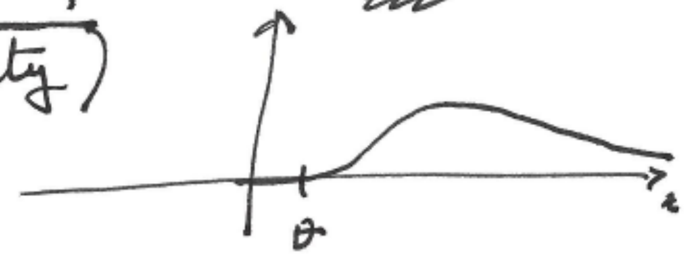
F-Score without regularity (no fixed support)

$p(x) = \frac{1}{2} x^2 e^{-x}$ $(x>0)$ (Gamma density)



$p(x;\theta) = \frac{1}{2}\, p(x-\theta)$ $(x>\theta)$

$f \in \mathcal{C}^\infty(\mathbb{R})$ $\qquad \theta \mapsto \int f(x)\, p(x;\theta)\, d\theta = \frac{1}{2} \int_\theta^{+\infty} f(x) (x-\theta)^2 e^{-(x-\theta)}\, dx$

$$\frac{d}{d\theta} \int f(x)\, p(x;\theta) = \frac{1}{2} \int_\theta^{+\infty} f(x) \left[2(x-\theta) e^{-(x-\theta)} + (x-\theta)^2 e^{-(x-\theta)} \right] dx$$

$$= \frac{1}{2} \int_\theta^{+\infty} f(x) \left[2(x-\theta) + (x-\theta)^2 \right] e^{-(x-\theta)}\, dx$$

$$= \int_\theta^{+\infty} f(x)\, \frac{2(x-\theta) + (x-\theta)^2}{(x-\theta)^2}\, \frac{1}{2} (x-\theta)^2\, \overset{e^{-(x-\theta)}}{\cancel{p(x-\theta)}}\, dx$$

$$= \int f(x) \left(\frac{2 + x - \theta}{x-\theta} \right) p(x;\theta)\, dx$$

$\overset{*}{p}(x;\theta) = \frac{2}{x-\theta} + 1$ but arbitrary on $(x<\theta)$