

ST2005: Applied Probability II

Lab Assignment 4

$X_1 \sim N(\theta_1, \sigma^2)$ and $X_2 \sim N(\theta_2, \sigma^2)$ are independent and σ^2 is known and equal to sum of the first two right digits of your student number.

Based on 1 sample from each distribution, and for the hypothesis test

$$H_0: \theta_1 = \theta_2 = 0$$

$$H_1: \theta_1^2 + \theta_2^2 > 0$$

if the critical region is given in form of $X_1^2 + X_2^2 > C$, using 100,000 samples from X_1 and X_2 and by assuming $\alpha = 0.05$, find the value of C and show that

$$\frac{C}{\sigma} = \chi_{2,0.05}^2$$

Hint.

If

$$X \sim N(\theta, \sigma^2)$$

then

$$Z = \frac{X - \theta}{\sigma} \sim N(0, 1)$$

and therefore

$$Y = Z^2 \sim \chi_1^2 \text{ (Chi-square with 1 degree of freedom)}$$