CAP 2017, HW 3 due February 7

Give complete explanations of what you are doing, written in full sentences. Solutions that have all the correct calculations and computations, but lack explanations, will not get full marks!

1. (5 points) Under certain circumstances a rumour spreads according to the equation

$$p(t) = \frac{1}{1 + ae^{-kt}},$$

where $p(t)$ is the proportion of the population that knows the rumor at time t (in days) and a and k are positive constants.

   (a) Find $\lim_{t\to\infty} p(t)$. What does this mean for the rumor?

   (b) Find the rate of spread of the rumor.

   (c) Find the inverse function of $p(t)$ and give an interpretation of the meaning.

   (d) Graph p for the case $a = 10, k = 0.5$ and use your graph to estimate how long it will take for 80% of the population to hear the rumor. Can you also calculate this time?

2. (5 points) Bismuth-210 has a half-life of 5.0 days.

   (a) A sample originally has a mass of 800 mg. Find a formula for the mass remaining after t days.

   (b) Find the mass remaining after 30 days.

   (c) When is the mass reduced to 1 mg?

   (d) Sketch a graph of the mass function.

3. (5 points) If f'' is continuous, show that

$$\lim_{h\to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x).$$

4. (5 points) Suppose that $3 \le f'(x) \le 5$ for all values of x, where f is a function defined on all of the real numbers and differentiable everywhere. Show that

$$18 \le f(8) - f(2) \le 30.$$