Problem Session 1

Chapter 1

17. Lot

$n = 100$ - lot size

k - number of defectives

$m = 4$ - sample size

- reject lot if 1 or more defective
- accept lot if 0 defective

X - number of defectives in sample

$$P(X=0) = \frac{\binom{k}{0}\binom{n-k}{m-0}}{\binom{n}{m}} = \frac{1 \cdot \binom{100-k}{4-0}}{\binom{100}{4}} = \frac{\binom{100-k}{4}}{\binom{100}{4}}$$

*** See Problem Session 1 R Markdown file for the rest of the solution ***

Bonus:

What if the problem told you to reject the lot if more than 1 item was defective in the sample?

- accept lot if $X=0 \vee X=1$

$$P(X=0)+P(X=1) = \frac{\binom{100-k}{4}}{\binom{100}{4}} + \frac{\binom{k}{1}\binom{100-k}{4-1}}{\binom{100}{4}}$$

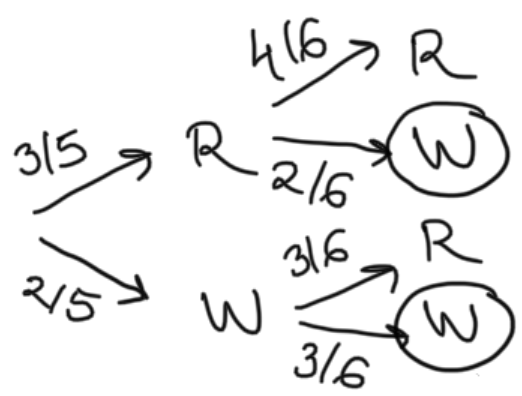
$$= \frac{\binom{100-k}{4}}{\binom{100}{4}} + \frac{k\binom{100-k}{3}}{\binom{100}{4}}$$

*** See Problem Session 1 R Markdown file for the rest of the solution ***

48.

	1st scenario	2nd scenario
3R	4R	3R
2W	2W	3W
5	6	6

a. $P(W_{2nd}) = ?$



$$P(W_{2nd}) = \frac{3}{5}\cdot\frac{2}{6} + \frac{2}{5}\cdot\frac{3}{6} = \frac{\cancel{3}}{5}\cdot\frac{1}{\cancel{3}} + \frac{\cancel{2}}{5}\cdot\frac{1}{\cancel{2}} = \frac{2}{5}$$

b. $P(R_{1st} \mid W_{2nd}) = ?$

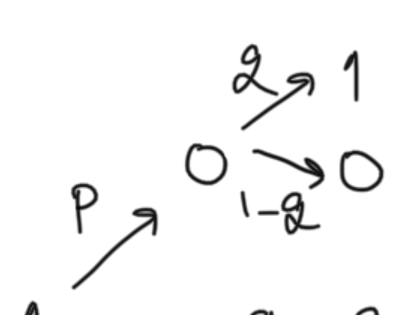
$$P(R_{1st} \mid W_{2nd}) = \frac{P(R_{1st} \wedge W_{2nd})}{P(W_{2nd})} = \frac{\frac{3}{5}\cdot\frac{2}{6}}{\frac{2}{5}} = \frac{\frac{1}{\cancel{5}}}{\frac{2}{\cancel{5}}} = \frac{1}{2}$$

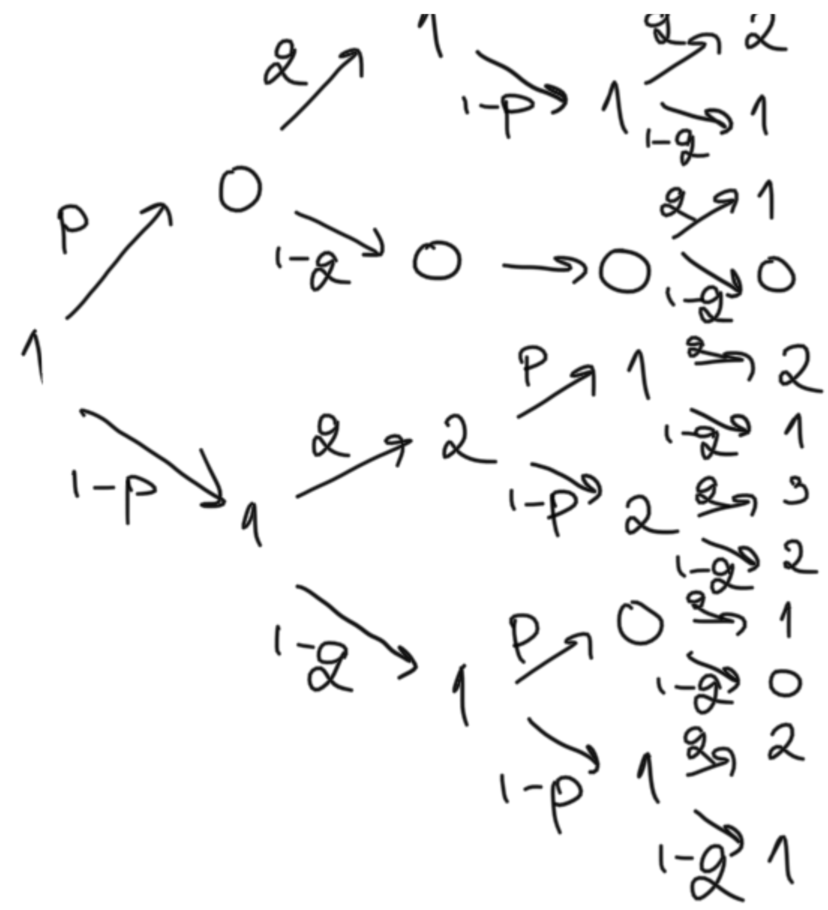
76.

$P(\text{served}) = p$
$P(\text{arrives}) = q$

- $t = 0$: $n = 1$
- $t = 2$: $P(n=0) = ?$
  $P(n=1) = ?$
  $P(n=2) = ?$
  $P(n=3) = ?$

$t=0$ $\qquad$ $t=1$ $\qquad$ $t=2$





$$P(n=0) = p \cdot q \cdot p \cdot (1-q) + p \cdot (1-q) \cdot (1-q) + (1-p) \cdot (1-q) \cdot p \cdot (1-q)$$

$$P(n=1) = p \cdot q \cdot p \cdot q + p \cdot q \cdot (1-p) \cdot (1-q) + p \cdot (1-q) \cdot q + (1-p) \cdot q \cdot p \cdot (1-q) + (1-p) \cdot (1-q) \cdot p \cdot q + (1-p) \cdot (1-q) \cdot (1-p) \cdot (1-q)$$

$$P(n=2) = p \cdot q \cdot (1-p) \cdot q + (1-p) \cdot q \cdot p \cdot q + (1-p) \cdot q \cdot (1-p) \cdot (1-q) + (1-p) \cdot (1-q) \cdot (1-p) \cdot q$$

$$P(n=3) = (1-p) \cdot q \cdot (1-p) \cdot q$$

Chapter 2

66. Let

$$f(x) = \begin{cases} \alpha \cdot x^{-\alpha-1} & , x \geq 1 , \alpha > 0 \\ 0 & , x < 1 \end{cases}$$

- generate RVs with this pdf using uniform random number generator

- Proposition D (Rice, p. 63)

$U \sim U(0,1)$
$X = F^{-1}(U)$
$\Rightarrow F$ cdf of X

- Step 1: compute F.
- Step 2: find F^{-1}.
- Step 3: generate RVs by plugging $U \sim U(0,1)$ into F^{-1}.

$$F(x) = \int_{-\infty}^{x} f(u)\,du = \int_{1}^{x} \alpha \cdot u^{-\alpha-1}\,du = \alpha \cdot \int_{1}^{x} u^{-\alpha-1}\,du$$

- Remember integration rules: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$$F(x) = \alpha \cdot \left[\frac{u^{-\alpha-1+1}}{-\alpha-1+1}\right]_1^x = \alpha \cdot \left[\frac{u^{-\alpha}}{-\alpha}\right]_1^x = -\left[u^{-\alpha}\right]_1^x$$

$$= -x^{-\alpha} + 1^{\alpha} = 1 - x^{-\alpha}$$

- find F^{-1}: $y = 1 - x^{-\alpha}$

$x = 1 - y^{-\alpha}$
$y^{-\alpha} = 1 - x \quad /^{-\frac{1}{\alpha}}$
$(y^{-\alpha})^{-1/\alpha} = (1-x)^{-1/\alpha}$
$y = (1-x)^{-1/\alpha}$
$F^{-1}(x) = (1-x)^{-1/\alpha}$

* See Problem Session 1 R Markdown file for the rest of the solution ***