

Deep Learning in Data Science $\mathrm{DD}2424$

SOLUTION TO ASSIGNMENT 2

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1 Introduction

This assignment aims at training a double-layer network using *gradient descent* method. The dataset used in this assignment is CIFAR-10.

2 Methods & Mechanisms

The network consists of two sets of linear classifiers W together with bias vectors \mathbf{b} . The classifying functions are similar as in assignment 1 and the only addition is that we applied a activation function

$$\mathbf{h} = max(0, \mathbf{s}_1) \tag{1}$$

2.1 Gradient Calculation

The gradient is now calculated by

$$\frac{\partial J}{\partial \mathbf{W}_2} = \frac{1}{|\mathcal{D}|} \sum \mathbf{g}_2 \tag{2}$$

$$\frac{\partial J}{\partial \mathbf{b}_2} = \frac{1}{|\mathcal{D}|} \sum \mathbf{g}_2^T \mathbf{x}^T + 2\lambda \mathbf{W}_2$$
 (3)

$$\frac{\partial J}{\partial \mathbf{W}_1} = \frac{1}{|\mathcal{D}|} \sum \mathbf{g}_1 \tag{4}$$

$$\frac{\partial J}{\partial \mathbf{b}_1} = \frac{1}{|\mathcal{D}|} \sum \mathbf{g}_1^T \mathbf{x}^T + 2\lambda \mathbf{W}_1 \tag{5}$$

(6)

where

$$\mathbf{g}_2 = -\frac{\mathbf{y}^T}{\mathbf{y}^T \mathbf{p}} \left(\operatorname{diag}(\mathbf{P}) - \mathbf{P} \mathbf{P}^T \right)$$
 (7)

$$\mathbf{g}_1 = \mathbf{g}_2 \cdot \mathbf{W}_2 \cdot (\operatorname{diag}(\operatorname{Ind}(\mathbf{s}_1) > 0)) \tag{8}$$

the implementation of the *gradient descent* can be seen on the file ComputeGradient.m. We have not tried the file ComputeGradientNum.m for verification of correctness but are quite convinced that our implementation is correct, since by choosing some hyper-parameters the loss function converges to a satisfactory level.

2.2 The effect of momentum

We set #(number of epochs) = 10 and kept every hyper-parameter else unchanged but trained the network with/without using momentum. We set $\rho = 0.9$. After 10 epochs the loss function converges to 2.3011 without momentum while 1.9359 with momentum which is much smaller than the former. The loss function becomes 1.8403 when $\rho = 0.99$.

Conclusion: Adding momentum would increase the speed of convergence when the gradient direction is correct. But too high momentum may lead the gradient 'oscillate' across the minimum when the training process plateaus.

| # | η | λ |
|---|--------|------------------------|
| 1 | 0.0035 | 0.0129 |
| 2 | 0.0033 | $3.2291 \cdot 10^{-5}$ |
| 3 | 0.0013 | 0.0079 |

Table 1: Coarse search.

| # | η | λ |
|---|------------------------|------------------------|
| 1 | $5.9438 \cdot 10^{-4}$ | $1.5142 \cdot 10^{-4}$ |
| 2 | 0.0028 | 0.0067 |
| 3 | $1.0385 \cdot 10^{-4}$ | $1.5504 \cdot 10^{-5}$ |

Table 2: Fine search.

2.3 Coarse search for lambda and eta

We set #(number of epochs) = 10 and kept every hyper-parameter else unchanged but trained the network with/without using momentum. We tried 100 sample where λ range from 10^{-1} to 10^{-6} and η from 10^{-1} to 10^{-4} . The top 3 combination of λ - η s are shown below. We will do the fine search using η range from 0.001 to 0.003 and λ from $1 \cdot 10^{-5}$ to $1 \cdot 10^{-2}$.

2.4 Fine search for lambda and eta

We set #(number of epochs) = 10 and kept every hyper-parameter else unchanged but trained the network with/without using momentum. We tried 100 sample where η range from 0.001 to 0.003 and λ from $1 \cdot 10^{-5}$ to $1 \cdot 10^{-2}$. The top 3 combination of λ - η s are shown below. Among the top 3 there are roughly two sets of magnitudes of η - λ : 10^{-4} and 10^{-3} . Basically those two parameters keep the same magnitude. Later on we will try those top 1 set of hyper-parameters.

2.5 Training result using best hyper-parameters (by fine search)

We tried $\eta=0.0028$, $\lambda=0.0067$ and $\eta=5.9438\cdot 10^{-4}$, $\lambda=1.5142\cdot 10^{-4}$ (the first 2 combinations) and trained the network by 30 epochs. Below are the training results. After 30 epochs they obtained 19.53% and 13.16% accuracy respectively.

2.6 Training result using best hyper-parameters (by grid search)

We are not satisfied with the result above since the accuracies are too low. 100 coarse searches could not cover the whole combination of λs and ηs from such a huge range. Using grid search (and with longer time) we have found a 'better' solution: $\eta = 0.02$, $\lambda = 1.5 \cdot 10^{-5}$. The test accuracy is 44.07% and the plot showing the training cost and the validation cost is shown below.

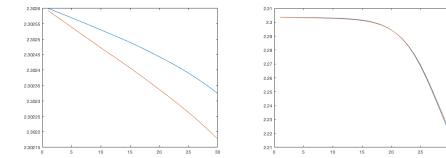


Figure 1: Training results using $\eta=5.9438\cdot 10^{-4},~\lambda=1.5142\cdot 10^{-4}$ (left) and $\eta=0.0028,~\lambda=0.0067$ (right).

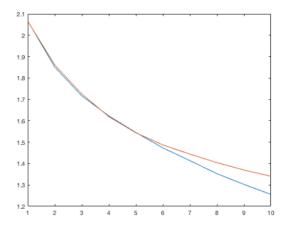


Figure 2: cost function of training (blue) and validation (red), lambda=1.5*10^{-5}, eta=0.02