Mass Media and Cultural Homogenization:

Broadcasting the American Dream on the Radio\*

Technical Appendix

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#### Abstract

In this appendix, I give technical details on radio signal propagation through amplitude modulation (AM). I briefly introduce the different types of radio waves that work with AM focusing on the mechanics of ground waves. I then present the model I implement to compute radio coverage of US stations during the interwar period which exclusively broadcasted using AM technology. Finally, I discuss the accuracy of my prediction model comparing it with tables published by the International Telecommunication Union (ITU).

<sup>\*</sup>For fruitful conversations on the technical aspect of radio propagation I thank David Kazdan, Adjunct Assistant Professor in Electrical Engineering and Computer Science at Case Western Reserve University, William Scott, expert of medium wave DXing and author of the blog Radio-Timetraveller and the members of the HamSCI Google group. I thank Caterina Mauri for digitizing the conductivity data.

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# 1 AM Radio Propagation

Radio propagation is the phenomenon through which radio waves travel from one point to another. Depending what radio frequency is used to broadcast, there are different ways to transmit the electromagnetic waves that constitute the radio signal. The vast majority of broadcasters use amplitude modulation (AM) when transmitting signal on frequencies associated with short and medium wave lengths. Here, I maintain this focus, discussing exclusively the propagation of radio signal on the medium frequency band through amplitude modulation.<sup>1</sup>

AM radio signal on the medium frequency band propagates using two type of waves: ground waves and sky waves. Sky waves rely on the ionosphere to bounce back on earth and reach receiver antennas. During the day the solar activity does not allow the ionosphere to reflect AM radio waves back on Earth. Instead, ground waves spread out of the transmitter along the surface of the Earth allowing their use at any hour of the day and the night. Hence, day time transmission of AM radio signal exclusively relies on ground waves.

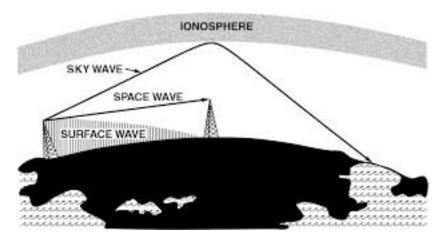


Figure 1: Sky Waves Versus Ground Waves

To reconstruct radio coverage of US stations during the interwar period, I exclusively

<sup>&</sup>lt;sup>1</sup>Whenever I talk about AM radio, I refer to AM radio on the medium frequency band. The engineering definition of the portion of radio spectrum characterized by medium wave length goes from 300 kHz to 3 MHz. In practice, what is usually devoted to broadcasting goes from approximately 500 kHz to 1700 kHz. Today the Federal Communication Commission has allocated the medium frequency band to be between 530 to 1700 kHz for the United States. Notice that there is a mathematical relationship between frequency and wave length: frequency f is equal to the speed of light C divided by wave length  $\lambda$ :  $f = C/\lambda$ .

focus on ground waves. The reason for this is twofold. First, sky waves are less reliable, which makes them difficult to consistently receive, especially historically, and harder to precisely predict. Second, in one of my empirical exercises I investigate the adoption of baseball players names. During the interwar period, games were for the vast majority played with the light of the day.<sup>2</sup> Thus, in what follows, I give details on the mechanics of ground wave propagation. For a discussion of sky wave propagation in economics see Gagliarducci et al. (2018).

### 1.1 Ground Waves Propagation

Ground waves encompass all waves that are transmitted by a radio antenna and propagate through the area between the surface of the earth and the ionosphere.<sup>3</sup> Given a transmitting antenna, the voltage (field strength) induced in the receiving antenna, at an arbitrary position can be expressed by the following expression:

$$V = QI \left\{ Q_1 \frac{exp(-jkr_1)}{r_1} + Q_2 R \frac{exp(-jkr_2)}{r_2} + S \frac{exp(-jkr_2)}{r_2} \right\}$$
 (1)

Where I is the current in the transmitting antenna, Q is a constant, Q1 and Q2 measure the transmitting- and receiving-antenna polar diagrams, R is a reflection coefficient, k is the radio wave number  $k = 2\pi/\lambda$ ,  $r_1$  is the straight line distance between transmitting and receiving antenna and  $r_2$  is the straight line distance between transmitting antenna and the ground. The parameter S embodies an attenuation factor.

Each one of the three components in equation (1) reflects a subset of ground waves that determines field strength at the receiving end: space waves, composed by direct line waves and reflected waves, and surface waves.

Space waves are typically formed by the direct wave — transmitting antenna to receiving antenna — and the waves that leave the antenna and bounce over natural obstacles (see

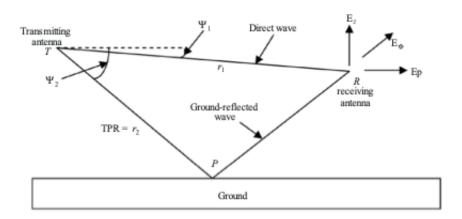
<sup>&</sup>lt;sup>2</sup>The first night game in Major League history was played in 1935. Until the 1940s only a very tiny fractions of the games were played at night.

<sup>&</sup>lt;sup>3</sup>This section is heavily based on the ITU handbook on radio ground waves (ITU 2014)

Figure 2). Space waves are effective in reaching the receiving antenna only if this is within line of sight. If the receiving end is beyond the horizon, or both antennas are very close to the ground the space waves are either zero or very negligible.

Figure 2: Propagation of the Space Wave Component of the Ground Wave

#### Geometry of direct and ground reflected waves

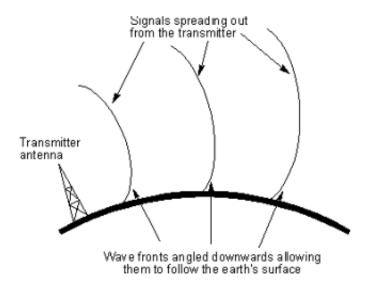


Surface waves are sometimes referred to as ground waves themselves. Instead of just following line of sight, like direct waves do, these waves tend to follow the curvature of the Earth. The reason for this is that currents in the surface of the earth cause the front of the wave to tilt downwards towards Earth (see Figure 3). This allows surface waves to curve around the Earth and reach receivers well beyond the horizon and beyond natural obstacles such as hills and mountains.

The rate of attenuation of the signal increases as it travels over a surface. This happens because the forward portion of the waves tilts even further. The speed with which this happens is regulated by an attenuation factor called complex dielectric permittivity. Large values of the attenuation factor correspond to a small forward tilt and therefore low attenuation and vice-versa.

The attenuation factor is determined by two elements: the electrical conductivity of the surface over which the wave travels and the broadcasting frequency. Sea water has an outstandingly high conductivity and the surface wave, with a near-vertical electric field,

Figure 3: Propagation of the Surface Wave Component of the Ground Wave



Ground wave radio propagation

propagates over it with relatively low attenuation. On the other hand, surface-wave attenuation is greatest over ground of low conductivity and at high radio frequencies. Within the realm of AM radio the role of the frequency band is limited. Hence, to predict field strength at the receiving end it is key to measure soil or sea conductivity accurately.

## 1.2 The Role of Soil Conductivity

Ground waves emanating out of the transmitting antenna pass over one or multiple surfaces before they reach a receiver. Different type of soils have different levels of electrical conductivity and affect the rate of attenuation factor of the ground wave. Hence, to confidently predict field strength at a certain distance from the transmitting antenna it is important to know the conductivity of the surface the wave is expected to travel over.

Table 1 shows broad definition of lands and their typical levels of conductivity expressed in siemens per meter  $(\sigma_{s/m})$  and their permittivity.<sup>4</sup> By far, sea, especially if with high

<sup>&</sup>lt;sup>4</sup>Permittivity is closely related to conductivity. It measures the ability of a substance to hold an electrical charge. Higher permittivity decreases attenuation of the wave

**Table 1:** Typical Conductivity Levels of Different Types of Soil

| Soil Description                     | Conductivity $(\sigma_{s/m})$ | Permittivity $(\epsilon_r)$ |  |
|--------------------------------------|-------------------------------|-----------------------------|--|
| Polar ice cap                        | 0.0001                        | 1                           |  |
| Polar ice                            | 0.0003                        | 3                           |  |
| Arctic land                          | 0.0005                        | 3                           |  |
| Sea ice                              | 0.001                         | 4                           |  |
| City industrial; average attenuation | 0.001                         | 5                           |  |
| Dry, sandy, coastal                  | 0.001                         | 10                          |  |
| Fresh water (low salinity)           | 0.001                         | 80                          |  |
| Mountainous hills                    | 0.002                         | 5                           |  |
| Fertile land                         | 0.002                         | 10                          |  |
| Rocky                                | 0.002                         | 15                          |  |
| Medium hills and forest              | 0.004                         | 13                          |  |
| Highly moist ground                  | 0.005                         | 30                          |  |
| Pastoral hills, rich soil            | 0.007                         | 17                          |  |
| Marshly land, densily wooded         | 0.0075                        | 12                          |  |
| Marshly, forested, flat              | 0.008                         | 12                          |  |
| Rich agric land, low hills           | 0.01                          | 15                          |  |
| Sea water                            | 5                             | 81                          |  |

Note: The CCIR (predecessor of the ITU) made available ground conductivity maps through their Atlas (CCIR 1992). A handy collection of their data can be found on Serge Stroobandt website.

salinity, is the most conductive surface with 5  $\sigma_{s/m}$ . Among land surfaces, wet grounds typically have the best conductivity. This makes farming areas on average very suitable for the propagation of ground waves. Dry and desert type lands are among the least conductive.

While Table 1 gives an idea of the conductivity for certain type of soils, measuring ground conductivity is cumbersome. Sampling soil is not enough because conductivity is determined by the type of soil even up to 25 meters below the immediate surface, requiring the use of vector network analyzers in conjunction with specific wire lines. For this reason, people interested in predicting field strength rely on conductivity contour maps published by ITU or, in the US, by the Federal Communication Commission.

# 2 Ground Wave Propagation Model

I explain the model I chose to predict signal strength and how I code it using R. I then compare my signal predictions with tables from the ITU.

### 2.1 Implementing the Model

Norton (1936) built on the theoretical work from Sommerfeld (1909) to provide the first method for calculations of field strength over a flat earth. Van der Pol and Bremmer (1937) made it possible to calculate field strength over a spherical body. Bremmer's contribution is based on a residue series and it is cumbersome to compute. The empirical model I use is a relatively recent simplification of his work (Schrank 1990).

The empirical model to compute electrical field (E mV/m) is the following:

$$E = (30WD)^{1/2} \frac{A A_1}{R} \tag{2}$$

where the definition of A depends on the values of two parameters,  $p_0$  and b, which in turn depend on observables like frequency and conductivity values:

$$p_0 = \frac{\pi (R/1000) f^2 cos(b)}{54 \cdot 10^2 \sigma}, \quad b = \frac{(\epsilon + 1) f}{18 \cdot 10^3 \sigma}.$$

For  $p_0 \leq 4.5$  and all b:

$$A = exp(-.43p_0 + .01p_0^2) - (\sqrt{p_0/2})sin(b)exp(-5/8p_0)$$

For  $p_0 > 4.5$  and all b:

$$A = \frac{1}{2p_0 - 3.7} - (\sqrt{p_0/2})sin(b)exp(-5/8p_0)$$

and  $A_1$  is defined as follow,

$$A_1 = \frac{-A_2 R}{10^4 \cdot 8/f^{1/3}}.$$

In order to compute field strength E at a distance of R meters, I need to fix the following parameters: transmitting power W in watts, frequency in mHz f, antenna directivity D, conductivity  $\sigma$  in siemens by meters, permittivity  $\epsilon$  and  $A_2$  a coefficient experimentally determined.

To do so, I collect novel data on the universe of US stations active between 1924 and 1940. For each station, in addition to their location and years of operation, I gather daytime wattage and frequency of broadcast.

Unfortunately, I am not able to systematically observe the type of antenna used to broadcast, nor to experimentally determine the coefficient  $A_2$ . Hence, I assume that all stations were broadcasting using a non-directional antenna with initial directivity D = 3.116468997. While this assumption might be crude, it is relatively less harmful for the historical period at hand when the range of different types of antenna used was much more narrow than today. I also assume  $A_2 = 0.225$ , an approximation that is considered to be good average for ground waves in the US.<sup>5</sup>

I use data that report the average ground conductivity at the county level to feed conductivity  $\sigma$  and permittivity  $\epsilon$ . Figure 4 shows the source of the data: the FFC map of ground conductivity produced in 1954. The FCC also provides a more detailed piecemeal version of the map which was digitized and averaged at the county level. Unfortunately, to the best of my knowledge, an older version of the map is not available. A rendering of the county level conductivity is available in figure 5.

Finally, I exploit the relationship between conductivity and permittivity shown in Table 1 to interpolate values of permittivity whenever I can not infer them directly. This is a good approximation because, excluding the important case of water surfaces, there is an almost

<sup>&</sup>lt;sup>5</sup>For the construction of the model William Scott provided extensive help, including good first order approximation values that could not be determined empirically.

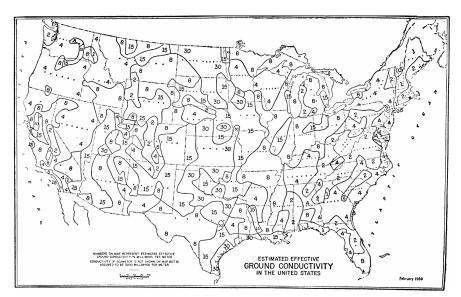


Figure 4: FCC Map of Ground Conductivity

Source: The zip file with the txt files for all the 48 portions and as well as other formats available on the FCC website

linear relationship between  $\sigma$  and  $\epsilon$ . Fortunately, only a very tiny fractions of the signal transmitted in the US travels over water before reaching a listener in the US.

#### 2.1.1 Software Implementation

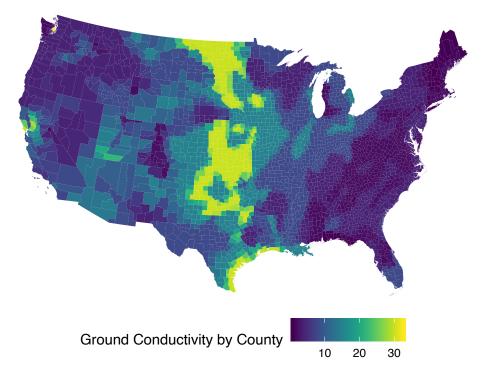
Consider a station located in a broadcasting on a frequency f with wattage W and a receiver located in b at distance R. We want to know field strength at b to determine if it is high enough to produce a good quality broadcast service. To determine field strength at b we are left to pin down the conductivity of all the possible soils that the wave will cross before reaching b. Since I only observe county averages of conductivity, this boils down to keeping track of all the times the signal crosses county borders before reaching b.

I do this by computing the average ground conductivity between a and b weighting each value of conductivity by distance in meters the wave travels on before reaching b. Using R, I implement this in two steps.

First, for each *sending* station I determine field strength at each possible *receiving* location assuming maximum (sea-level) conductivity.<sup>6</sup> I then keep only sending-receiving connections

 $<sup>^6</sup>$ The set of all possible receiving locations is determined by locations of residence of American households





Source: Calculations of the author based on the average conductivity at the county level.

that have field strength higher than the lowest possible threshold for a decent radio broadcast.

Second, I recompute field strength at b using as ground conductivity the weighted average of all the values of conductivity the waves crosses times the number of meters the wave travels on them.

The final step to determine radio coverage at b is to transform the electrical field value E into an indicator equal to one if that signal connection is strong enough to produce a good signal to noise ratio. The simplicity of AM transmission makes it vulnerable to radio interferences and radio noise created by both natural atmospheric electrical activity and electrical and electronic equipment. In large urban centers, AM radio signals can be severely disrupted by metal structures and tall buildings.

To account for interferences, I follow Schrank (1990) in specifying three separate cutoffs based on the type of area the receiving household was listening from. For the signal to I recover. More details on this in the data section of the paper.

produce a good quality broadcast the electrical field has to be larger than 25 mV/m in urban areas, 5 mV/m in residential areas and 0.5 mV/m in rural areas. I use the US census definitions of urban, rural and type of metropolitan status to define in what of these three kind of areas was the household located.

Ultimately, the indicator for station a's signal reception will be equal to one in location b if the field strength is larger than 25, 5 or .5 depending on whether b is considered urban, residential or rural respectively. I operate this procedure for the universe of broadcasting stations to obtain the matrix with all the active connections between stations and receiving locations in a given year.

### 2.2 Testing Accuracy

I have explained how I predict signal coverage of US radio stations during the interwar period. Now, I test the accuracy of my predictions against similar type of simulation done by the ITU.

To check the compatibility of amateur stations on incumbent services, ITU estimated field strength for the frequency range of medium and short waves (ITU 2010). In the report, ITU published tables that show predicted ground waves field strength holding constant transmitting antenna and soil characteristics. Here, I simulate signal strength based on the observables they use and check how far off is my prediction from theirs.

Table 2 shows the original ITU simulations for distances between ten and two hundred kilometers, holding constant ground conductivity at an average level of  $\sigma = 0.03$ . They also fix the frequency of broadcast at 500 kHz and the transmitting power of the antenna is to 20 watts.

The ITU simulates field strength for different heights at which the receiver could be situated. In my case, I am not able to recover such information for American households, although it is conceivable to assume that the vast majority of them was located in the first column, that is the lowest height from soil. Column 5 of Table 2 shows that the standard

**Table 2:** ITU Simulations of Field Strength E (Db/m) by Distance and Receiver Height,  $\sigma = 0.03, f = 500 \text{ kHz}$ 

|               | Receiver Height |       |       |       |       |                     |
|---------------|-----------------|-------|-------|-------|-------|---------------------|
| Distance (Km) | 5/10 m          | 15 m  | 20 m  | 50 m  | Mean  | $\operatorname{Sd}$ |
|               | (1)             | (2)   | (3)   | (4)   | (5)   | (6)                 |
| 10            | 72.17           | 72.45 | 72.45 | 72.1  | 72.29 | 0.18                |
| 20            | 65.9            | 66.39 | 66.39 | 65.83 | 66.13 | 0.3                 |
| 30            | 62.13           | 62.81 | 62.81 | 62.06 | 62.45 | 0.41                |
| 40            | 59.38           | 60.25 | 60.25 | 59.31 | 59.80 | 0.52                |
| 50            | 57.22           | 58.3  | 58.3  | 57.15 | 57.74 | 0.64                |
| 60            | 55.39           | 56.63 | 56.63 | 55.31 | 55.99 | 0.74                |
| 70            | 53.79           | 55.2  | 55.2  | 53.71 | 54.48 | 0.84                |
| 80            | 52.37           | 53.95 | 53.95 | 52.29 | 53.14 | 0.94                |
| 90            | 51.07           | 52.82 | 52.82 | 51    | 51.93 | 1.03                |
| 100           | 49.88           | 51.79 | 51.79 | 49.81 | 50.82 | 1.12                |
| 110           | 48.78           | 50.85 | 50.85 | 48.71 | 49.80 | 1.22                |
| 120           | 47.74           | 49.97 | 49.97 | 47.67 | 48.84 | 1.31                |
| 130           | 46.77           | 49.15 | 49.15 | 46.69 | 47.94 | 1.4                 |
| 140           | 45.84           | 48.38 | 48.39 | 45.77 | 47.10 | 1.49                |
| 150           | 44.95           | 47.64 | 47.63 | 44.88 | 46.28 | 1.57                |
| 160           | 44.1            | 46.93 | 46.93 | 44.03 | 45.50 | 1.65                |
| 170           | 43.29           | 46.26 | 46.26 | 43.21 | 44.76 | 1.74                |
| 180           | 42.5            | 45.61 | 45.61 | 42.42 | 44.04 | 1.82                |
| 190           | 41.73           | 44.99 | 44.99 | 41.66 | 43.34 | 1.9                 |
| 200           | 40.99           | 44.39 | 44.39 | 40.91 | 42.67 | 1.99                |

Note: This table is a replication of Table 1 in ITU (2010). It shows ITU simulations of field strength at by distance (from 10 to 200 km) and receiver height (5 to 50 meters) for a value of conductivity  $\sigma = 0.03$  and frequency f = 500 kHz (.5 mHz). The values of conductivity and frequency are chosen to be the most representative of the average AM broadcaster. Notice that height is referred as height from soil not as in altitude. Column 5 reports the standard deviation for different heights.

**Table 3:** Comparison between ITU simulations and S of Field Strength E (Db/m) by Distance and Receiver Height,  $\sigma = 0.03, f = 500 \text{ kHz}$ 

| Distance (Km) | ITU Mean | Own-Simulations | $\Delta$ Mean Height | $\Delta$ Min Height | $\mathrm{Max}\ \Delta$ |
|---------------|----------|-----------------|----------------------|---------------------|------------------------|
|               | (1)      | (2)             | (3)                  | (4)                 | (5)                    |
| 10            | 72.29    | 72.311          | -0.021               | -0.141              | 0.139                  |
| 20            | 66.13    | 65.903          | 0.227                | -0.003              | 0.487                  |
| 30            | 62.45    | 61.998          | 0.452                | 0.132               | 0.812                  |
| 40            | 59.80    | 59.117          | 0.683                | 0.263               | 1.133                  |
| 50            | 57.74    | 56.799          | 0.941                | 0.421               | 1.501                  |
| 60            | 55.99    | 54.837          | 1.153                | 0.553               | 1.793                  |
| 70            | 54.48    | 53.121          | 1.359                | 0.669               | 2.079                  |
| 80            | 53.14    | 51.584          | 1.556                | 0.786               | 2.366                  |
| 90            | 51.93    | 50.185          | 1.745                | 0.885               | 2.635                  |
| 100           | 50.82    | 48.895          | 1.925                | 0.985               | 2.895                  |
| 110           | 49.80    | 47.693          | 2.107                | 1.087               | 3.157                  |
| 120           | 48.84    | 46.564          | 2.276                | 1.176               | 3.406                  |
| 130           | 47.94    | 45.495          | 2.445                | 1.275               | 3.655                  |
| 140           | 47.10    | 44.479          | 2.621                | 1.361               | 3.911                  |
| 150           | 46.28    | 43.509          | 2.771                | 1.441               | 4.131                  |
| 160           | 45.50    | 42.577          | 2.923                | 1.523               | 4.353                  |
| 170           | 44.76    | 41.680          | 3.080                | 1.610               | 4.580                  |
| 180           | 44.04    | 40.813          | 3.227                | 1.687               | 4.797                  |
| 190           | 43.34    | 39.974          | 3.366                | 1.756               | 5.016                  |
| 200           | 42.67    | 39.160          | 3.510                | 1.830               | 5.230                  |
| Mean          |          |                 | 1.919                | 0.979               | 2.904                  |
| Sd            |          |                 | 1.089                | 0.587               | 1.573                  |

Note: Column 1 is identical to Column 5 of Table 2. The delta symbol is an abbreviation for "difference with". The first value of the difference is always intended as the ITU value minus column 2. Column 3 is Column 1 minus column 2. Column 4 is the ITU prediction for the lowest height and my prediction. Column 5 is the maximum difference I could produce between ITU values and mine.

deviation for each distance value is very small. Even at its maximum value this is just four percent of the average field strength. It is reassuring that even when unsure of the actual height of the receiving antenna the maximum error is small.

In Table 3, I compare my field strength simulations to ITU's. I show that my simulations are an almost perfect replication of the ITU's for close receivers. As distance increases, the simulation becomes less precise and generally underestimates the value of field strength at a given distance but always by a reasonably small margin. For convenience, column 1 presents the average field strength as calculated by ITU from column 5 of Table 1. Column 2 shows my own predictions based on the same observables used in ITU (2010): frequency at 500

kHz, wattage set to 20 watts, conductivity level equal to 0.03 s/m.

To evaluate the quality of my simulations, columns 3 to 6 present four distinct differences between the ITU's numbers and my own prediction.

In column 3, I show the difference between the average ITU simulation and my own simulation. This column shows that I generally underestimate the value of field strength for all distances except 10 km. The magnitude of the difference between the two simulations ranges from a minimum of -0.02 Db/m to a maximum of 3.51 Db/m, with an average value of 1.919 Db/m. The distance is smallest for the close receivers and becomes larger monotonically as distance increases. As a percentage of the simulation value the percentage of the discrepancies are quite small especially for field strength below 100 km where the value ranges between 0 to 3 percent.

Column 4 shows that my simulations are more precise when I compare them to ITU's field strength at the minimum height, which most likely represent the values closest to reality. In this case the average discrepancy is below 1 Db/m.

In column 5, I construct the worst case scenario, generating the maximum difference possible between ITU's numbers and my simulation. In this case the discrepancy increases faster becoming larger than 5% of the simulated value at 100 km and about 12% at maximum distance.

## 3 Conclusion

To study the role of radio networks on American households I need to reconstruct the radio coverage of American radio stations during the interwar period. Here, I have briefly discussed the technical aspects of radio broadcasting with amplitude modulation, the predominant broadcasting technology until the 1940s.

I have started by outlining the different type of radio waves used with amplitude modulation and justified my focus on ground waves. I explained the theory of ground waves and the factors affecting them, especially ground conductivity. I then went into details on the empirical model I used to bridge the gap between theory and the data I was able to retrieve. I shortly depicted the way I implemented the model on R. Finally, I have shown that my simulations almost perfectly replicate official ITU tables for short distances. While the simulation become less precise with larger distances, they always do so by a reasonably small margin.

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