GRADIENT FLOW OF THE DIVERGENCE (DEFECTIVE)

$$\frac{d}{dt} D(f \| q(t)) = \langle 1 - 1_{X_0} f\, q(t)^{-1}, \overset{*}{q}(t) \rangle_{q(t)}$$

$$\operatorname{Grad} D(f \| q) = (1 - 1_{X_0} f q^{-1})$$

$$\boxed{\overset{*}{q}(t) = - \operatorname{grad} D(f \| q(t))}$$

$$\overset{*}{q}(t) = -(1 - 1_{X_0} f q^{-1})$$

$$\dot{q}(t) = 1_{X_0} f - q(t)$$

$$\frac{d}{dt}\left(1_{X_0} f - q(t)\right) = -(1_{X_0} f - q(t))$$

$$(x \in X_0) f(x) - q(t) = e^{-t}\left((x \in X_0) f(x) - q_0(x)\right)$$

$$q(t) = (x \in X_0) f(x) - e^{-t} (x \in X_0) f(x) - q_0(x))$$

$$= q_0(x) e^{-t} + (1 - e^{-t})(x \in X_0) f(x)$$

$$= \begin{cases} q_0(x) e^{-t} & x \notin X_0 \\ q_0(x) e^{-t} + (1 - e^{-t}) f(x) & x \in X_0 \end{cases}$$

$f \in \mathcal{P}(X) \qquad X_0 = \operatorname{supp} f = \{x \mid f(x) > 0\}$

$q \in \mathcal{P}_>(X) \qquad q = e^{u - K_p(u)} \cdot p \qquad p \in \mathcal{P}_>(X) \quad u \in B_p$

$$D(f \| q) = \sum_{x \in X_0} f(x) \log \frac{f(x)}{q(x)} = \sum_{x \in X} (x \in X_0) f(x) \log f(x)$$

$$- \sum_{x \in X} (x \in X_0) f(x) \log q(x)$$

$t \to u(t) \qquad t \to q(t) = e^{u(t) - K_p(u(t))} \cdot p$

$$\log q(t) = u(t) - K_p(u(t)) + \log p$$

$$\frac{d}{dt} \log q(t) = \dot{u}(t) - dK_p(u(t))[\dot{u}(t)] = \dot{u}(t) - E_{q(t)}(\dot{u}(t))$$

$$= \overset{*}{q}(t)$$

$$\frac{d}{dt} D(f \| q(t)) = - \sum_{x \in X} (x \in X_0) f(x) \overset{*}{q}(t)$$

$$= \sum_{X} - \left[(x \in X_0) f(x) q(x;t)^{-1} \right] \overset{*}{q}(x;t)\, q(x;t)$$

$$E_q\left(1_{X_0} f q^{-1}\right) = \sum_{x \in X_0} f(x) = 1$$

$$D(f \,\|\, q(t)) = \sum_{x \in \mathcal{X}_0} f(x) \log f(x)$$

$$- \sum_{x \in \mathcal{X}_0} f(x) \log \left(q_0(x) e^{-t} + (1 - e^{-t}) f(x) \right)$$

$$= \sum_{x \in \mathcal{X}_0} f(x) \log \frac{f(x)}{q_0(x) e^{-t} + (1 - e^{-t}) f(x)}$$

$$\to 0$$

GRADIENT FLOW OF TH DIVERGENCE

$p, q \in \mathcal{P}_{>}(X)$ $\qquad$ $q = e^{u - K_p(u)} \cdot p$

$$D(p \| q) = E_p\left(\log \frac{p}{q}\right) = E_p(-(u - K_p(u)))$$

$$= K_p(u)$$

$t \mapsto u(t)$ $\qquad$ $t \mapsto e_p(u(t))$

$$\frac{d}{dt} K_p(u(t)) = dK_p(u(t))[\dot{u}(t)] = E_{q(t)}(\dot{u}(t))$$

$$= E_{q(t)}(\overset{*}{q}(t) - E_p(\overset{*}{q}))$$

$$= -E_p(\overset{*}{q}(t)) = E_{q(t)}\left(\frac{p}{q(t)} \overset{*}{q}(t)\right)$$

$$= \left\langle \left(\frac{p}{q(t)} - 1\right), \overset{*}{q}(t) \right\rangle_{q(t)}$$

is a mixture

$$\overset{*}{q}(t) = - \frac{p}{q(t)} - 1$$

$$\dot{q}(t) = p - q(t)$$

$$q(t) = (1 - e^{-t})p + e^{-t} q_0$$

$$\frac{d}{dt}(p - q(t)) = -(p - q(t))$$

$$p - q(t) = e^{-t}(p - q_0)$$

$$\phi(q) = \sum 1 - q^2 \qquad \frac{d}{dt}\phi(q(t)) = \sum -2q(t)\dot{q}(t) =$$

$$\sum \quad -2q^{*}(t)\,\overset{*}{q}(t)\,q(t) =$$

$$\sum \left(-2q(t) - E_{q(t)}(-2q(t))\right)\overset{*}{q}(t)\,q(t)$$

$$\text{Grad}\,\phi(q) = -2q + 2E_q(q) = -2q + 2\left(E_q(q) - 1\right) + 2$$

$$= -2(q-1) \div 2\phi(q)$$