$$\left(1-\eta_1\xi\right)\frac{\eta_2(1-\eta_1)}{\eta_1(1-\eta_2)} = 1+(1-\eta_1)\xi$$

$$\frac{\eta_2(1-\eta_1)}{\eta_1(1-\eta_2)} - 1 = \left((1-\eta_1)+\frac{\eta_2(1-\eta_1)}{1-\eta_2}\right)\xi$$

$$\frac{\eta_2(1-\eta_1)-\eta_1(1-\eta_2)}{\eta_1(1-\eta_2)} = \frac{(1-\eta_1)(1-\eta_2)+\eta_2(1-\eta_1)}{1-\eta_2}\xi$$

$$\frac{\eta_2-\eta_1}{\eta_1(1-\eta_2)} = \frac{1-\eta_1-\eta_2+\eta_1\eta_2+\eta_2-\eta_1\eta_2}{1-\eta_2}\xi$$

$$\boxed{\frac{\eta_2^j-\eta_1^j}{\eta_1^j(1-\eta_1^j)} = \xi}$$

$$S_{P\eta_2}(P\eta_2) = \sum_j \frac{\eta_2^j-\eta_1^j}{\eta_1^j(1-\eta_1^j)}\left(u^j-\eta_1^j\right)$$