$$\frac{d}{d\theta} \mathbb{E}_\theta(\varphi) = \frac{d}{d\theta} \sum_x \varphi(x)\, p(x;\theta)$$

$$= \sum_x \varphi(x)\, \frac{d}{d\theta} p(x;\theta)$$

$$= \sum_x \varphi(x)\, \frac{d}{d\theta} \log p(x;\theta)\; p(x;\theta)$$

$$= \sum_x \Big(\varphi(x) - \sum_y \varphi(y)\, p(y;\theta) \Big) \frac{d}{d\theta} \log p(x;\theta)\; p(x;\theta)$$

$$= \mathrm{Cov}_\theta\Big(\varphi, \frac{d}{d\theta} \log p(x;\theta)\Big)$$

$\theta \mapsto \Big(p_\theta, \frac{d}{d\theta} \log p_\theta\Big)$ is a LIFTING of $\theta \mapsto p(\theta)$ to the STATISTICAL BUNDLE