Control and Operation of Complex Networks

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Cyber-Physical systems

Engineered systems integrating a large number of heterogeneous cyber devices with physical interactions.

- Distributed robotics
- Satellite networks
- Power networks
- Industrial plants

Challenges in Cyber-Physical systems

Operating CPSs is a challenging task because of:

- Dymensionality
- Unknown dynamics
- Uncertain parameters
- Complex interconnection structure

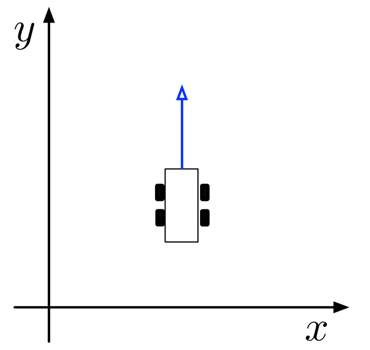
As network size and complexity increase, the analysis of the system behavior becomes extremely challenging.

System controllability

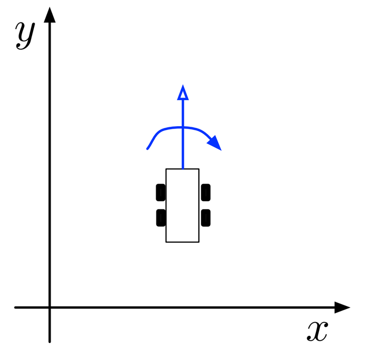
Discrete time linear system models complex networks dynamics:

$$x(t+1) = Ax(t) + Bu(t)$$

For dynamical systems, controllability is the ability to drive the state to arbitrary configurations through external inputs:



Not controllable



Controllable

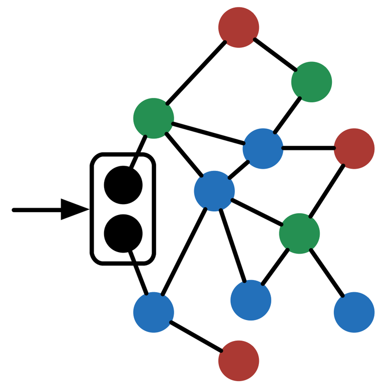
For Complex networks:

Controllability Matrix: $\mathcal{C} = [B \; AB \dots A^{n-1}B]$

Controllability Gramian: $\mathcal{W}_T = \sum_{t=0}^{T-1} A^t BB^\mathsf{T} A^t$

Energy to control networks

- Drive the state from x_0 to x_f
- Input affects a given set of nodes
- Worst case input energy $= \frac{1}{\lambda_{\min}(\mathcal{W}_T)}$
- ? Can we control the network to the state x_f with a finite energy input



Small	$\lambda_{\min}(\mathcal{W}_T)$	$\Leftrightarrow$	**Large input energy**
Large	$\lambda_{\min}(\mathcal{W}_T)$	$\Leftrightarrow$	**Small input energy**

Difficulty in controlling networks

The geometry of the network determines its controllability degree:

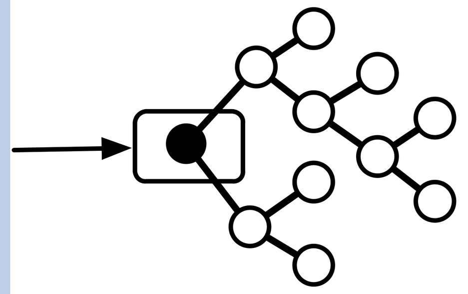
$$\lambda_{\min}(\mathcal{W}_{\mathcal{K},T}) \leq \text{cond}^2(V) \frac{\mu^{2\lceil n(\mu)/m \rceil - 2}}{1-\mu^2}$$

- $|\mu| \in [0, 1]$
- $n(\mu)$ grows linearly with the network cardinality
- V is an eigenvector matrix of A
- cond(V) is the condition number of V

If cond(V) remains bounded with the network dimension:

- The controllability degree decreases exponentially with the network cardinality
- The network remains difficult to control

Controllability of acyclic networks



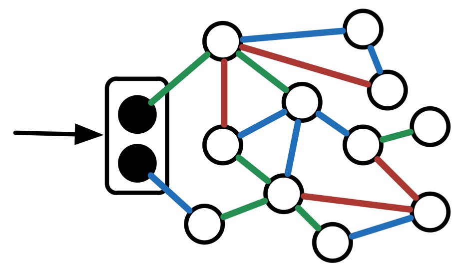
$$\lambda_{\min}(\mathcal{W}_{\mathcal{K},T}) \leq \left(\frac{a_{\max}}{a_{\min}}\right)^{\text{diameter}} \frac{\mu^{2\lceil n(\mu)/m \rceil - 2}}{1-\mu^2}$$

Acyclic networks with diameter $\mathcal{O}(n)$ are difficult to control (by constant number of control nodes).

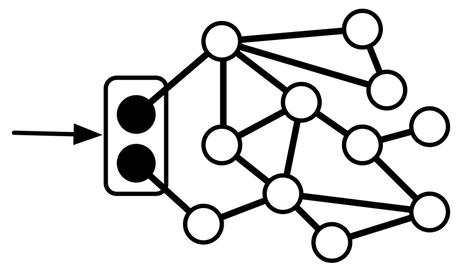
Network features that quantify controllability

G. Bianchin et al. (2015). "The Role of Diameter in the Controllability of Complex Networks". In: *IEEE Conference on Decision and Control*. Osaka, Japan

Strong Structural Controllability



(A, B)



$(\mathbf{A}, \mathbf{B})$

- Network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with self loops
- We look at conditions in the network topology that guarantee controllability for all choices of edge weights
- ? Can we characterize topological features of Strongly Structurally Controllable networks

Necessary and sufficient condition

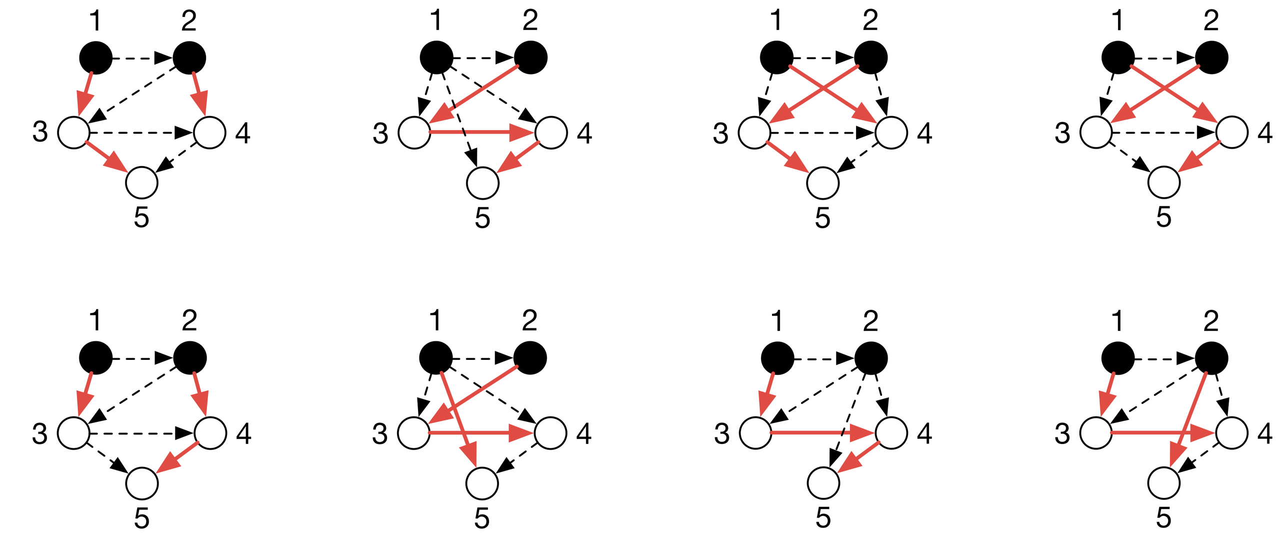
The system is Strongly structurally controllable **if and only if** the nodes can be labeled such that for all $i \in \{m+1, \dots, n\}$ there exists $j \in \{1, \dots, i-1\}$ such that

$$\text{supp}(\mathbf{A}(i : \text{end}, j)) = \{1\}.$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{e}_1 \\ \vdots \\ \mathbf{e}_m \\ \vdots \\ 0 \end{bmatrix}^\mathsf{T} \qquad \mathbf{A} = \begin{bmatrix} \mathbf{a}_{11} & 0 & \mathbf{a}_{13} & 0 & 0 & 0 \\ 0 & \mathbf{a}_{22} & \mathbf{a}_{23} & \mathbf{a}_{24} & 0 & 0 \\ \mathbf{a}_{31} & \mathbf{a}_{32} & \mathbf{a}_{33} & \mathbf{a}_{34} & \mathbf{a}_{35} & \mathbf{a}_{36} \\ 0 & \mathbf{a}_{42} & \mathbf{a}_{43} & \mathbf{a}_{44} & \mathbf{a}_{45} & 0 \\ 0 & 0 & \mathbf{a}_{53} & \mathbf{a}_{54} & \mathbf{a}_{55} & 0 \\ 0 & 0 & \mathbf{a}_{63} & 0 & 0 & \mathbf{a}_{66} \end{bmatrix}$$

Control paths

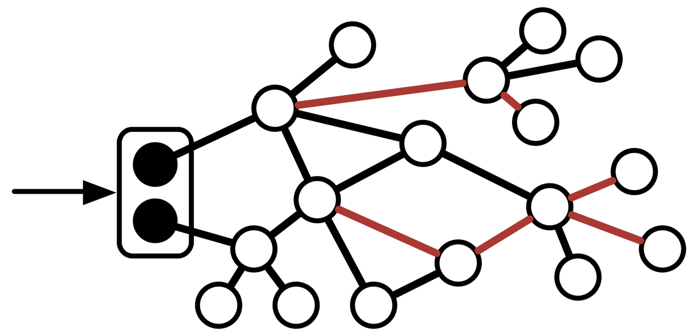
Every strongly structurally controllable network admits a set of control paths comprehending the entire set of nodes:



Control patterns for controllability

T. Menara **G. Bianchin** et al. (2016b). "Control Patterns for Strong Structural Controllability". In: *IEEE Conference on Decision and Control [Submitted]*. Las Vegas, NE, USA

System under attack



- Network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ controlled by control nodes
- Adversaries can manipulate some edge weights
- ? Can the adversary make the dynamics uncontrollable

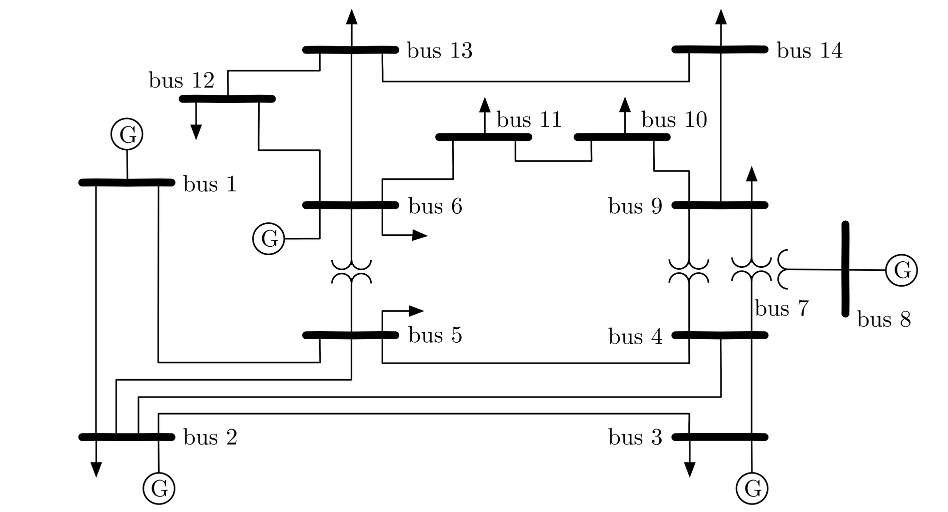
Problem formulation

Before perturbation, (A, B) is controllable

$$\begin{aligned} &\min_{\Delta,\lambda,x} \; \|\Delta\|_F^2 && \text{Frobenius norm} \\ &\text{s.t.} \quad (A+\Delta)x = \lambda x, && \text{(eigenvalue constraint)}, \\ &\qquad \|x\|_2 = 1, && \text{(eigenvector constraint)}, \\ &\qquad x^\mathsf{T} B = 0, && \text{(uncontrollability)}, \\ &\qquad \Delta \in \mathcal{A}_\mathcal{H}, && \text{(structural constraint)}, \end{aligned}$$

Topology attacks against power systems

IEEE 14 power grid, with 5 generators and 14 load buses:



Attacker effect	$\|\Delta\|_F$	Unobservable mode
Disconnect load 1	4.60	10.92
Stop generator 1	2.59	$10.92 \pm 20.95j$
Modify impedances	2.34	$10.92 \pm 10^4 j$

Controllability radius of network systems

G. Bianchin et al. (2016a). "The Observability Radius of Network Systems". In: *IEEE Transactions on Automatic Control [Submitted]*

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