Stability and Robustness of Traffic Networks with App-Informed Vehicle Routing

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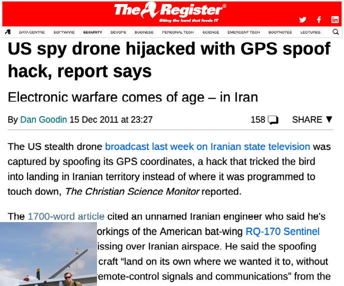
Robustness in Transportation (and More)

Robustness = operate efficiently despite perturbations

Non-nominal conditions and component failures

Changes in user behavior

Malicious attacks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US spy drone hijacked with GPS spoof hack, report says

Electronic warfare comes of age – in Iran

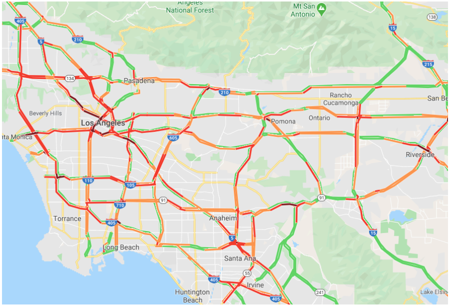
By Dan Goodin 15 Dec 2011 at 23:27 158 SHARE ▼

The US stealth drone broadcast last week on Iranian state television was captured by spoofing its GPS coordinates, a hack that tricked the bird into landing in Iranian territory instead of where it was programmed to touch down, *The Christian Science Monitor* reported.

The 1700-word article cited an unnamed Iranian engineer who said he's
orkings of the American bat-wing RQ-170 Sentinel
issing over Iranian airspace. He said the spoofing
craft "land on its own where we wanted it to, without
emote-control signals and communications" from the

Things can go terribly bad if (design) $\not\Rightarrow$ (robustness)

Transportation and Needs for Robustness



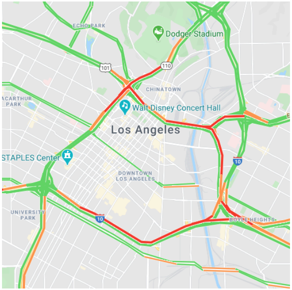
(source: Google)

- Transportation: 9% US GDP
- Congestion: wastes 3B Gallons of fuel every year
- Large-scale, complex, rich nonlinear dynamics

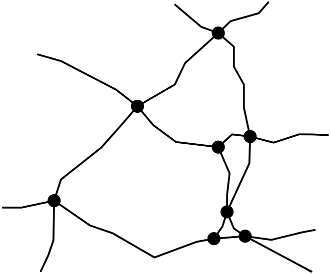
Robustness is extremely relevant problem

- 100 years old and operating at capacity limits
- Things can go really bad (Atlanta 2014, Beijing 2010, Houston 2005, NY 2001)

Modeling Traffic



(source: Google)



Traffic network topology:

(1) Highways  each transfers traffic flows

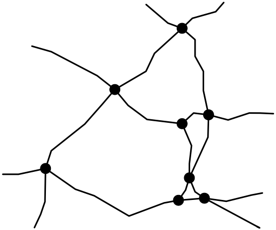
(2) Junctions ● exchange traffic flows between highways

(1) **Highways**

Modeled as vehicle accumulators

$\dot{x}_\ell = f_\ell^{\text{in}}(x) - f_\ell^{\text{out}}(x_\ell)$

Classical models: each highway has a single flow variable

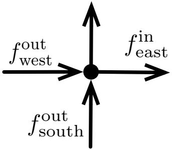


(2) **Junctions**

Transfer flows between highways

$f_{\text{east}}^{\text{in}} = r_{\text{west}\to\text{east}} f_{\text{west}}^{\text{out}} + r_{\text{south}\to\text{east}} f_{\text{south}}^{\text{out}}$

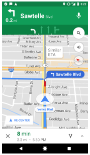
Routing is the result of human preferences



The Open Problem of Real-Time Information

$$\dot{x}_\ell = f_\ell^{\text{in}}(x) - f_\ell^{\text{out}}(x_\ell)$$

$$f_{\text{east}}^{\text{in}} = r_{\text{west}\to\text{east}} f_{\text{west}}^{\text{out}} + r_{\text{south}\to\text{east}} f_{\text{south}}^{\text{out}}$$



- Effective optimal-route algorithms
- Real-time congestion information

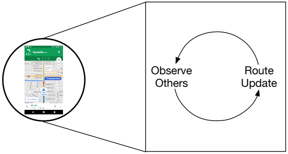
Open problem

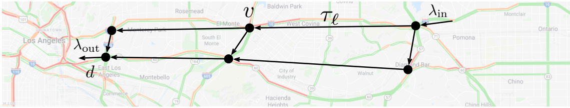
Real-time congestion information $\leftrightarrow$ Robustness of transportation system

Robustness:

- Transfer largest traffic flows
- Despite noncooperative human behaviors

Modeling Navigation Apps





Microscopic: at **every node** drivers minimize travel time to destination

$$\boxed{\text{minimize } \tau_\ell + (\text{time from } v \text{ to dest.})}$$

π_ℓ **:= perceived cost** $\rightarrow$ economic cost that drivers associate to each highway

Macroscopic: all drivers minimize their perceived costs

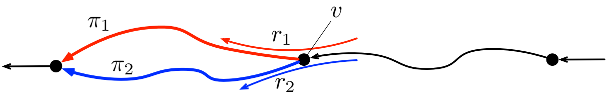
$$\boxed{\dot{r}_{\ell m} = r_{\ell m}\left(\textstyle\sum_q r_{\ell q}\pi_q - \pi_m\right)}$$

"Replicator dynamics"

Evolutionary Model of Routing Apps

"Replicator dynamics"

$$\boxed{\dot{r}_{\ell m} = r_{\ell m}\left(\sum_q r_{\ell q}\pi_q - \pi_m\right)}$$



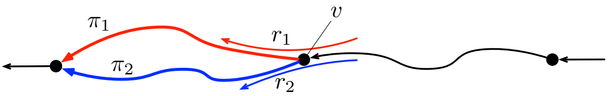
- $r_1 \to \%$ of drivers chosing path 1
- $\underbrace{\boxed{r_1\pi_1 + r_2\pi_2}}_{\triangle} \to$ Average cost from v to dest.

  - if $\pi_1 > \pi_2 \quad \Rightarrow \quad \dot{r}_1 = r_1(\triangle - \pi_1) < 0$
  - if $\pi_1 < \pi_2 \quad \Rightarrow \quad \dot{r}_1 = r_1(\triangle - \pi_1) > 0$

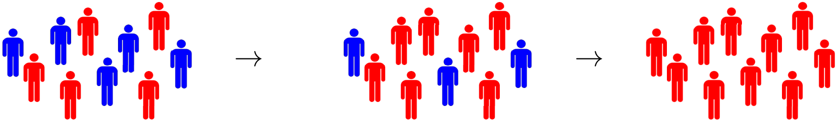
Evolutionary Model of Routing Apps (2)

"Replicator dynamics"

$$\boxed{\dot{r}_{\ell m} = r_{\ell m}\left(\sum_q r_{\ell q}\pi_q - \pi_m\right)}$$



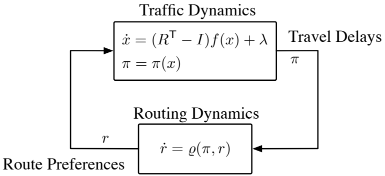
- Red is more convenient than blue ($\pi_1 > \pi_2$)



But changes in the user behavior will change congestion

Coupled Traffic and Routing Dynamics

- Congestion affects route choices
- Routing affects congestion



- Nonlinear $\rightarrow$ trajectories difficult to characterize
- We study equilibria, dynamical behavior

Does the system admit equilibrium points?

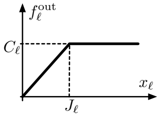
Are the equilibrium points stable?

Existence of Equilibria

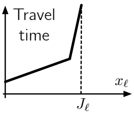
Equilibria (x^*, r^*): if system starts at these points will remain at all times

Technical assumptions:

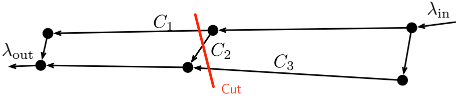
Roads have flow capacities



Drivers avoid jammed roads



Min-cut capacity: capacity of smallest cut that disconnects λ_{in} from λ_{out}



(Bianchin, Pasqualetti, TAC 2020)

Networks with app-routing admit equilibrium $\Leftrightarrow$ $\lambda_{\text{in}} <$ min-cut capacity

Existence of Equilibria: Implications

(Bianchin, Pasqualetti, TAC 2020)

Networks with app-routing admit equilibrium $\Leftrightarrow$ $\lambda_{\text{in}} <$ min-cut capacity

Implications:

(1) Routing apps $\rightarrow$ maximum network throughput

(2) $\lambda_{\text{in}} \gg 1 \rightarrow$ no equilibria (congestion grows unbounded)

(1) If routing is "free"

Max-flow theorem $\Rightarrow$ exists maximum flow with finite travel times

(2) If travel times are finite and "fixed"

$\Rightarrow$ Replicator equation admits equilibrium

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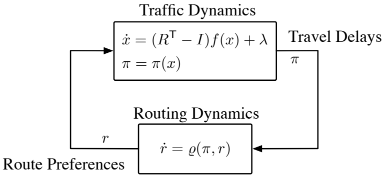

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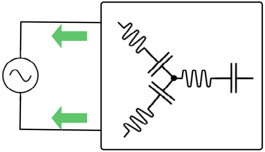
Are the equilibrium points stable?

Detour: Passivity in Nonlinear Dynamical Systems

Passivity: the system does not generate energy
but instead dissipates, stores, and releases it

Theory inspired from electrical circuits:

When energy is not supplied
$\Rightarrow$ system releases



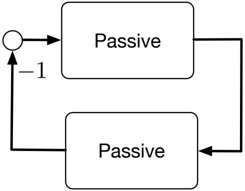
In control systems, a system is passive if

There exists storage function $V \geq 0$
such that $\dot{V} \leq u^{\mathsf{T}} y$

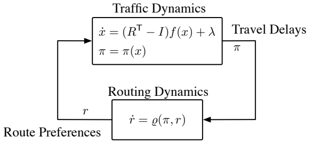
Detour: Passivity in Nonlinear Dynamical Systems

Passivity: the system does not generate energy
but instead dissipates, stores, and releases it

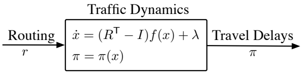
The negative feedback interconnection between
two passive nonlinear systems is passive



Detour: Passivity in Nonlinear Dynamical Systems (2)

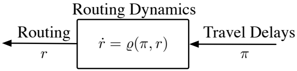


The **open-loop systems are passive**:



The traffic dynamics are passive

- suboptimal routing $\Rightarrow$ network stores congestion
- optimal routing $\Rightarrow$ congestion is released



The routing dynamics are passive

Stability of Equilibria

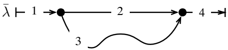
Stability: if system starts near equilibrium
will remain near that operating point

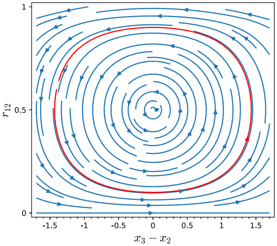
Stability $\Rightarrow$ measure of robustness of the system

The answer is positive, but only partially:

(Bianchin, Pasqualetti, TAC '20)

Equilibria with app-informed drivers are stable,
but not necessarily asymptotically stable





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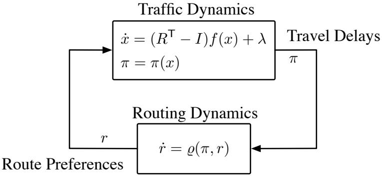

- Nonlinear $\rightarrow$ trajectories difficult to characterize
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Does the system admit equilibrium points?

Yes, if $\lambda_{\text{in}} <$ min-cut capacity

Are the equilibrium points stable?

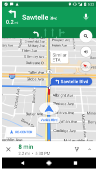
Not necessarily asymptotically stable

Directions

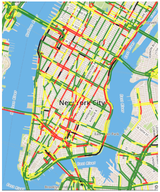
(1) Design navigation apps for better robustness

$$\delta_{\ell m}^{-1} \dot{r}_{\ell m} = r_{\ell m} \left(\sum_q r_{\ell q} \pi_q - \pi_m \right)$$

$\delta_{\ell m} \rightarrow$ congestion-dependent reaction



(2) Design control policies with system-level performance



Given: Real-Time traffic conditions

Objective: Minimize city overall congestion

Control: Automated Intersections

G. Bianchin and F. Pasqualetti, "Gramian-based optimization for the analysis and control of traffic networks," *IEEE Transactions on Intelligent Transportation Systems*, pp. 1–12, 2019