Resilience of Traffic Networks with Partially Controlled Routing

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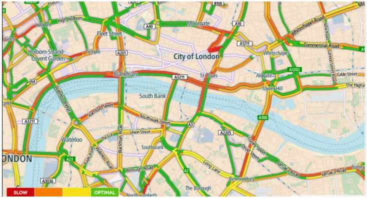
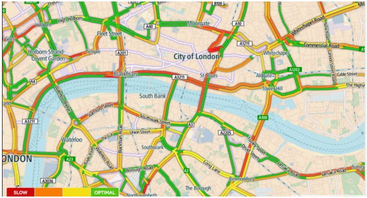
Routing in Traffic Networks



Network routing: captures how travelers respond to congestion

Routing in Traffic Networks



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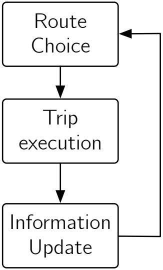

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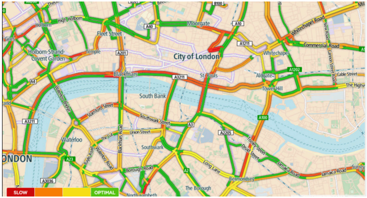

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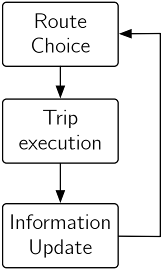
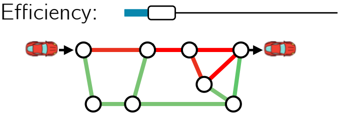
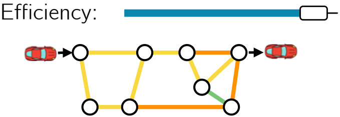




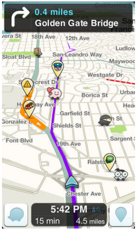
- Minimize individual delay
- Braess Paradox

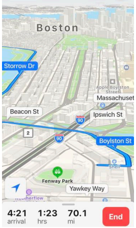


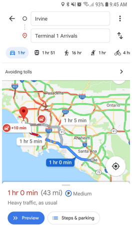
- Do what's best for network
- Can do longer commute

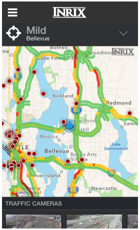
The Introduction of Real-Time Traffic Information

- Commute as fast as we can









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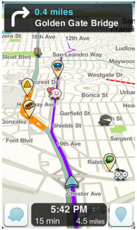

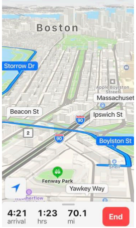

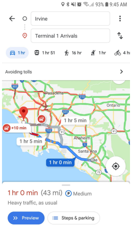

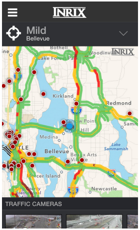

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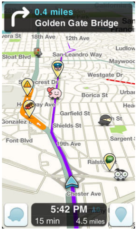
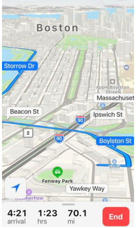
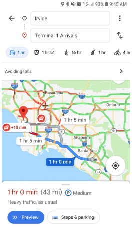
Researcher: Hackers can cause traffic jams by manipulating real-time traffic data

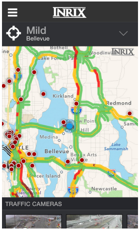
Hackers can influence real-time traffic-flow-analysis systems to make people drive into traffic jams or to keep roads clear in areas where a lot of people use Google or Waze navigation systems, a German researcher demonstrated at BlackHat Europe.

- Commute as fast as we can









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STORY · Published January 29, 2009 · Last Update May 15, 2015

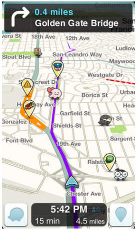
Hackers Crack Into Texas Road Sign, Warn of Zombies Ahead

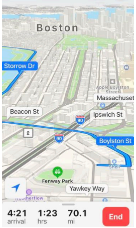
Transportation officials in Texas are scrambling to prevent hackers from changing messages on digital road signs after one sign in Austin was altered to read, "Zombies Ahead."

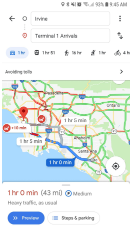
Chris Lippincott, director of media relations for the Texas Department of Transportation, confirmed that a portable traffic sign at Lamar Boulevard and West 15th Street, near the University of Texas at Austin, was hacked into during the early hours of Jan. 19.

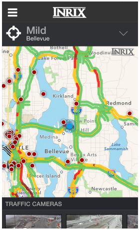
The Introduction of Real-Time Traffic Information

- Commute as fast as we can









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APPLICATIONS
Navigation App Waze Vulnerable to Hackers
Researchers have found that hackers can break into users accounts, track the users in real time, issue instructions and provide an inaccurate picture of traffic at any given time.
(TNS) -- A serious security breach has been discovered in the Google-owned navigation and traffic monitoring app Waze used by millions of drivers worldwide. It seems that Waze, whose slogan is "Outsmarting Traffic, Together" can be outsmarted by hackers, researchers at the University of California Santa Barbara have found, with serious privacy implications for users and scope for criminal abuse.

Two Emerging Questions

1. Do real-time routing suggestions improve performance?

✓ Yes, under appropriate design

2. What is the impact on the network robustness?

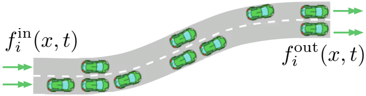
✗ Controlled routing can increase network fragility

Model of Network and Routing

Model of Traffic Network

- Roads dynamics

$$\dot{x}_i = f_i^{\text{in}}(x,t) - f_i^{\text{out}}(x,t)$$



x_i = density [veh/mile] f_i = flow [veh/h]

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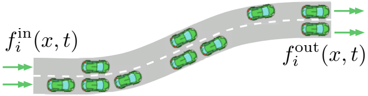
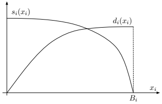


- Cell Transmission Model

$$f_i^{\text{in}}(x,t) \leq s_i(x_i) \quad f_i^{\text{out}}(x,t) \leq d_i(x_i)$$



B_i = max occupancy

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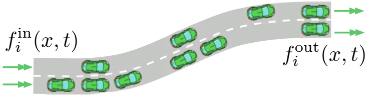

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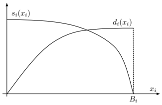
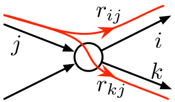


B_i = max occupancy

- Flow conservation at intersections

$$f_i^{\text{in}}(x,t) = \sum_j r_{ij} f_j^{\text{out}}(x,t)$$



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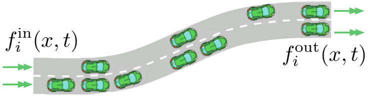

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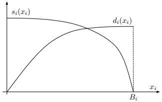

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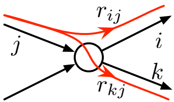
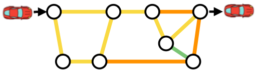


$$f_i^{\text{in}}(x,t) = \sum_j r_{ij} f_j^{\text{out}}(x,t)$$

- Network dynamics:



$$\dot{x} = (R - I) f(x,t) + \lambda$$

Partially Controlled Routing

$$R = \Sigma R^{\mathsf{C}} + (I - \Sigma) R^{\mathsf{S}}, \quad R^{\mathsf{S}} \in \mathcal{R}_{\mathcal{G}}, \quad R^{\mathsf{C}} \in \mathcal{R}_{\mathcal{G}}$$

- R^{C} = design parameter
- R^{S} = fixed routing
- $\Sigma = \mathrm{Diag}\{\sigma_1, \dots, \sigma_n\}$

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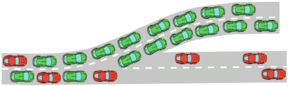
- $\sigma_i \in [0, 1]$ describes penetration level at road i
- σ_i will follow suggestions r_{ij}^{c}
- $(1 - \sigma_i)$ will follow fixed routing r_{ij}^{s}



σ_i are link dependent and time-varying

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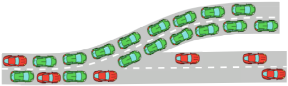

σ_i are link dependent and time-varying

Routing Design

1. Do real-time routing suggestions improve performance?

$$
\begin{aligned}
\min_{R^{\mathsf{C}}} \quad & \int_0^{\mathcal{H}} x_1 + \cdots + x_n \; dt \\
\text{s.t} \quad & \dot{x} = (R - I) f(x,t) + \lambda \\
& R = \Sigma R^{\mathsf{C}} + (I - \Sigma) R^{\mathsf{S}} \qquad (\sigma = \sigma_0) \\
& R^{\mathsf{C}} \in \mathcal{R}_{\mathcal{G}}, \quad x \leq B
\end{aligned}
$$

- Optimizing network total travel time
- Design parameter: R^{C} (routing suggestions)

Network Resilience

2. What is the impact real-time information on the network robustness?

$$\begin{aligned} \rho(\mathcal{G}, x_0) := \min_{\tilde{\sigma}} \quad & \|\tilde{\sigma} - \sigma_0\|_1 \\ \text{s.t.} \quad & x_i = B_i, \ \text{for some } i \text{ and some } t \end{aligned}$$

- Smallest change in penetration levels that results in spillbacks
- The variable parameter is: σ (penetration rate)

Routing Suggestions Design

Numerically Solving the Optimization

1. Discretization: $x_{k+1} = x_k + T_s((R_k - I)f(x_k) + \lambda_k) := \mathcal{F}(x_k, r_k, \lambda_k)$
2. Vectorization: $r_k = (\Sigma_k^\top \otimes I)r^{\mathsf{c}} + ((I - \Sigma_k)^\top \otimes I)r^{\mathsf{s}} := \Psi(\sigma_k, r^{\mathsf{s}}, r^{\mathsf{c}})$
3. Sparsity: $\sum_i r_{ij}^{\mathsf{c}} = b_j,\ 0 \leq r_{ij}^{\mathsf{c}} \leq 1, \quad (i,j) \in \mathcal{E}$

Optimization on discretized dynamics:

$$
\begin{aligned}
\min_{r^{\mathsf{c}}} \quad & \sum_{k=1}^{h} \phi(x_k) \\
\text{s.t.} \quad & x_{k+1} = \mathcal{F}(x_k, r_k, \lambda_k), && k = 1, \dots, h, \\
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Because of nonlinearity of $\mathcal{F}$: **nonlinear programming opt. problem**

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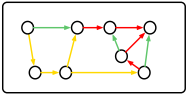

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\end{aligned}
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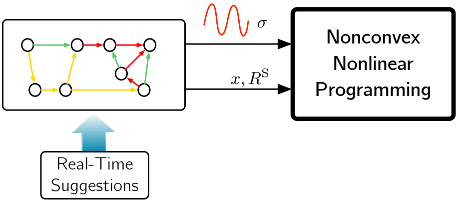
Because of nonlinearity of $\mathcal{F}$: **nonlinear programming opt. problem**

Numerically Solving the Optimization



Optimization on discretized dynamics:

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\min_{r^{\mathsf{c}}} \quad & \sum_{k=1}^{h} \phi(x_k) & \\
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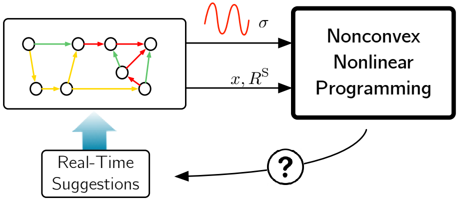

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Solving Nonlinear Programming in Real Time

$$\boxed{\begin{aligned} \min_{r^{\mathsf{c}}} \quad & f_0(r^{\mathsf{c}}, \hat{x}, \sigma) \\ \text{s.t.} \quad & g_i(r^{\mathsf{c}}, \hat{x}, \sigma) \leq 0 \\ & h_j(r^{\mathsf{c}}, \hat{x}, \sigma) = 0 \end{aligned}}$$

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2. Compose the Lagrangian

$$\mathcal{L}(r^{\mathsf{c}}, \hat{x}, \sigma, w, u) = f_0(r^{\mathsf{c}}, \hat{x}, \sigma) + u^\top g(r^{\mathsf{c}}, \hat{x}, \sigma) + w^\top h(r^{\mathsf{c}}, \hat{x}, \sigma)$$

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3. Set of KKT conditions

$$\nabla \mathcal{L}(r^{\mathsf{c}*}, \hat{x}^*, \sigma_0, w^*, u^*) = 0$$
$$u_i g_i(r^{\mathsf{c}*}, \hat{x}^*, \sigma_0) = 0$$
$$h_j(r^{\mathsf{c}*}, \hat{x}^*, \sigma_0) = 0$$

Implicit Equation

$$F(r^{\mathsf{c}*}, \hat{x}^*, \sigma_0, u^*, w^*) = 0,$$

Describes Relation Between Variables At Optimality

Solving Nonlinear Programming in Real Time (2)

$F(r^{c*}, \hat{x}^*, \sigma_0, u^*, w^*) = 0$

④ Let $y = [r^c(\sigma)\ u(\sigma)\ w(\sigma)]$ and derive w.r.t. σ

$$M(\sigma)\frac{dy}{d\sigma} + N(\sigma) = 0,$$

$M(\sigma) = [\partial F_i/\partial y_j]$, $dy/d\sigma = [dy_i/d\sigma_j]$, $N(\sigma) = [\partial F_i/\partial \sigma_j]$

④ If $r^{c*}(\sigma_0)$ is a local isolated minimizing point

Describes variation of optimal solution to changes in σ

$$\frac{dy}{d\sigma} = -M^{-1}(\sigma_0)N(\sigma_0)$$

⑤ Let $dy/d\sigma = [\eta_1,\ \eta_2,\ \eta_3]^\mathsf{T}$

$$r^c(\sigma) = r^{c*}(\sigma_0) + \eta_1(\sigma - \sigma_0)$$

Linear update: significantly faster than solving nonlinear programming

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Fast Update Error Bound

Theorem

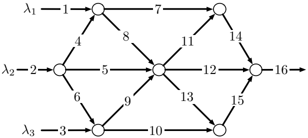
Assume

1. All constraints are linearly independent
2. Second order KKT conditions hold at optimizer
3. Strict complementary slackness

Then,

$$r^{\mathsf{c}*}(\sigma) = r^{\mathsf{c}*}(\sigma_0) + \eta_1(\sigma - \sigma_0) + o(\|\sigma - \sigma_0\|^2)$$

Numerical Results



- Network: $n = 15$ roads, $m = 7$ junctions
- Roads: $L_i = 5\text{mi}$, capacity $B_i = 200\text{veh}$
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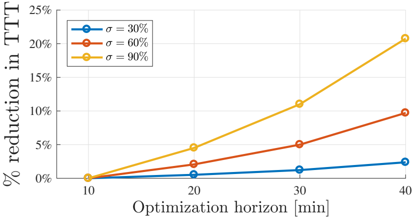

Consistent reduction in travel time not only requires significant penetration levels, but also appropriately large control horizons

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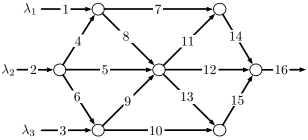

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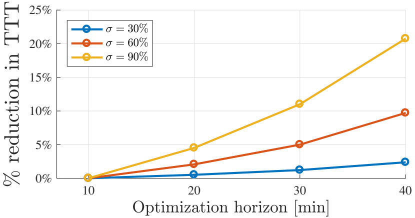

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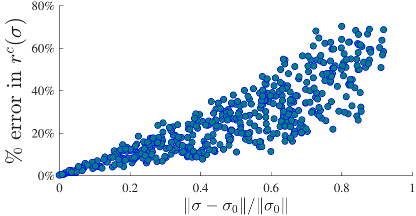

Error with real-time update = $o(\|\sigma - \sigma_0\|^2)$

Network Resilience

Resilience of a Link

The Margin of Resilience of link i is

Smallest change in σ that generates its failure due to a jam

$$\begin{aligned} \rho_i(x_0) := \min_{\sigma} \quad & \|\sigma - \sigma_0\|_1 \\ \text{s.t.} \quad & \dot{x} = (R - I) f(x, t) + \lambda \\ & R = \Sigma R^{\mathsf{C}} + (I - \Sigma) R^{\mathsf{S}} \\ & x_i \geq B_i, \ \text{for some } t \in [0, \mathcal{H}] \end{aligned}$$

Estimating the Margin of Resilience

1. Constraint violation is $\mathcal{F}_i(x_k, r_k(\sigma), \lambda_k) \geq B_i$
2. For small changes of σ, take Taylor expansion

$$\mathcal{F}_i(x_k, r_k(\sigma), \lambda_k) = \mathcal{F}_i(x_k, r_k(\sigma_0), \lambda_k) + \underbrace{\frac{d\mathcal{F}_i}{d\sigma}(x_k, r_k(\sigma), \lambda_k)\Big|_{\sigma=\sigma_0}}_{\Psi_i(r_k, x_k, \lambda_k, \sigma)}(\sigma - \sigma_0) + o(\|\delta_\sigma\|^2) \geq B_i$$

3. By rearranging the terms and by taking the norms

$$\rho_i(\mathcal{G}, x_0) \geq \min_k \frac{B_i - \mathcal{F}_i(x_k, r_k, \lambda_k)}{\|\Psi_i(k, \lambda, \sigma_0)\|_\infty}$$

Lower Bound on Margin of Resilience

(Can be quickly computed from real-time update)

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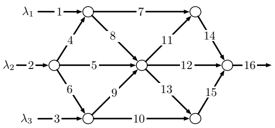

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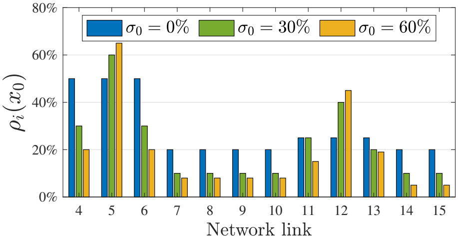
Lower bound on resilience:



Networks where the routing is partially controlled by a system planner are more prone to traffic jam phenomena originated by changes in fluctuations in routing choices

Conclusions

Contribution:

- Real-time mechanism based on first-order sensitivity analysis for NLP
- Technique to estimate margin of resilience

Outcomes:

- (Expected) Routing control leads to improved network performance
- (Counterintuitive) Performance comes at the cost of higher fragility

Directions:

✓ Need of a framework that allows us to formalize these observations

✎ Increasing need to capture human responses to control policies

Thanks: Control of Complex Systems Initiative (CCSI) at PNNL & NSF

Resilience of Traffic Networks with Partially Controlled Routing

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University of California, Riverside

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2019 American Control Conference
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