Modelling the Spread of COVID-19 Cases

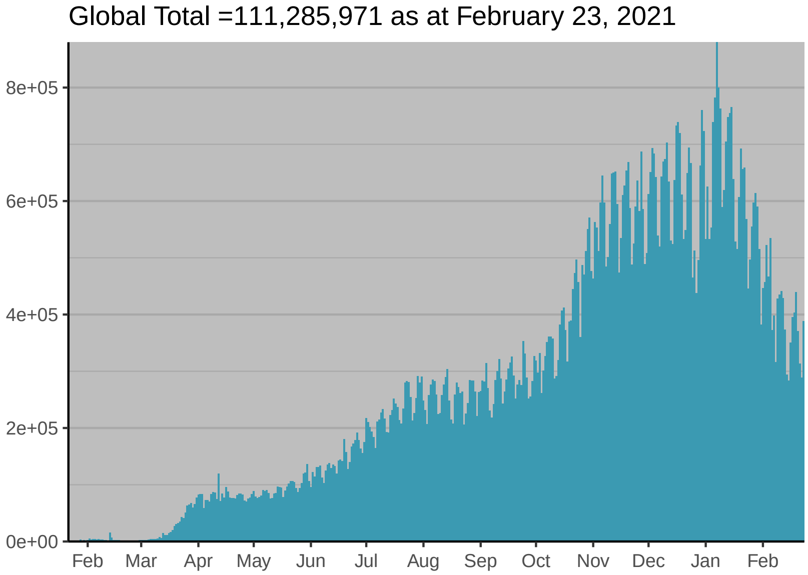
MAU44M00: Mathematics Project Presentation

Anthony Gibbons
Student number: 17322353
Supervisor: Athanasios Georgiadis

February 2021

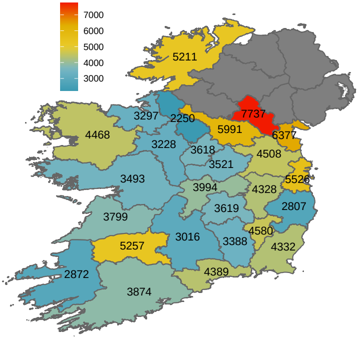
The Coronavirus disease (COVID-19) was first characterized by the World Health Organisation as pandemic on 11th March 2020. The outbreak has affected almost every aspect of human life throughout 2020, and is expected to continue for much of 2021.

Daily Cases Worldwide [**countrydata**]

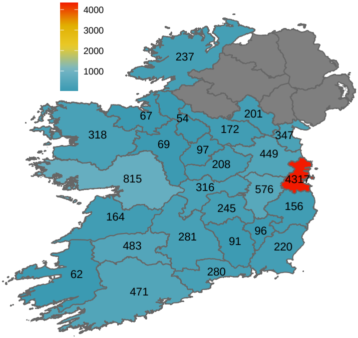


Situation in Ireland [**irelanddata**]

Cases in Ireland per 100,000 population by county
Cumulative, up to February 21, 2021



Cases in Ireland by county
From February 08, 2021 to February 21, 2021



Basic Model - Assumptions [**grigor20**]

(I) Any infected person becomes ill (symptomatic) and infectious on the *q*-th day after infection.[1]

(A) During each day, each ill person unconfined infects on average *a* other persons.

(B) During each day, a fraction *b* of ill people loose gets isolated (hospitalized or otherwise) and withdrawn from a further spread of the epidemic.

[1] The number of days before an infected person becomes infectious is called the latent period, and before he/she becomes symptomatically ill – the incubation period. Here we assume for simplicity that these two periods are equal.

Basic Model - Implementation

x_n^* is the actual number of reported cases on day n
x_n is the (according to the model) number of infected people that are detected and isolated during the day n

$$x_{n+1} = (1-b)x_n + ax_{n-q}.$$

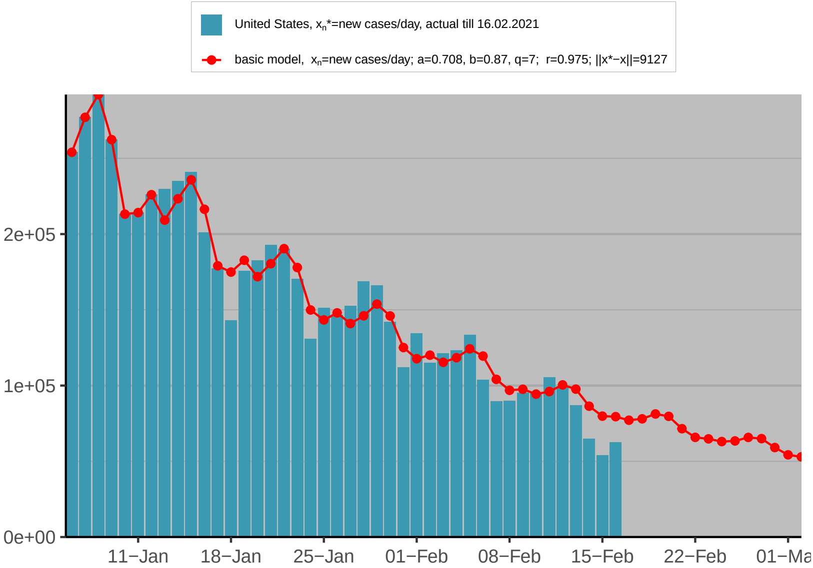
We let the model equal the actual data for the first $q+1$ days

$$x_n = x_n^* \text{ for } n = 0, 1, \ldots, q,$$

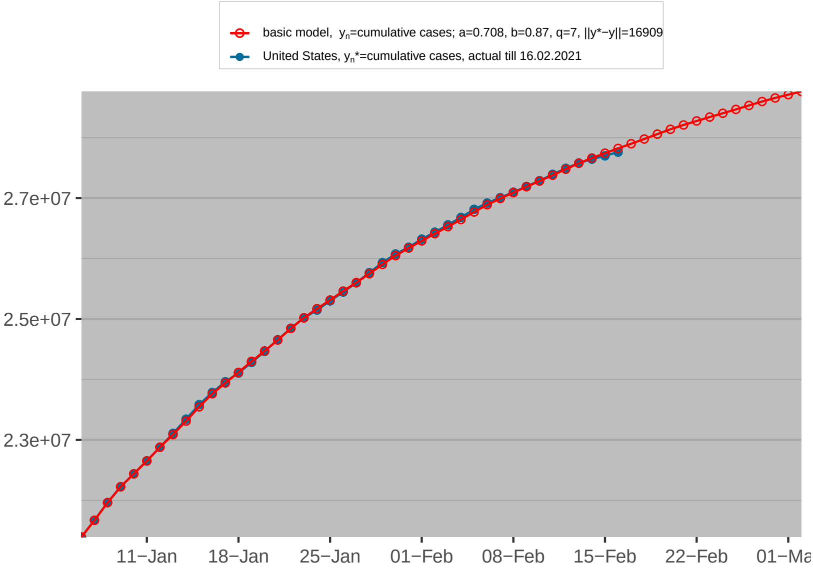
To fit our model we optimize against the normalized 1-norm:

$$||x^* - x|| := \frac{1}{N+1} \sum_{n=0}^{N} |x_n - x_n^*|.$$

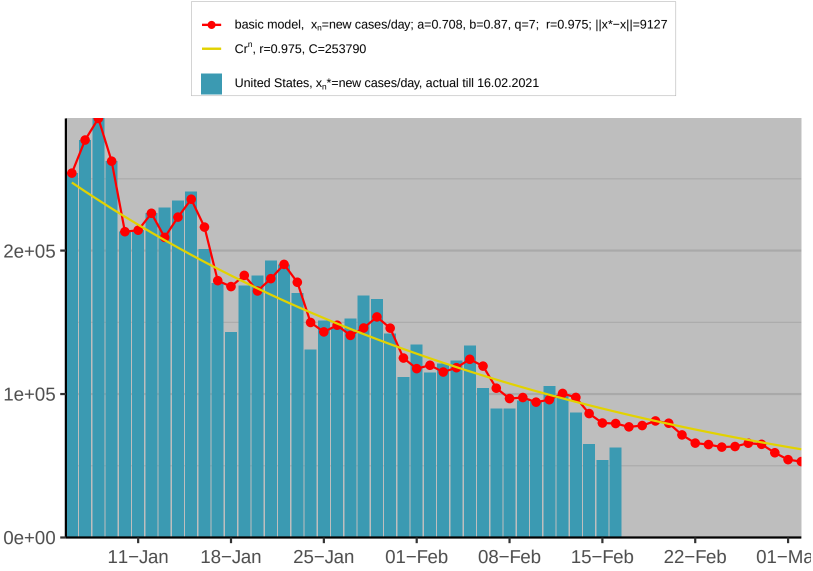
United States - Basic Model



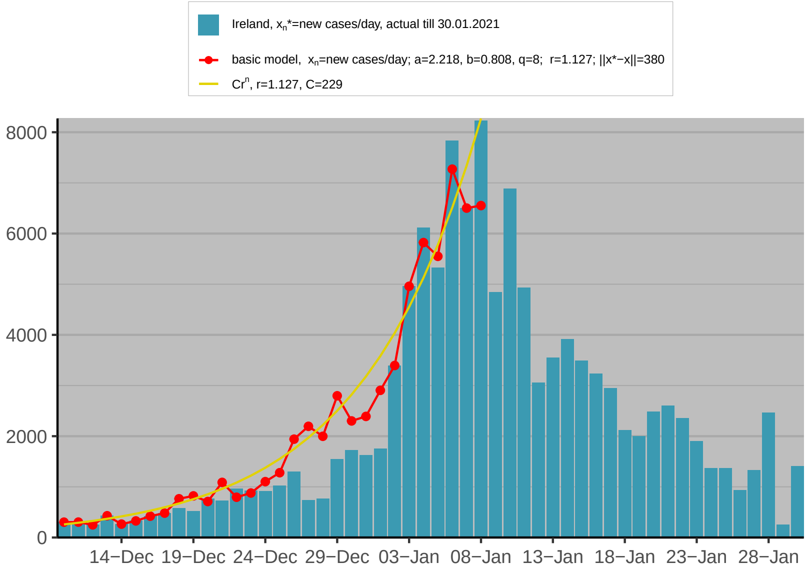
United States - Basic Model



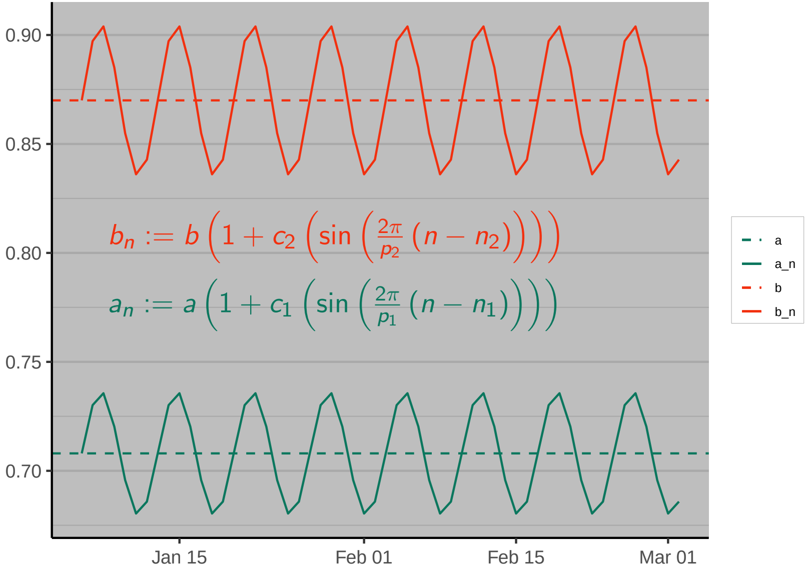
United States - Limiting Curve



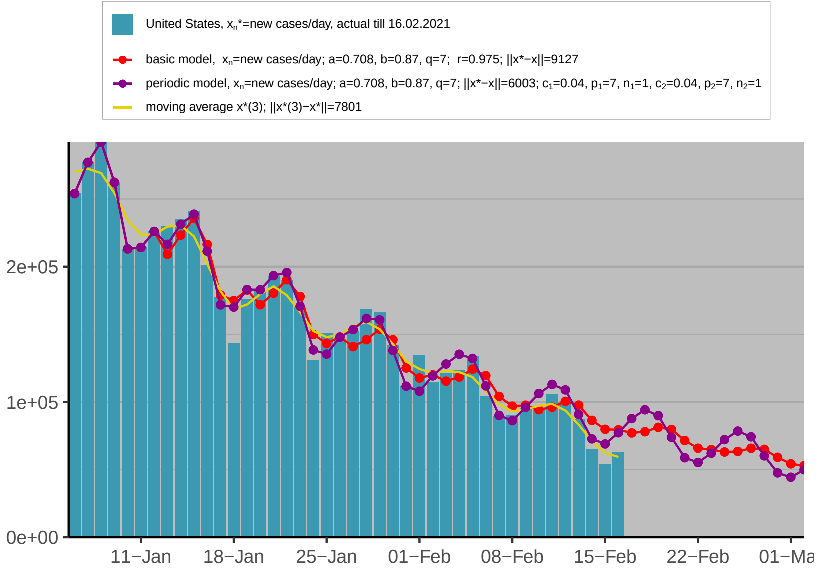
Ireland - Limiting Curve Growing Exponentially



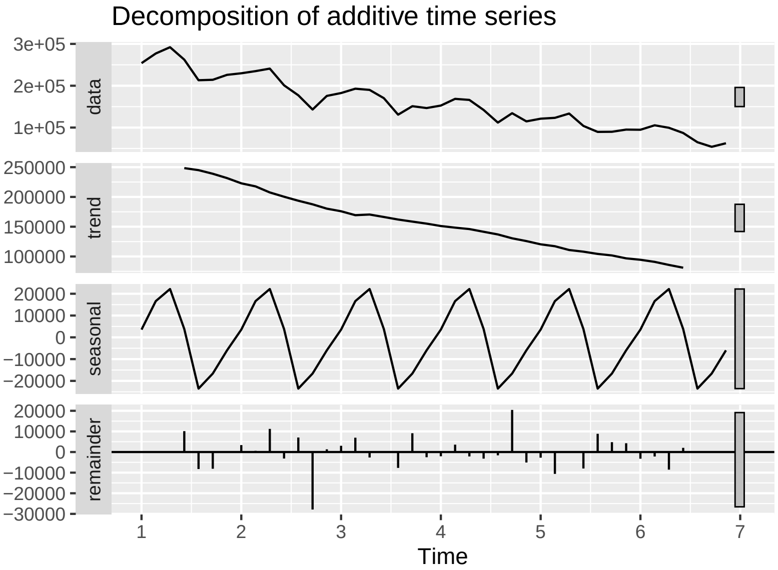
United States - Periodic Parameters



United States - Periodic Model



Time series decomposition [**Hyndman-et-al-2018**]



Holt-Winters Model

Suppose there are N observations.

Initial step:

$$
\begin{aligned}
& L_s = \frac{1}{s}\sum_{i=1}^{s} x_i \\
& b_s = \frac{1}{s}\left[\frac{x_{s+1}-x_1}{s} + \frac{x_{s+2}-x_2}{s} + \cdots + \frac{x_{2s}-x_s}{s}\right] \\
& S_n = x_n - L_s, \;\; n = 1, \ldots, s
\end{aligned}
$$

and choose parameters $0 \leq \alpha, \beta, \gamma \leq 1$

Then compute for $s < n \leq N$:

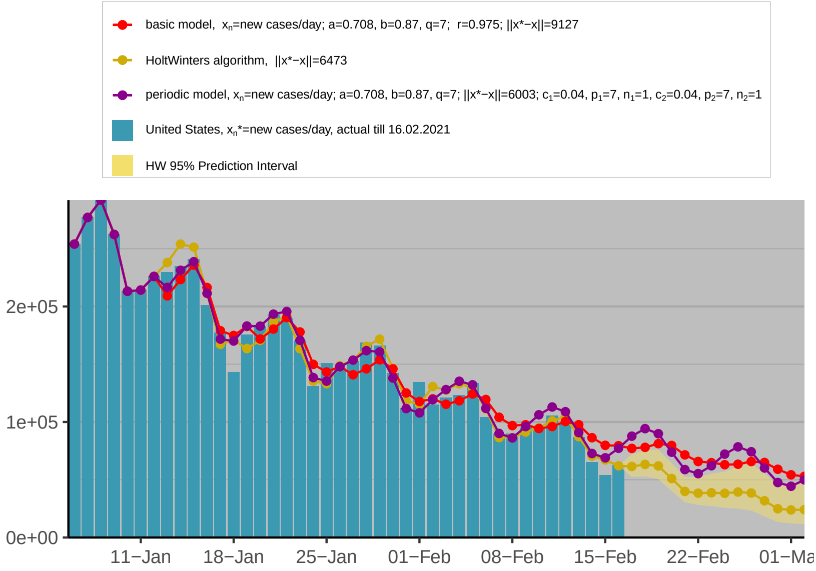
$$
\begin{array}{lll}
\text{Level} & L_n & = \alpha(x_n - S_{n-s}) + (1-\alpha)(L_{n-1} + b_{n-1}) \\
\text{Trend} & b_n & = \beta(L_n - L_{n-1}) + (1-\beta)b_{n-1} \\
\text{Seasonal} & S_n & = \gamma(x_n - L_n) + (1-\gamma)S_{n-s} \\
\text{Forecast} & F_{n+1} & = L_n + b_n + S_{n+1-s}
\end{array}
$$

For subsequent observations,

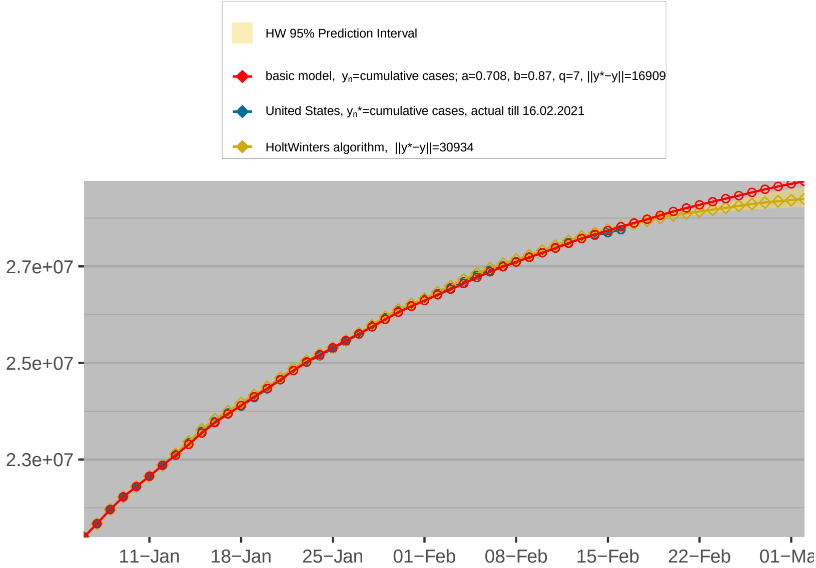
$$F_{N+k} = L_N + k \cdot b_N + S_{N+k-s}$$

Figure: Seasonal Holt Winter's Additive Model Algorithm (denoted SHW_+)

United States - Holt-Winters Model



United States - Holt-Winters Model



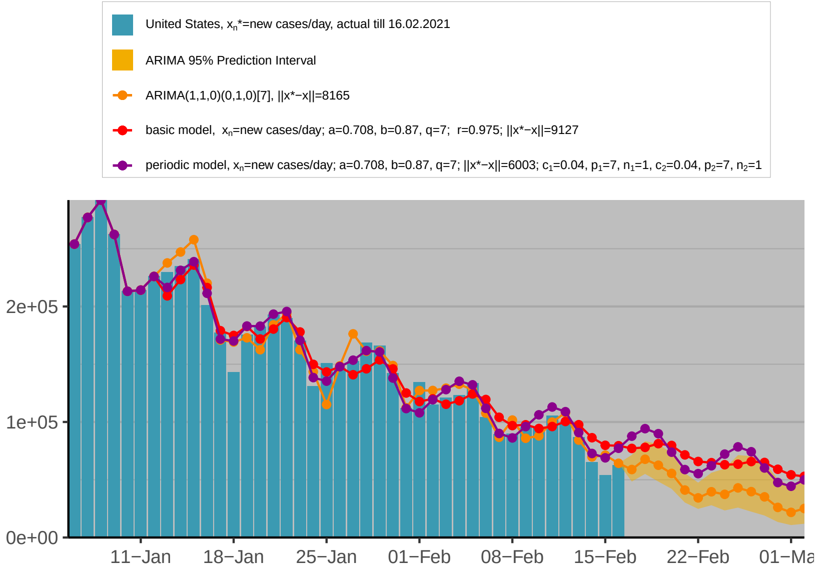
ARIMA(p, d, q) Model

A non-seasonal AutoRegressive Integrated Moving Average Model is defined as

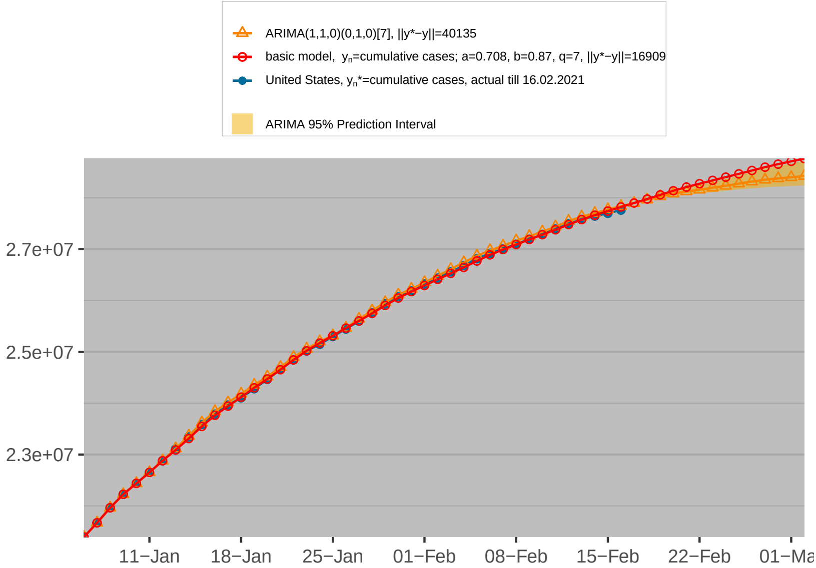
$$\underbrace{\left(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p\right)}_{AR(p)} \underbrace{(1 - B)^d}_{I(d)} x_n = c + \underbrace{\left(1 - \psi_1 B - \psi_2 B^2 - \cdots - \psi_q B^q\right)}_{MA(q)} \varepsilon_n$$

where $B^k x_n = x_{n-k}$
The orders p, d, q are computed by analysing the correlation functions (ACF and PACF).
The $p + q + 1$ coefficients $c, \phi_1, \ldots, \phi_p, \psi_1, \ldots, \psi_q$ are computed using Maximum Likelihood Estimation

United States - ARIMA$(p, d, q)(P, D, Q)$ Model



United States - ARIMA$(p, d, q)(P, D, Q)$ Model



Regression or ARIMA$(p, 0, 0)$ Model

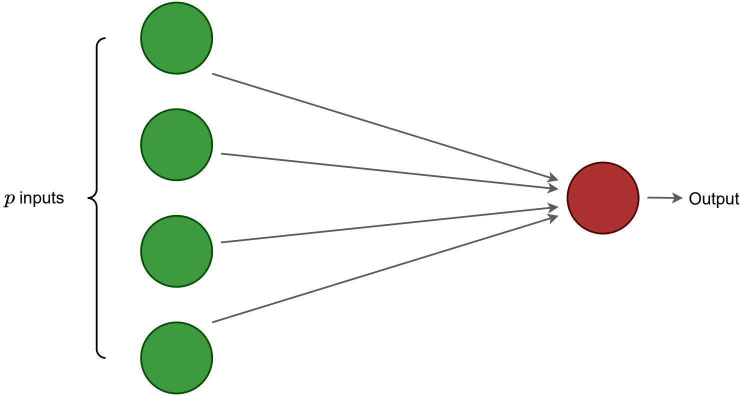


Figure: A linear regression model, or ARIMA$(p, 0, 0)$ model.

Neural Network NNAR($p, k, 0$) Model

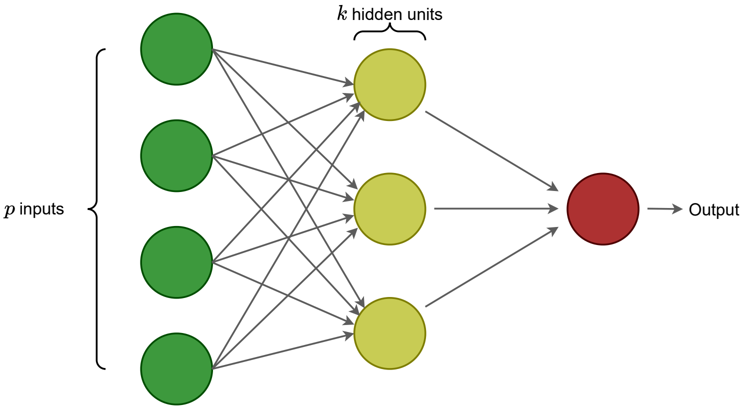
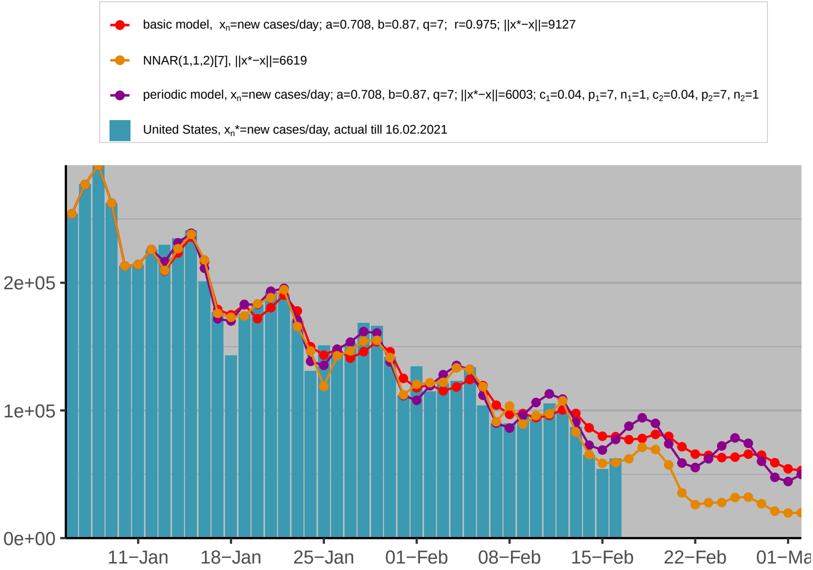
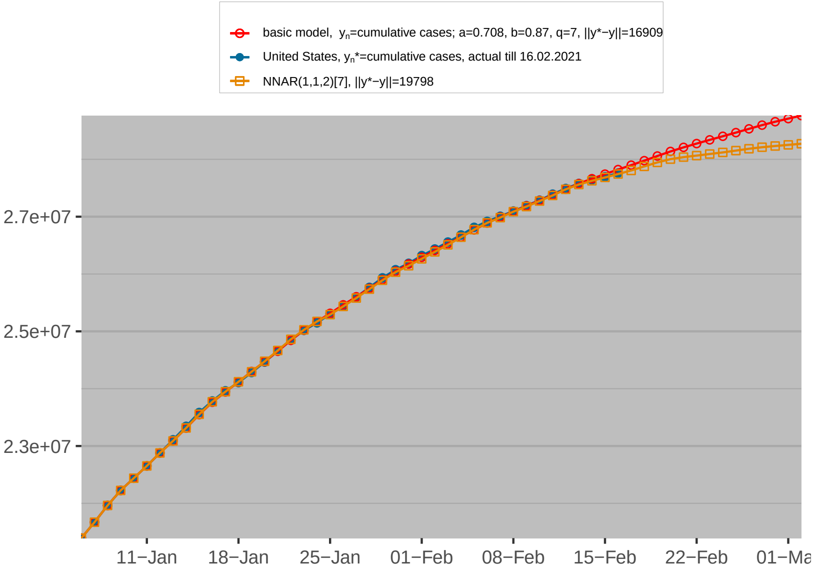


Figure: A neural network with p inputs and one hidden layer with k hidden neurons.

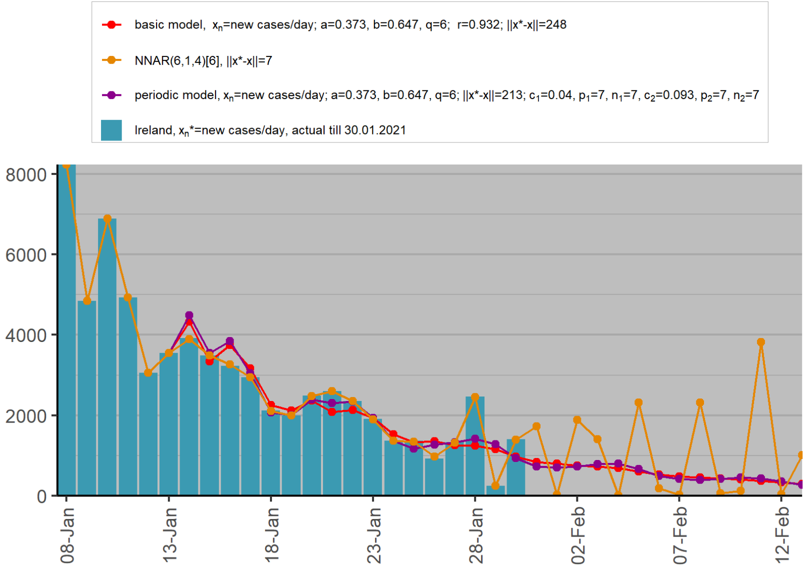
United States - Neural Network Autoregression Model NNAR(p, k, P)



United States - Neural Network Autoregression Model NNAR(p, k, P)

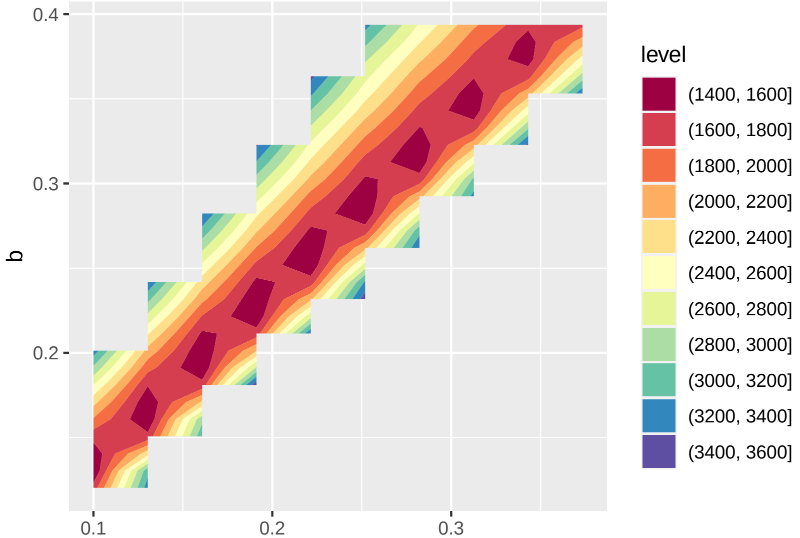


Ireland - Overfitting the Neural Network Model



Future work

- Improve optimisation algorithm (danger of local optima, like with Ireland below)



Future work

What's left to do:

- **Train** and **Test** datasets to rigorously compare models.
- Factor in distance $||y^* - y||$ to ensure the basic model matches **cumulative cases** as well as daily cases
- Improving code **efficiency**

References