CAP 2017, HW 7 due March 14

Each question is worth 5 points.

1. (a) Calculate the following integrals

   - $$\int_{1/3}^{3} \frac{\sqrt{x}}{x^2+x}\,dx$$
   - $$\int \frac{\cos(x)}{\sin^2(x)+\sin(x)}\,dx$$
   - $$\int \frac{1}{x^4+1}\,dx$$

   (b) Determine whether the following integrals are convergent or divergent, and evaluate those that are convergent.

   - $$\int_{-\infty}^{0} 2^r\,dr$$
   - $$\int_{-\infty}^{\infty} \cos(\pi t)\,dt$$

2. Astronomers use a technique called *stellar stereography* to determine the density of stars in a star cluster from the observed (two-dimensional) density that can be analysed from a photograph. Suppose that in a spherical cluster of radius R the density of stars depends only on the distance r from the centre of the cluster. If the perceived star density is given by $y(s)$, where s is the observed planar distance from the centre of the cluster, and $x(r)$ is the actual density, it can be shown that

   $$y(s) = \int_s^R \frac{2r}{\sqrt{r^2-s^2}} x(r)\,dr.$$

   If the actual density of stars in a cluster is $x(r) = \frac{1}{2}(R-r)^2$, show that the perceived density is

   $$y(s) = s^2 R\left[\ln(s) - \ln(R+\sqrt{R^2-s^2})\right] + \sqrt{R^2-s^2}\left(\frac{2}{3}s^2 + \frac{1}{3}R^2\right).$$

3. Use the comparison theorem to determine whether the integral

   $$\int_0^{\infty} \frac{\arctan(x)}{2+e^x}\,dx$$

   is convergent or divergent.

4. (a) Find the number a such that the line $x = a$ bisects the area under the curve $y = 1/x^2$, $1 \le x \le 4$.

   (b) Find the number b such that the line $y = b$ bisects the area in part (a).