Problem 3

1. The set of all possible graphs generated by a $G(n, p)$ model, where n is the total number of nodes of each graph and p is the probability of there being an edge between any two nodes of each graph. The set has $\sum_{i=0}^{(n\, nCr\, 2)} \ ((n\, nCr\, 2)\ nCr\ i)$ elements in total.

2. The probability of each element of the set is a function P of n, p and the total number of edges of the element, e.
   $P(n,\ p,\ e)\ =\ p^{e}(1\ -\ p)^{(n\, nCr\, 2)\,-\,e}$

3. Two cycles of size $\frac{n}{2}$ will take up n edges of the graph. The probability of generating a graph with n edges is:
   $P(n,\ p,\ n)\ =\ p^{n}(1\ -\ p)^{(n\, nCr\, 2)\,-\,n}$
   The numbers of graphs of n nodes with two $\frac{n}{2}$ cycles and no other edges that can be generated is:
   $N\ =\ \left(n\, nCr\ \frac{n}{2}\right)$
   Multiplying the previous two results will yield us the probability of generating a graph of n nodes with two $\frac{n}{2}$ cycles and no other edges:
   $N\ \cdot\ P(n,\ p,\ n)\ =\ \left(n\, nCr\ \frac{n}{2}\right)\ \cdot\ p^{n}(1\ -\ p)^{(n\, nCr\, 2)\,-\,n}$

4. My brain hurts.

5. $(n\ -\ 1)\ \cdot\ p$

6. $(n \, nCr \, 2) \cdot p$

7. Probability of 5 nodes from the graph forming a house subgraph:

   $P = p^6(1 - p)^{(5 \, nCr \, 2)-6}$

   Number of possible arrangements of 5 node groups inside the graph:

   $N = \prod_{i=0}^{\frac{n}{5}} ((n - 5i) \, nCr \, 5)$

   Multiplying both will yeld the expected amount of house subgaphs on a given randomly generated graph:

   $P \cdot N = p^6(1 - p)^{(5 \, nCr \, 2)-6} \cdot \prod_{i=0}^{\frac{n}{5}} ((n - 5i) \, nCr \, 5)$ <-- at least I tried