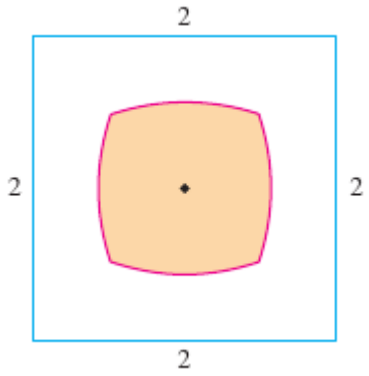
CAP 2017, HW 6 due March 7

Give complete explanations of what you are doing, written in full sentences. Solutions that have all the correct calculations and computations, but lack explanations, will not get full marks!

1. If f' is continuous on $[a, b]$, show that
$$2\int_a^b f(x)f'(x)\,dx = [f(b)]^2 - [f(a)]^2.$$

2. The figure shows the region inside a square consisting of those points that are closer to the centre than to the sides of the square. The sides of the square have length two. Find the area of the region.
Hint: use the fact that the region is symmetric.



3. Evaluate the integrals
   - $$\int_1^4 \sqrt{t}\ln(t)\,dt$$
   - $$\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$$
   - $$\int_0^1 e^{\sqrt{x}}\,dx$$
   - $$\int \tan^2(x)\sec(x)\,dx$$
   - $$\int \frac{1}{\sqrt{t^2-6t+13}}\,dt$$

4. A rocket accelerates by burning its onboard fuel, so its mass decreases with time. Suppose the initial mass of the rocket at liftoff (including its fuel) is m, the fuel is consumed at rate r, and the exhaust gases are ejected with constant velocity v_e (relative to the rocket). A model for the velocity of the rocket at time t is given by the equation
$$v(t) = -gt - v_e \ln\left(\frac{m-rt}{m}\right),$$
where g is the acceleration due to gravity and t is not too large. If $g = 9.8$ m/s^2, $m = 30,000$ kg, $r = 160$ kg/s, and $v_e = 3000$ m/s, find the height above ground of the rocket one minute after liftoff.