CAP 2017, HW 1 due January 24

Give complete explanations of what you are doing, written in full sentences. Solutions that have all the correct calculations and computations, but lack explanations, will not get full marks!

1. A cell phone plan has a basic charge of £25 a month. The plan includes 400 free minutes and charges 5 pence for each additional minute of usage. Write the monthly cost C as a function of the number x of minutes used and graph C as a function of x for $0 \leq x \leq 600$.

2. Let $f(x) = x - \lfloor x \rfloor$, where $\lfloor x \rfloor$ denotes the greates integer function.

   a) Sketch the graph of f.

   b) If n is an integer, evaluate (i) $\lim_{x \to n^-} f(x)$, (ii) $\lim_{x \to n^+} f(x)$.

   c) For what values of a does $\lim_{x \to a} f(x)$ exist?

3. a) Suppose $f(x)$ and $g(x)$ are even functions. What can you say about $f(x) + g(x)$ and $f(x)g(x)$?

   b) What if $f(x)$ and $g(x)$ are both odd?

   c) Prove that every function defined on the real line can be written as a sum of an even and an odd function.

4. Use the Intermediate Value Theorem to prove that there is a positive number c such that $c^2 = 2$.