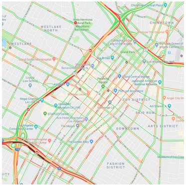
A Network Optimization Framework for the Control of Traffic Dynamics and Intersection Signaling

Gianluca Bianchin, Fabio Pasqualetti

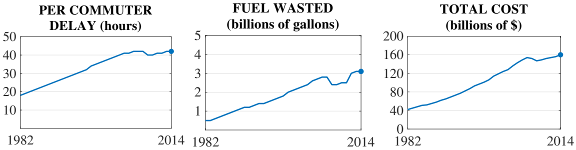
Department of Mechanical Engineering
University of California, Riverside

IEEE Conference on Decision and Control
December 17, 2018 | Miami Beach, FL, USA

How Do We Operate Modern Traffic Networks?



- Transportation systems: vital for urban development
- Huge economical and societal impact
- Undergoing increasing travel demand, but limited infrastructure grow



Source: 2015 Urban Mobility Storecard, Texas A&M Transportation Institute

Solution: more-efficient operation of the infrastructure

Network-Wide Control of Signalized Intersections

Traffic lights: Manhattan, NY

Control parameters: traffic lights

minimize (network congestion)

subject to (traffic conditions)
(network interconnection)

- Network-wide control is a massive optimization problem
  $\Rightarrow$ MPC-based, limited optimization horizons
- Traditional approaches: control at single-intersection level
  $\Rightarrow$ SCOOT, RHODES, OPAC, Max-Pressure

Tractable models of overall interconnection can provide insights to overcome suboptimalities of distributed (local) controllers

Network-Wide Control of Signalized Intersections

Traffic lights: Manhattan, NY

Control parameters: traffic lights

minimize (network congestion)

subject to (traffic conditions)

(network interconnection)

- Network-wide control is a massive optimization problem
  $\Rightarrow$ MPC-based, limited optimization horizons
- Traditional approaches: control at single-intersection level
  $\Rightarrow$ SCOOT, RHODES, OPAC, Max-Pressure

Tractable models of overall interconnection can provide insights to overcome suboptimalities of distributed (local) controllers

Network-Wide Control of Signalized Intersections

Traffic lights: Manhattan, NY

Control parameters: traffic lights

minimize (network congestion)

subject to (traffic conditions)

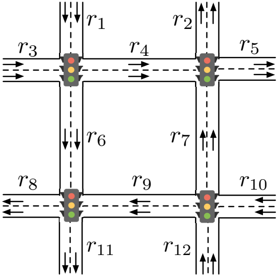
(network interconnection)

- Network-wide control is a massive optimization problem
  $\Rightarrow$ MPC-based, limited optimization horizons
- Traditional approaches: control at single-intersection level
  $\Rightarrow$ SCOOT, RHODES, OPAC, Max-Pressure

Tractable models of overall interconnection can provide insights to overcome suboptimalities of distributed (local) controllers

Model of Traffic Network

- Network $\mathcal{N} = (\mathcal{R}, \mathcal{I})$
  - $\mathcal{R} = \{r_1, \dots, r_{n_r}\}$ one-way roads
  - $\mathcal{I} = \{\mathcal{I}_1, \dots, \mathcal{I}_{n_\mathcal{I}}\}$ intersections
- Inflows enter at roads $\mathcal{S} \subseteq \mathcal{R}$
- Outflows exit at roads $\mathcal{D} \subseteq \mathcal{R}$



Standard Connectivity Assumption

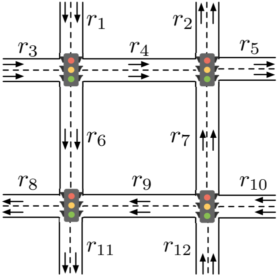
There exists at least one path in $\mathcal{N}$ from every r_i to a destination $r_j \in \mathcal{D}$

Exogenous Flows Assumption

Exogenous inflows and outflows are not known a priori

Model of Traffic Network

- Network $\mathcal{N} = (\mathcal{R}, \mathcal{I})$
  - $\mathcal{R} = \{r_1, \ldots, r_{n_r}\}$ one-way roads
  - $\mathcal{I} = \{\mathcal{I}_1, \ldots, \mathcal{I}_{n_{\mathcal{I}}}\}$ intersections
- Inflows enter at roads $\mathcal{S} \subseteq \mathcal{R}$
- Outflows exit at roads $\mathcal{D} \subseteq \mathcal{R}$



Standard Connectivity Assumption

There exists at least one path in $\mathcal{N}$ from every r_i to a destination $r_j \in \mathcal{D}$

Exogenous Flows Assumption

Exogenous inflows and outflows are not known a priori

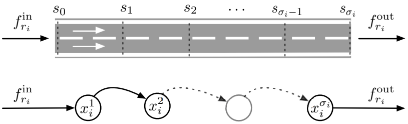
Model of Roads

Start from hydrodynamic model in free flow:

$$\frac{\partial \rho}{\partial t} + \frac{\partial f}{\partial s} = 0$$

$\Rightarrow$

Discretize in space



If speed is constant along the road (regimes of free flow):

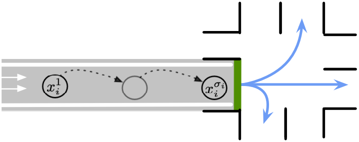
$$\begin{bmatrix} \dot{x}_i^1 \\ \dot{x}_i^2 \\ \vdots \\ \dot{x}_i^{\sigma_i} \end{bmatrix} = \frac{\gamma_i}{h} \begin{bmatrix} -1 & & & \\ 1 & -1 & & \\ & \ddots & \ddots & \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x_i^1 \\ x_i^2 \\ \vdots \\ x_i^{\sigma_i} \end{bmatrix} + \begin{bmatrix} f_{r_i}^{\text{in}} \\ 0 \\ \vdots \\ -f_{r_i}^{\text{out}} \end{bmatrix}$$

- x_i^k density of segment k, in road r_i
- γ_i (average) flow speed
- h spatial discretization step

Model of Interconnection Flows

Road outflow

$$f_{r_i}^{\text{out}} = \sum_{r_k} s(r_k, r_i, t)\, c(r_i, r_k)\, x_k^{\sigma_k}$$

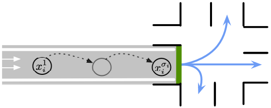


- $x_k^{\sigma_k}$ density at intersection proximity
- $c(r_i, r_k)$ models routing ratios, transmission rates, enforces conservation of flows
- $s(r_k, r_i, t) \in \{0, 1\}$ green splits

Model of Interconnection Flows

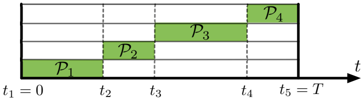
Road outflow

$$f^{\text{out}}_{r_i} = \sum_{r_k} s(r_k, r_i, t)\, c(r_i, r_k)\, x_k^{\sigma_k}$$



- $x_k^{\sigma_k}$ density at intersection proximity
- $c(r_i, r_k)$ models routing ratios, transmission rate
- $s(r_k, r_i, t) \in \{0, 1\}$ green splits

Alternate right of way



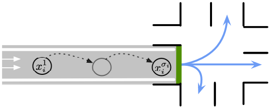
Control of intersections

Green splits $s(r_k, r_i, t)$ are the design parameters

Model of Interconnection Flows

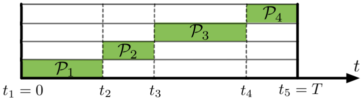
Road outflow

$$f_{r_i}^{\text{out}} = \sum_{r_k} s(r_k, r_i, t)\, c(r_i, r_k)\, x_k^{\sigma_k}$$



- $x_k^{\sigma_k}$ density at intersection proximity
- $c(r_i, r_k)$ models routing ratios, transmission rate
- $s(r_k, r_i, t) \in \{0, 1\}$ green splits

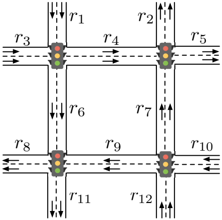
Alternate right of way



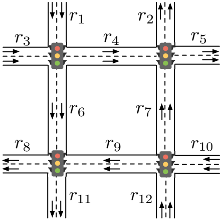
Control of intersections

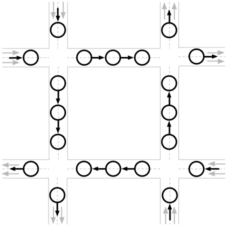
Green splits $s(r_k, r_i, t)$ are the design parameters

Network Model

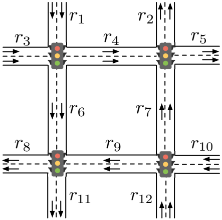


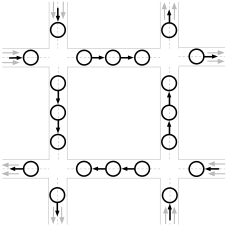
Network Model





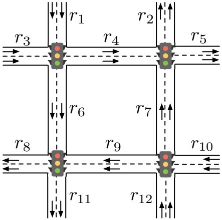
Network Model

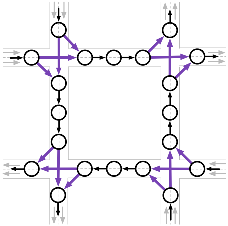




$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n_\mathsf{r}} \end{bmatrix} = \begin{bmatrix} A_{11} & & & \\ & A_{22} & & \\ & & \ddots & \\ & & & A_{n_\mathsf{r} n_\mathsf{r}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n_\mathsf{r}} \end{bmatrix}
$$

Network Model

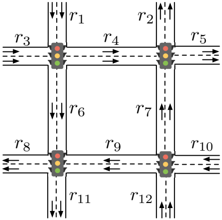


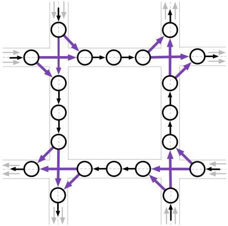


$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n_\mathrm{r}} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n_\mathrm{r}} \\ A_{21} & A_{22} & \ddots & A_{2n_\mathrm{r}} \\ \vdots & \ddots & \ddots & \vdots \\ A_{n_\mathrm{r}1} & A_{n_\mathrm{r}2} & \cdots & A_{n_\mathrm{r}n_\mathrm{r}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n_\mathrm{r}} \end{bmatrix}$$

Binary green split functions
$\Rightarrow$
switching dynamics

Network Model





$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n_r} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n_r} \\ A_{21} & A_{22} & \ddots & A_{2n_r} \\ \vdots & \ddots & \ddots & \vdots \\ A_{n_r1} & A_{n_r2} & \cdots & A_{n_rn_r} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n_r} \end{bmatrix}$$

Binary green split functions
$\Rightarrow$
switching dynamics

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n_r} \end{bmatrix} = \begin{bmatrix} e_{\sigma_1}^{\mathsf{T}} & \dots & 0 \\ \vdots & \ddots & \\ 0 & \dots & e_{\sigma_{n_r}}^{\mathsf{T}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n_r} \end{bmatrix}$$

Congestion due to intersections
$=$
density of downstream segments

Problem Formulation

Network-wide intersections control

- Given a certain network state x_0, unknown inflows
- Determine green splits that minimize congestion due to intersections

Evacuate the network as fast as possible, final condition is empty system

$$\begin{aligned} \min_{s(r_i, r_k, t)} \quad & \int_0^\infty \|y\|_2^2 \, dt \\ \text{s.t.} \quad & \dot{x} = A_{s(r_i, r_k, t)} x \\ & y = Cx \\ & x(0) = x_0 \\ & s(r_i, r_k, t) \text{ is a feasible set of green splits} \end{aligned}$$

- Measurements enter the optimization updating x_0
- We optimize over $[0, \infty]$ and adopt a "receding horizon" approach

Problem Formulation

Network-wide intersections control

- Given a certain network state x_0, unknown inflows
- Determine green splits that minimize congestion due to intersections

Evacuate the network as fast as possible, final condition is empty system

$$\begin{aligned}
\min_{s(r_i, r_k, t)} \quad & \int_0^\infty \|y\|_2^2 \, dt \\
\text{s.t.} \quad & \dot{x} = A_{s(r_i, r_k, t)} x \\
& y = Cx \\
& x(0) = x_0 \\
& s(r_i, r_k, t) \text{ is a feasible set of green splits}
\end{aligned}$$

- Measurements enter the optimization updating x_0
- We optimize over $[0, \infty]$ and adopt a "receding horizon" approach

Problem Formulation

Network-wide intersections control

- Given a certain network state x_0, unknown inflows
- Determine green splits that minimize congestion due to intersections

Evacuate the network as fast as possible, final condition is empty system

$$
\begin{aligned}
\min_{s(r_i,r_k,t)} \quad & \int_0^\infty \|y\|_2^2 \, dt \\
\text{s.t.} \quad & \dot{x} = A_{s(r_i,r_k,t)} x \\
& y = Cx \\
& x(0) = x_0 \\
& s(r_i, r_k, t) \text{ is a feasible set of green splits}
\end{aligned}
$$

- Measurements enter the optimization updating x_0
- We optimize over $[0, \infty]$ and adopt a “receding horizon” approach

Solving the Optimization

(1) Handling Switching Dynamics

$$\dot{x} = A_{s(r_i,r_k,t)}x$$
$$s(r_i,r_k,t) = \text{piecewise constant}$$

Define $\{d_1,\ldots,d_m\}$ durations, where $s(r_i,r_k,t) = \text{constant}$

$$\dot{x}_{\mathsf{av}} = A_{\mathsf{av}}x_{\mathsf{av}}$$
$$A_{\mathsf{av}} = \frac{1}{T}(A_1d_1 + \cdots + A_md_m)$$

"Average" network dynamics
T = signals period

(1) Handling Switching Dynamics

$$\dot{x} = A_{s(r_i, r_k, t)} x$$
$$s(r_i, r_k, t) = \text{piecewise constant}$$

Define $\{d_1, \ldots, d_m\}$ durations,
where $s(r_i, r_k, t) = \text{constant}$

$$\dot{x}_{\mathsf{av}} = A_{\mathsf{av}} x_{\mathsf{av}}$$
$$A_{\mathsf{av}} = \frac{1}{T}(A_1 d_1 + \cdots + A_m d_m)$$

"Average" network dynamics
$T = \text{signals period}$

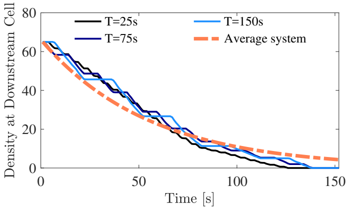
(1) Handling Switching Dynamics

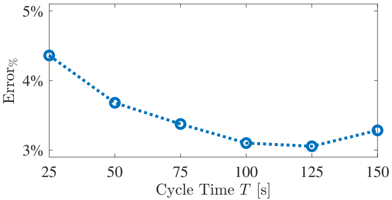
$$\dot{x} = A_{s(r_i, r_k, t)} x$$
$$s(r_i, r_k, t) = \text{piecewise constant}$$

Define $\{d_1, \dots, d_m\}$ durations, where $s(r_i, r_k, t) = \text{constant}$

$$\dot{x}_{\mathsf{av}} = A_{\mathsf{av}} x_{\mathsf{av}}$$
$$A_{\mathsf{av}} = \frac{1}{T}(A_1 d_1 + \dots + A_m d_m)$$

"Average" network dynamics
T = signals period





Network mode durations $\{d_1, \dots, d_m\}$ are the new design parameters

The Optimization Problem on Average Dynamics

$$
\begin{aligned}
\min_{d_1,\dots,d_m} \quad & \int_0^\infty \|y_{\mathsf{av}}\|_2^2 \, dt \\
\text{subject to} \quad & \left.\begin{aligned}
&\dot{x}_{\mathsf{av}} = A_{\mathsf{av}} x_{\mathsf{av}} \\
&y_{\mathsf{av}} = C_{\mathsf{av}} x_{\mathsf{av}} \\
&A_{\mathsf{av}} = \frac{1}{T}\left(d_1 A_1 + \cdots + d_m A_m\right)
\end{aligned}\right\} \text{Avg. network dynamics} \\
& x_{\mathsf{av}}(0) = x_0 \\
& \left.\begin{aligned}
&T = d_1 + \cdots + d_m \\
&d_i \geq 0 \quad i \in \{1, \dots, m\}
\end{aligned}\right\} \text{Feasible splits}
\end{aligned}
$$

The optimization will return a set of duration for the green splits that are compatible with the cycle time T

(2) Relationship to Controllability Metrics

1. Controllability Gramian for a dynamical system $\dot{x} = Ax + Bu$ is

$$\mathcal{W}(A, B) = \int_0^\infty e^{At} B B^\mathsf{T} e^{A^\mathsf{T} t} \, dt$$

Quantitative measure of the degree of controllability of the dynamical sys.

2. Equivalent optimization problem:

$$\begin{aligned}
\min_{d_1,\dots,d_m} \quad & \mathrm{Trace}\left(C_{\mathsf{av}} \, \mathcal{W}(A_{\mathsf{av}}, x_0) \, C_{\mathsf{av}}^\mathsf{T} \right) \\
\text{subject to} \quad & A_{\mathsf{av}} = \frac{1}{T} (d_1 A_1 + \dots + d_m A_m) \\
& T = d_1 + \dots + d_m \\
& d_i \geq 0, \quad i \in \{1, \dots, m\}
\end{aligned}$$

Optimal split durations minimize a controllability metric (trace of weighted controllability Gramian)

(2) Relationship to Controllability Metrics

1. Controllability Gramian for a dynamical system $\dot{x} = Ax + Bu$ is

$$\mathcal{W}(A, B) = \int_0^\infty e^{At} B B^\mathsf{T} e^{A^\mathsf{T} t} \, dt$$

Quantitative measure of the degree of controllability of the dynamical sys.

2. Equivalent optimization problem:

$$\begin{aligned}
\min_{d_1, \ldots, d_m} \quad & \mathrm{Trace}\left(C_{\mathsf{av}} \, \mathcal{W}(A_{\mathsf{av}}, x_0) \, C_{\mathsf{av}}^\mathsf{T} \right) \\
\text{subject to} \quad & A_{\mathsf{av}} = \frac{1}{T} (d_1 A_1 + \cdots + d_m A_m) \\
& T = d_1 + \cdots + d_m \\
& d_i \geq 0, \quad i \in \{1, \ldots, m\}
\end{aligned}$$

Optimal split durations minimize a controllability metric (trace of weighted controllability Gramian)

Optimizing Network Controllability

$$\min_{d_1,\dots,d_m} \quad \mathrm{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}}, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right)$$

Difficulties:

- A_{av} and $\mathcal{W}(A_{\mathsf{av}}, x_0)$ are related by the (**nonlinear**) relation

$$A_{\mathsf{av}}\ \mathcal{W} + \mathcal{W}\ A_{\mathsf{av}}^{\mathsf{T}} = -x_0 x_0^{\mathsf{T}}$$

- Similar problems: consider stability $\alpha(A_{\mathsf{av}})$
  - Captures steady state rates (not transient overshoots)
  - Nonconvex in A_{av} and "very hard to optimize"

J. Vanbiervliet, B. Vandereycken, W. Michiels, S. Vandewalle, and M. Diehl, "The smoothed spectral abscissa for robust stability optimization," in *SIAM Journal on Optimization*, vol. 20, no. 1, 2009.

1. For a certain A_{av} the associated network performance is

$$\text{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}}, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right) = 1/\epsilon$$

2. Now assume we desire better performance $\bar{\epsilon} > \epsilon$

3. $\bar{\epsilon}$ can be obtained by making the system "faster":
   - We shift: $A_{\mathsf{av}} \to A_{\mathsf{av}} - sI$ (s variable)
   - Then: $\text{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}} - sI, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right) = 1/\bar{\epsilon}$

If we can "change" A_{av} so that $s = 0$, then that network will have performance cost $1/\bar{\epsilon}$

4. $s := \tilde{\alpha}(\bar{\epsilon}, A_{\mathsf{av}})$ "smoothed spectral abscissa"

1. For a certain A_{av} the associated network performance is

$$\mathrm{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}}, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right) = 1/\epsilon$$

2. Now assume we desire better performance $\bar{\epsilon} > \epsilon$

3. $\bar{\epsilon}$ can be obtained by making the system "faster":
   - We shift: $A_{\mathsf{av}} \to A_{\mathsf{av}} - sI$ (s variable)
   - Then: $\mathrm{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}} - sI, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right) = 1/\bar{\epsilon}$

If we can "change" A_{av} so that $s = 0$, then that network will have performance cost $1/\bar{\epsilon}$

4. $s := \tilde{\alpha}(\bar{\epsilon}, A_{\mathsf{av}})$ "smoothed spectral abscissa"

1. For a certain A_{av} the associated network performance is

$$\mathrm{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}}, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right) = 1/\epsilon$$

2. Now assume we desire better performance $\bar{\epsilon} > \epsilon$
3. $\bar{\epsilon}$ can be obtained by making the system "faster":
   - We shift: $A_{\mathsf{av}} \to A_{\mathsf{av}} - sI$ (s variable)
   - Then: $\mathrm{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}} - sI, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right) = 1/\bar{\epsilon}$

If we can "change" A_{av} so that $s = 0$, then that network will have performance cost $1/\bar{\epsilon}$

4. $s := \tilde{\alpha}(\bar{\epsilon}, A_{\mathsf{av}})$ "smoothed spectral abscissa"

1. For a certain A_{av} the associated network performance is

$$\text{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}}, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right) = 1/\epsilon$$

2. Now assume we desire better performance $\bar{\epsilon} > \epsilon$

3. $\bar{\epsilon}$ can be obtained by making the system "faster":
   - We shift: $A_{\mathsf{av}} \to A_{\mathsf{av}} - sI$ (s variable)
   - Then: $\text{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}} - sI, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right) = 1/\bar{\epsilon}$

If we can "change" A_{av} so that $s = 0$, then that network will have performance cost $1/\bar{\epsilon}$

4. $s := \tilde{\alpha}(\bar{\epsilon}, A_{\mathsf{av}})$ "smoothed spectral abscissa"

1. For a certain A_{av} the associated network performance is

$$\text{Trace}\left(C_{\text{av}}\ \mathcal{W}(A_{\text{av}}, x_0)\ C_{\text{av}}^{\mathsf{T}}\right) = 1/\epsilon$$

2. Now assume we desire better performance $\bar{\epsilon} > \epsilon$
3. $\bar{\epsilon}$ can be obtained by making the system "faster":
   - We shift: $A_{\text{av}} \to A_{\text{av}} - sI$ (s variable)
   - Then: $\text{Trace}\left(C_{\text{av}}\ \mathcal{W}(A_{\text{av}} - sI, x_0)\ C_{\text{av}}^{\mathsf{T}}\right) = 1/\bar{\epsilon}$

> If we can "change" A_{av} so that $s = 0$, then that network will have performance cost $1/\bar{\epsilon}$

4. $s := \tilde{\alpha}(\bar{\epsilon}, A_{\text{av}})$ "smoothed spectral abscissa"

Optimizing Controllability Metrics: Numerical Methods

$$
\begin{aligned}
\min_{d_1,\dots,d_m} \quad & |\alpha_{\bar{\epsilon}}(A_{\mathsf{av}})| \\
\text{subject to} \quad & A_{\mathsf{av}} = \frac{1}{T}\left(d_1 A_1 + \cdots + d_m A_m\right) \\
& T = d_1 + \cdots + d_m \\
& d_i \geq 0, \quad i \in \{1,\dots,m\}
\end{aligned}
$$

Gradient Descent

Descent direction for cost function

$$
\frac{\partial \alpha_\epsilon(A_{\mathsf{av}})}{\partial d} = \mathrm{vec}\left(\frac{QP}{\mathrm{Trace}\,(QP)}\right)\frac{\partial A_{\mathsf{av}}}{\partial d}
$$

where P, Q solve the Lyapunov equations

$$
XP + PX^{\mathsf{T}} + x_0 x_0^{\mathsf{T}} = 0, \qquad X^{\mathsf{T}}Q + QX + C_{\mathsf{av}}C_{\mathsf{av}}^{\mathsf{T}} = 0
$$

and $X = A_{\mathsf{av}} - \alpha_\epsilon(A_{\mathsf{av}})I$

Optimizing Controllability Metrics: Numerical Methods

$$
\begin{aligned}
\min_{d_1,\ldots,d_m} \quad & |\alpha_{\bar{\epsilon}}(A_{\mathsf{av}})| \\
\text{subject to} \quad & A_{\mathsf{av}} = \frac{1}{T}\left(d_1 A_1 + \cdots + d_m A_m\right) \\
& T = d_1 + \cdots + d_m \\
& d_i \geq 0, \quad i \in \{1, \ldots, m\}
\end{aligned}
$$

Gradient Descent

Descent direction for cost function

$$\frac{\partial \alpha_\epsilon(A_{\mathsf{av}})}{\partial d} = \operatorname{vec}\left(\frac{QP}{\operatorname{Trace}(QP)}\right)\frac{\partial A_{\mathsf{av}}}{\partial d}$$

where P, Q solve the Lyapunov equations

$$XP + PX^{\mathsf{T}} + x_0 x_0^{\mathsf{T}} = 0, \qquad X^{\mathsf{T}}Q + QX + C_{\mathsf{av}}C_{\mathsf{av}}^{\mathsf{T}} = 0$$

and $X = A_{\mathsf{av}} - \alpha_\epsilon(A_{\mathsf{av}})I$

Optimizing Controllability via Gradient-Descent

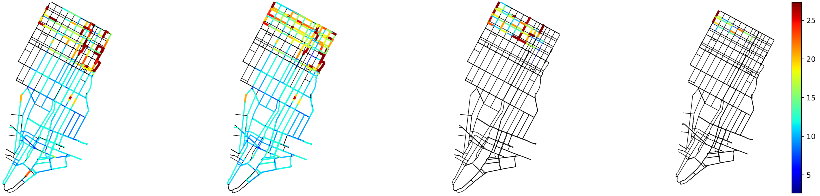
$$\begin{aligned} \min_{d_1,\dots,d_m} \quad & |\alpha_{\bar\epsilon}(A_{\text{av}})| \\ \text{s.t.} \quad & A_{\text{av}} = \frac{1}{T}(d_1 A_1 + \cdots + d_m A_m) \\ & T = d_1 + \cdots + d_m \\ & d_i \geq 0, \quad i \in \{1,\dots,m\} \end{aligned}$$

$+$

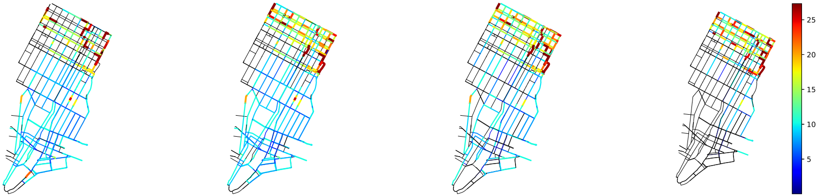
Line search over $\bar\epsilon$

Input: Matrix C_{av}, vector x_0, scalars ξ, μ
Output: $\{d_1^*, \dots d_m^*, \epsilon^*\}$
Initialize: $d^{(0)}$, $\bar\epsilon = 0$, $k = 1$
while $\tilde\alpha_{\bar\epsilon}^{(k)} = 0$ **do**
  repeat
    Compute $\tilde\alpha_{\bar\epsilon}^{(k)}$;
    Solve for P and Q:
    $(A_{\text{av}}^{(k)} - \alpha_{\bar\epsilon}^{(k)} I)P + P(A_{\text{av}}^{(k)} - \alpha_{\bar\epsilon}^{(k)} I)^\mathsf{T} + x_0 x_0^\mathsf{T} = 0$;
    $(A_{\text{av}}^{(k)} - \alpha_{\bar\epsilon}^{(k)} I)^\mathsf{T} Q + Q(A_{\text{av}}^{(k)} - \alpha_{\bar\epsilon}^{(k)} I) + C_{\text{av}} C_{\text{av}}^\mathsf{T} = 0$;
    $\frac{\partial \alpha_{\bar\epsilon}^{(k)}}{\partial d} \leftarrow \frac{QP}{\text{Trace}(QP)}$;
    $\nabla \leftarrow \tilde\alpha_{\bar\epsilon} \frac{\partial \alpha_{\bar\epsilon}^{(k)}}{\partial d}$;
    Compute projection matrix $\mathcal{P}^{(k)}$;
    $d^{(k)} \leftarrow d^{(k)} - \mu\, \mathcal{P}^{(k)} \nabla$;
    $A_{\text{av}}^{(k)} \leftarrow \frac{1}{T}(d_1 A_1 + \cdots + d_m A_m)$;
    $k \leftarrow k + 1$;
  until $\mathcal{P}^{(k)} \nabla = 0$;
  $\bar\epsilon \leftarrow \bar\epsilon + \xi$;
end
return d;

Test Case: Manhattan, NY

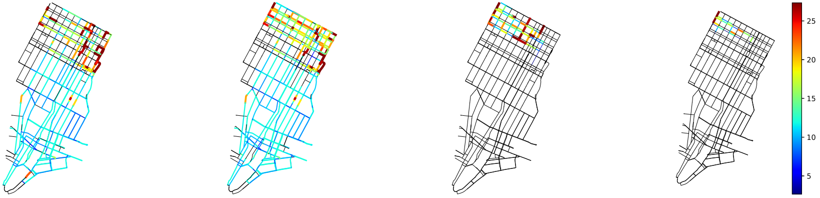


Top: green split timing from network-wide optimization

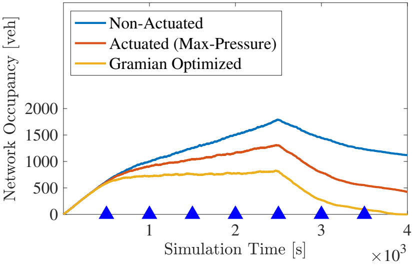


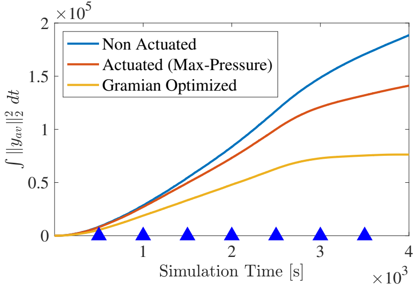
Bottom: intersection-level distributed control

Test Case: Manhattan, NY



Top: green split timing from network-wide optimization





Summary

Motivation: extremely challenging to perform network-wide control

Approximate model: tradeoff between complexity and accuracy

- Give insights to overcome suboptimalities of distributed control

Approximations lead to optimization that performs very well in practice

- Manhattan, NY
- SUMO online code: github.com/gianlucaBianchin

Directions:

- Include effects of congestion
- Use framework for design other control variables (speed limits)

Summary

Motivation: extremely challenging to perform network-wide control

Approximate model: tradeoff between complexity and accuracy

- Give insights to overcome suboptimalities of distributed control

Approximations lead to optimization that performs very well in practice

- Manhattan, NY
- SUMO online code: github.com/gianlucaBianchin

Directions:

- Include effects of congestion
- Use framework for design other control variables (speed limits)

Summary

Motivation: extremely challenging to perform network-wide control

Approximate model: tradeoff between complexity and accuracy

- Give insights to overcome suboptimalities of distributed control

Approximations lead to optimization that performs very well in practice

- Manhattan, NY
- SUMO online code: github.com/gianlucaBianchin

Directions:

- Include effects of congestion
- Use framework for design other control variables (speed limits)

A Network Optimization Framework for the Control of Traffic Dynamics and Intersection Signaling

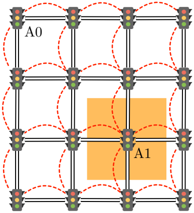
Gianluca Bianchin, Fabio Pasqualetti

Department of Mechanical Engineering
University of California, Riverside

IEEE Conference on Decision and Control
December 17, 2018 | Miami Beach, FL, USA

Distributed Computation of Descent Direction

- We want to solve: $\Lambda X + X\Lambda^{\mathsf{T}} + D = 0$ where $\Lambda = \Lambda_1 + \cdots + \Lambda_\nu$
- each agent i knows Λ_i



Each agent i constructs a local estimate $\hat{X}_i^{(k)}$ by performing the following operations in order:

1. Receive $\hat{w}_j^{(k)}$ and $K_j^{(k)}$ from neighbor j;
2. $\hat{w}_i^{(k+1)} = \hat{w}_i^{(k)} + [K_i^{(k)} \; 0][K_i^{(k)} \; K_j^{(k)}]^\dagger(\hat{w}_i^{(k)} - \hat{w}_j^{(k)})$;
3. $K_i^{(k+1)} = \mathrm{Basis}(\mathsf{Im}(K_i^{(k)}) \cap \mathsf{Im}(K_i^{(k)}))$;
4. Transmit $\hat{w}_i^{(k+1)}$ and $K_i^{(k+1)}$ to neighbor j;

Controllability of Traffic Networks in Free Flow

$$\min_{d_1,\ldots,d_m} \qquad \text{Trace}\left(C_{\mathsf{av}}\ \mathcal{W}(A_{\mathsf{av}}, x_0)\ C_{\mathsf{av}}^{\mathsf{T}}\right)$$

$\Rightarrow$ Cost function is finite if $\{d_1, \ldots, d_m\}$ leads to A_{av} that is Hurwitz

(Thm) Stability of optimal solutions = Graph-theoretic property

If there exists a path in $\mathcal{N}$ between any source $s \in \mathcal{S}$ and some destination $d \in \mathcal{D}$, then there exists $\{d_1, \ldots, d_m\}$:

$$\alpha(A_{\mathsf{av}}) < 0$$

Spectral abscissa of A_{av}

$$\alpha(A_{\mathsf{av}}) := \sup\{\Re(s) : s \in \mathbb{C}, \det(sI - A_{\mathsf{av}}) = 0\}$$