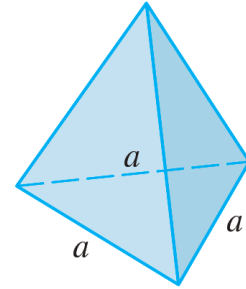


CAP 2017, HW 8 due March 21

Give complete explanations of what you are doing, written in full sentences. Solutions that have all the correct calculations and computations, but lack explanations, will not get full marks!

1. Find the volume of a pyramid with height h and base an equilateral triangle with side a (i.e. a tetrahedron).



2. A bowl is shaped like a hemisphere with a diameter of 30 cm. A ball with diameter 10 cm is placed in the bowl and water is poured into the bowl to a depth of h cm. The ball does not float. Find the volume of water in the bowl.
3.
 - (a) Sketch the curve $y^3 = x^2$.
 - (b) Use Formulas 3 and 4 of Chapter 7.4 (page 388)¹ to set up two integrals for the arc length from $(0, 0)$ to $(1, 1)$. Observe that one of these is an improper integral and evaluate both of them.
 - (c) Find the length of the arc of this curve from $(-1, 1)$ to $(8, 4)$.
4. The Great Pyramid of King Khufu was built of limestone in Egypt over a 20-year time period from 2580 BC to 2560 BC. Its base is square with side length 756 ft and its height when built was 481 ft. The density of the limestone is about 150 lb/ft³.
 - (a) Estimate the total work done in building the pyramid. Express your answer in units of foot-pounds. Hint: Look at the solutions to Workshop sheet 8.
 - (b) If a labourer worked 10 hours a day for 20 years, for 340 days a year, and did 200 ft-lb of work per hour in lifting the limestone blocks into place, about how many labourers were needed to construct the pyramid?

¹i.e., $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ and $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$