to be modified since, when $w = w_l + 1$, the P week is not yet filled (so, the p period is not entirely filled either). Table 1 in Sect. 1 shows an example of a solution found by **EnASS**₃ for $T = 8$: For instance, scheduling the $(3, 4)$ match from week 3

in period 2 forces the (5, 6) match from week 5 ($5 = 7 - 3 + 1$) to be also in period 2.

In Table 3, we show for $6 \leq T \leq 50$ comparisons of our new **EnASS₃** variant (as well as another new **EnASS₄** variant discussed in the next section), against the **EnASS₁** algorithm which solves all the instances for $T \leq 50$ within 3 hours per T value. The reported statistics include execution times (in seconds in all tables) and number of backtracks (columns labeled “|BT|”) needed to find a first solution.

In Table 4, we show for $52 \leq T \leq 70$ comparisons between the new variant **EnASS₃** (and **EnASS₄**) and the **EnASS₂** algorithm from [9] which solves *some* instances with $T \leq 70$ where $T \bmod 4 \neq 0$. “–” marks in the “Time” (respectively “|BT|”) columns indicate that the method found no solution within 3 hours (resp. that |BT| exceeds the maximal integer value authorized by the compiler/system, i.e., 4 294 967 295). All **EnASS** variants were coded in C and all computational results were obtained on an Intel PIV processor (2 Ghz) Linux station with 2 Gb RAM.

Table 3: Solving all prob026 instances up to $T = 50$.

T	EnASS₁ [9]		EnASS₃ (Sect. 3)		EnASS₄ (Sect. 4)	
	Time	BT	Time	BT	Time	BT
6	< 1	6	< 1	1	–	–
8	< 1	16	< 1	6	< 1	5
10	< 1	715	< 1	350	–	–
12	< 1	86	< 1	25	< 1	111
14	< 1	451	< 1	65	< 1	125
16	< 1	557	< 1	713	< 1	560
18	< 1	1 099	< 1	772	< 1	465
20	< 1	2 811	< 1	708	< 1	227
22	< 1	11 615	< 1	1 142	< 1	3 237
24	< 1	12 623	< 1	5 332	< 1	736
26	< 1	37 708	< 1	5 313	< 1	2 311
28	< 1	35 530	< 1	16 365	< 1	85 315
30	< 1	650 811	< 1	49 620	< 1	68 033
32	< 1	332 306	< 1	91 094	< 1	22 407
34	< 1	1 342 216	< 1	131 169	< 1	21 696
36	< 1	2 160 102	< 1	524 491	< 1	248 184
38	5.34	13 469 359	< 1	763 317	< 1	83 636
40	6.25	16 393 039	1.70	7 335 775	< 1	220 480
42	107.69	256 686 929	2.74	11 575 637	< 1	612 423
44	876.91	1 944 525 360	19.80	79 587 812	1.02	2 489 017
46	1 573.31	3 565 703 651	10.22	38 865 293	1.59	3 430 033
48	542.79	1 231 902 706	1 112.55	4 289 081 568	5.69	12 080 931
50	6 418.52	–	4 018.20	–	17.38	34 639 665

From Table 3–4, one observes that **EnASS₃** solves more prob026 instances than **EnASS₁** within 3 hours. Indeed, while **EnASS₁** is limited to $T \leq 50$, **EnASS₃**

Table 4: Solving all prob026 instances when $50 < T \leq 70$.

T	EnASS ₂ [9]		EnASS ₃ (Sect. 3)		EnASS ₄ (Sect. 4)	
	Time	BT	Time	BT	Time	BT
52	–	–	377.84	1 345 460 512	50.11	101 432 823
54	10.59	29 767 940	763.08	2 802 487 580	101.74	196 808 595
56	–	–	2 552.65	–	334.26	753 747 164
58	269.88	827 655 311	13 715.33	–	878.96	1 851 547 682
60	–	–	198 250.44	–	2 364.47	–
62	279.38	494 071 117	–	–	9 866.51	–
64	–	–	–	–	32 386.67	–
66	7 508.51	1 614 038 658	–	–	85 989.73	–
68	–	–	–	–	518 194.31	–
70	8 985.05	–	–	–	1 512 574.41	–

finds solutions for T up to 56 in at most 67 minutes (see the $T = 50$ case in Table 3). Moreover, except two cases ($T \in \{16, 48\}$), the number of backtracks required to find a solution is much smaller for **EnASS**₃ than for **EnASS**₁.

Table 4 shows that the comparison between **EnASS**₃ and **EnASS**₂ is somewhat mitigated. Indeed, **EnASS**₃ is able to find solutions for **all** T up to 56 within 3 hours while **EnASS**₂ solves the instances up to $T = 70$, but only when $T \bmod 4 \neq 0$. For the cases that are solved by both **EnASS**₃ and **EnASS**₂, **EnASS**₂ finds a solution much faster. On the other hand, **EnASS**₃ finds solutions for $T \in \{52, 56, 60\}$ for which **EnASS**₂ fails. Finally, one notices that **EnASS**₃ requires much more time to solve the $T \in \{58, 60\}$ instances (about 55 hours for $T = 60$).

4. Solving all prob026 instances when $50 < T \leq 70$

The rule r'_I used to solve **all** prob026 instances for $50 < T \leq 70$ is similar to the original r_I requirement introduced in [9, Sect. 7]. Indeed, like r_I , r'_I inverses two weeks and keeps them invariant during the search. The only difference between r_I and the new r'_I rule is the weeks that are concerned: While r_I considers weeks 2 and W , the r'_I constraint inverses weeks 2 and $W - 1$. More formally, $\forall w \in \{2, W - 1\}, r'_I(w) \Leftrightarrow \forall 1 \leq p \leq P, \langle p, w \rangle = \bar{s} \langle P - p + 1, w \rangle$.

This leads to **EnASS**₄ which comes from **EnASS**₃ by adding in \mathcal{R}_3 the new r'_I rule: $\mathcal{R}_4 = \{c_P, c_D, r'_\Rightarrow, r'_I\}$. Since the first two weeks are now invariant (and the last two due to r'_\Rightarrow), w_f has to be set to 3 before running **EnASS**₄. Table 1 in Sect. 1 shows an example of a solution found by **EnASS**₄ (and **EnASS**₃) for $T = 8$: For instance, the first match in week 2 is $\bar{s} \langle 4 - 1 + 1, 2 \rangle$, i.e., $\langle 1, 2 \rangle = (6, 8)$.

The computational performance of the **EnASS**₄ variant is provided in Table 3 for $6 \leq T \leq 50$ and in Table 4 for $50 < T \leq 70$ ¹. One notices that **EnASS**₄ is faster than **EnASS**₃ and **EnASS**₁ (see the “|BT|” columns in Table 3) to solve instances when $T \geq 12$ (and for $T = 8$), except for the $T \in \{12, 14, 16, 22, 28, 30\}$ cases. Furthermore, within 3 hours per T value, **EnASS**₄ is capable of solving larger instances (up to $T = 62$, see Table 4) than **EnASS**₁ ($T \leq 50$) and **EnASS**₃ ($T \leq 56$). While **EnASS**₂ solves only some instances for $50 < T \leq 70$ (those verifying $T \bmod 4 \neq 0$, see Table 4), **EnASS**₄ finds solutions for all these cases. This is achieved within 3 hours for T up to 62, but larger instances can require more execution time (about 18 days for $T = 70$). Finally, note that adding the new r'_I rule excludes solutions for $T \in \{6, 10\}$.

5. Conclusion

We provided in this short note two enhancements to an **Enumerative Algorithm for Sports Scheduling (EnASS)** previously proposed in [9]. These enhancements are based on additional properties (identified in *some* solutions) as new constraints to reduce the search tree constructed by the algorithm. With these

¹The first solution found by **EnASS**₄ for $50 < T \leq 70$ is available on-line from <http://www.info.univ-angers.fr/pub/hamiez/EnASS4/Sol152-70.html>.

enhancements, all prob026 instances with $T \leq 70$ can be solved for the first time. Since the main idea behind the enhancements is to add refined requirement rules in the **EnASS** method, we expect that the method can be further improved to solve prob026 instances for $T > 70$.

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