

qbs121_hw6_gibran

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GEE

Using the data sets you used for for Week 5 for LMM and GLMM models:

1. Refit the models for both LMM and GLMM using generalized estimating equations:

```
library(tidyverse)
library(geepack)
library(lme4)
library(mice)

# LMM
data <- read.csv('HousePrices.csv')

# data preprocessing
boolean_convert <- function(data) {
  if (data == "yes") {
    return(1)
  } else {
    return(0)
  }
}

data$prefer <- sapply(data$prefer, boolean_convert)

df <- data %>%
  select('price', 'lotsize', 'bedrooms', 'stories', 'garage', 'prefer')

# binary
data_binary <- read.csv('https://stats.idre.ucla.edu/stat/data/hdp.csv')
```

1a. Use the family/link function assumptions and working correlation structures that closest to the assumptions used in the previous LMM and GLMM fits. For instance, a random cluster effect in LMM/GLMM would be closest to an exchangeable working correlation matrix. Contrast the fitted coefficients for fixed effects from the GEE models to those you got from LMM and GLMM, and comment on any differences that might be seen between the coefficient estimates and standard errors.

```
# modeling
model <- lmer(log(price) ~ log(lotsize) + bedrooms + garage + stories +
              (1 | prefer), data = df)
print(summary(model))
```

```

## Linear mixed model fit by REML ['lmerMod']
## Formula: log(price) ~ log(lotsize) + bedrooms + garage + stories + (1 |
##     prefer)
## Data: df
##
## REML criterion at convergence: 66.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.3665 -0.6408  0.0572  0.6145  2.7973
##
## Random effects:
##   Groups   Name                Variance Std.Dev.
##   prefer   (Intercept)  0.01627   0.1275
##   Residual                0.06244   0.2499
## Number of obs: 546, groups: prefer, 2
##
## Fixed effects:
##              Estimate Std. Error t value
## (Intercept)   7.27369    0.26446  27.504
## log(lotsize)   0.39387    0.02962  13.299
## bedrooms       0.07422    0.01610   4.609
## garage         0.07185    0.01339   5.366
## stories        0.12618    0.01353   9.326
##
## Correlation of Fixed Effects:
##              (Intr) lg(lt) bedrms garage
## log(lotsiz) -0.924
## bedrooms    -0.079 -0.066
## garage       0.301 -0.340 -0.098
## stories      0.035 -0.059 -0.399  0.035

model_gee <- geeglm(log(price) ~ log(lotsize) + bedrooms + garage + stories,
                    id=prefer, family=gaussian, data = df)
print(summary(model_gee))

##
## Call:
## geeglm(formula = log(price) ~ log(lotsize) + bedrooms + garage +
##     stories, family = gaussian, data = df, id = prefer)
##
## Coefficients:
##              Estimate Std. err   Wald Pr(>|W|)
## (Intercept)  6.875502 0.223410  947.12 < 2e-16 ***
## log(lotsize)  0.433564 0.026173  274.41 < 2e-16 ***
## bedrooms     0.078892 0.019673   16.08 6.07e-05 ***
## garage       0.072795 0.020323   12.83 0.000341 ***
## stories      0.126298 0.008707  210.42 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation structure = independence
## Estimated Scale Parameters:
##

```

```
##           Estimate Std.err
## (Intercept) 0.06742 0.00539
## Number of clusters: 7 Maximum cluster size: 336
```

```
model_binary <- glmer(remission ~ IL6 + CRP + CancerStage + Experience +
  (1 | DID), data = data_binary, family = binomial, nAGQ = 10)
print(summary(model_binary))
```

```
## Generalized linear mixed model fit by maximum likelihood (Adaptive
## Gauss-Hermite Quadrature, nAGQ = 10) [glmerMod]
## Family: binomial ( logit )
## Formula: remission ~ IL6 + CRP + CancerStage + Experience + (1 | DID)
## Data: data_binary
##
##      AIC      BIC   logLik deviance df.resid
##    7408    7465   -3696    7392    8517
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.531 -0.444 -0.199  0.401  7.142
##
## Random effects:
## Groups Name      Variance Std.Dev.
## DID      (Intercept) 4.04      2.01
## Number of obs: 8525, groups: DID, 407
##
## Fixed effects:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -2.6264    0.5066   -5.18 2.2e-07 ***
## IL6            -0.0564    0.0115   -4.90 9.6e-07 ***
## CRP            -0.0223    0.0102   -2.18 0.029 *
## CancerStageII  -0.4874    0.0730   -6.68 2.4e-11 ***
## CancerStageIII -1.1259    0.0924  -12.19 < 2e-16 ***
## CancerStageIV  -2.5088    0.1508  -16.64 < 2e-16 ***
## Experience      0.1193    0.0274    4.35 1.3e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) IL6      CRP      CncSII CnSIII CncSIV
## IL6          -0.082
## CRP          -0.097 0.000
## CancerStgII  -0.069 0.009 0.000
## CancrStgIII  -0.047 0.013 0.010 0.446
## CancerStgIV  -0.020 0.036 0.009 0.282 0.249
## Experience   -0.963 -0.006 -0.002 -0.008 -0.014 -0.019
```

```
model_binary_gee <- geeglm(remission ~ IL6 + CRP + CancerStage + Experience,
  id=DID, data = data_binary, family = binomial)
print(summary(model_binary_gee))
```

```
##
## Call:
```

```
## geeglm(formula = remission ~ IL6 + CRP + CancerStage + Experience,
##       family = binomial, data = data_binary, id = DID)
##
## Coefficients:
##             Estimate Std.err   Wald Pr(>|W|)
## (Intercept)  -1.73707  0.35147  24.43  7.7e-07 ***
## IL6          -0.03823  0.00807  22.44  2.2e-06 ***
## CRP          -0.01774  0.00785   5.11   0.024 *
## CancerStageII -0.33205  0.05863  32.08  1.5e-08 ***
## CancerStageIII -0.71077  0.07920  80.53 < 2e-16 ***
## CancerStageIV -1.72709  0.12925 178.57 < 2e-16 ***
## Experience     0.08401  0.01923  19.09  1.2e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation structure = independence
## Estimated Scale Parameters:
##
##             Estimate Std.err
## (Intercept)   0.996  0.0574
## Number of clusters:  407 Maximum cluster size: 40
```

Between the lmer() and geeglm() model for continuous outcome above, the coefficients differ by varying degree. For the intercept, bedrooms, garage and stories variables, the coefficient we got from lmer() is higher than geeglm(), while the log(lotsize) coefficient is lower in lmer() compared to geeglm(). For standard error values between the two models, all variables have lower standard errors except for bedrooms and garage variables.

For the binary outcome, the coefficients of intercept, IL6, CRP, CancerStageIII and CancerStageIV are higher in geeglm() compared to glmer(). For the standard errors, all variables have lower standard error values in geeglm() compared to glmer() except for CancerStageII.

1.b Use the alternative working correlation structures available in GEE to see if that results in are any further differences in the estimated regression coefficients and standard errors.

```
model_gee_exc <- geeglm(log(price) ~ log(lotsize) + bedrooms + garage + stories,
                        id=prefer, family=gaussian, data = df,
                        corstr='exchangeable')
print(summary(model_gee_exc))
```

```
##
## Call:
## geeglm(formula = log(price) ~ log(lotsize) + bedrooms + garage +
##       stories, family = gaussian, data = df, id = prefer, corstr = "exchangeable")
##
## Coefficients:
##             Estimate Std.err   Wald Pr(>|W|)
## (Intercept)   7.54609  0.21003 1290.8 < 2e-16 ***
## log(lotsize)  0.36814  0.02358  243.8 < 2e-16 ***
## bedrooms      0.07376  0.01526   23.4  1.4e-06 ***
## garage        0.07346  0.02065   12.7  0.00037 ***
## stories       0.12161  0.00876  192.6 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Correlation structure = exchangeable
## Estimated Scale Parameters:
##
##           Estimate Std.err
## (Intercept)  0.0769  0.0125
##   Link = identity
##
## Estimated Correlation Parameters:
##           Estimate Std.err
## alpha      0.259   0.155
## Number of clusters: 7 Maximum cluster size: 336

model_gee_ar1 <- geeglm(log(price) ~ log(lotsize) + bedrooms + garage + stories,
                        id=prefer, family=gaussian, data = df,
                        corstr='ar1')
print(summary(model_gee_ar1))
```

```
##
## Call:
## geeglm(formula = log(price) ~ log(lotsize) + bedrooms + garage +
##   stories, family = gaussian, data = df, id = prefer, corstr = "ar1")
##
## Coefficients:
##           Estimate Std.err      Wald Pr(>|W|)
## (Intercept)  9.20123 0.25564 1295.53 < 2e-16 ***
## log(lotsize)  0.18708 0.02086  80.42 < 2e-16 ***
## bedrooms     0.05402 0.00705  58.76 1.8e-14 ***
## garage       0.04236 0.02200   3.71  0.054 .
## stories      0.06165 0.01286  22.97 1.6e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation structure = ar1
## Estimated Scale Parameters:
##
##           Estimate Std.err
## (Intercept)  0.0866 0.00416
##   Link = identity
##
## Estimated Correlation Parameters:
##           Estimate Std.err
## alpha      0.854  0.0541
## Number of clusters: 7 Maximum cluster size: 336
```

```
model_binary_gee_exc <- geeglm(remission ~ IL6 + CRP + CancerStage + Experience,
                              id=DID, data = data_binary, family = binomial,
                              corstr='exchangeable')
print(summary(model_binary_gee_exc))
```

```
##
## Call:
## geeglm(formula = remission ~ IL6 + CRP + CancerStage + Experience,
```

```
## family = binomial, data = data_binary, id = DID, corstr = "exchangeable")
##
## Coefficients:
##           Estimate Std.err Wald Pr(>|W|)
## (Intercept) -1.43713 0.33023 18.94 1.3e-05 ***
## IL6          -0.03588 0.00685 27.42 1.6e-07 ***
## CRP          -0.01420 0.00639 4.94 0.02617 *
## CancerStageII -0.30598 0.04746 41.57 1.1e-10 ***
## CancerStageIII -0.70791 0.06750 109.98 < 2e-16 ***
## CancerStageIV -1.63002 0.11811 190.45 < 2e-16 ***
## Experience    0.06502 0.01778 13.37 0.00026 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation structure = exchangeable
## Estimated Scale Parameters:
##
##           Estimate Std.err
## (Intercept)  0.981 0.047
## Link = identity
##
## Estimated Correlation Parameters:
##           Estimate Std.err
## alpha      0.368 0.0368
## Number of clusters: 407 Maximum cluster size: 40
```

```
model_binary_gee_ar1 <- geeglm(remission ~ IL6 + CRP + CancerStage + Experience,
                               id=DID, data = data_binary, family = binomial,
                               corstr='ar1')
print(summary(model_binary_gee_ar1))
```

```
##
## Call:
## geeglm(formula = remission ~ IL6 + CRP + CancerStage + Experience,
##        family = binomial, data = data_binary, id = DID, corstr = "ar1")
##
## Coefficients:
##           Estimate Std.err Wald Pr(>|W|)
## (Intercept) -1.06814 0.52148 4.20 0.041 *
## IL6          -0.05095 0.00867 34.56 4.1e-09 ***
## CRP          -0.01040 0.00691 2.27 0.132
## CancerStageII -0.29748 0.05529 28.95 7.4e-08 ***
## CancerStageIII -0.72336 0.08229 77.28 < 2e-16 ***
## CancerStageIV -1.46257 0.12250 142.55 < 2e-16 ***
## Experience    0.04654 0.02824 2.72 0.099 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation structure = ar1
## Estimated Scale Parameters:
##
##           Estimate Std.err
## (Intercept)  0.972 0.053
## Link = identity
```

```
##
## Estimated Correlation Parameters:
##      Estimate Std.err
## alpha    0.876  0.0143
## Number of clusters: 407 Maximum cluster size: 40
```

I tried two correlation structures from GEE: exchangeable and independence.

Seeing the results of `geeglm()` with exchangeable as the working correlation structure for continuous outcome, I observed similar pattern as the default `geeglm()` result from previous question except for the intercept coefficient, where it has a higher coefficient value in `geeglm()` compared to `lmer()`. For standard error values between the two models, all variables have lower standard errors in `geeglm()` except for garage variable.

Seeing the results of `geeglm()` with `ar1` as the working correlation structure for continuous outcome, the intercept and stories variables have higher coefficient values in `geeglm()` compared to `lmer()`, while the rest of the variables have lower coefficient values in `geeglm()` compared to `lmer()`. For standard error values between the two models, all variables have lower standard errors in `geeglm()` except for garage variable.

Looking at the results of `geeglm()` with exchangeable as the working correlation structure for binary outcome, all variables have higher coefficient values in `geeglm()` compared to `glmer()` except for Experience variable. For standard error values between the two models, all variables have lower standard errors in `geeglm()` compared to `glmer()`.

Looking at the results of `geeglm()` with `ar1` as the working correlation structure for binary outcome, all variables have higher coefficient values in `geeglm()` compared to `glmer()`. For standard error values between the two models, all variables have lower standard errors in `geeglm()` compared to `glmer()`, except for the intercept and Experience variables.

MISSING DATA: MICE

2. For missing data, choose online data sets suitable for both binary and continuous outcome regressions involving multiple predictor variables using `glm`.

Data Dictionary:

GPA - First-year college GPA on a 0.0 to 4.0 scale

HSGPA - High school GPA on a 0.0 to 4.0 scale

SATV - Verbal/critical reading SAT score

SATM - Math SAT score

Male - 1= male, 0= female

HU - Number of credit hours earned in humanities courses in high school

SS - Number of credit hours earned in social science courses in high school

FirstGen - 1= student is the first in her or his family to attend college, 0=otherwise

White - 1= white students, 0= others

CollegeBound - 1=attended a high school where $\geq 50\%$ students intended to go on to college, 0=otherwise

```
data <- read.csv('FirstYearGPA.csv')
```

2.b Using similar techniques discussed in class, create MAR and MCAR missingness in the outcome variable. These are specified in the .rmd file posted for the class. For MAR, this involves assigning outcome or predictor variable values to be missing depending on the value of an variable that is completely nonmissing. For MCAR, this can be just randomly assigning values of the outcome variable to be missing.

```

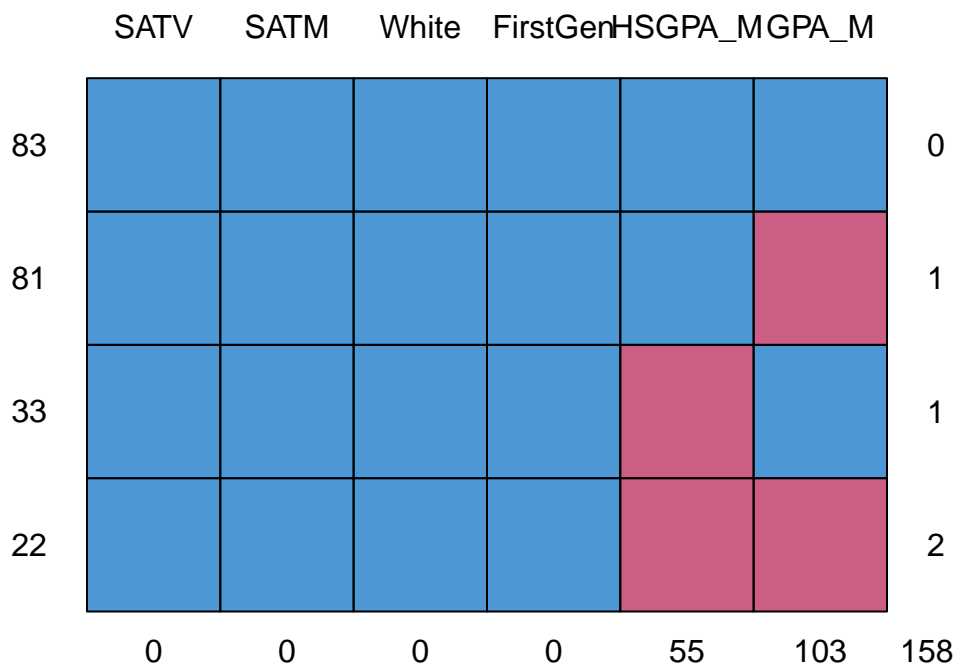
n <- dim(data)[1]

# missing at random (MAR)
# A case where the probability of being missing is the same only within
# groups defined by the observed data
selectht<-data$HU<median(data$HU)
ns<-sum(selectht)
data$HSGPA_M<-data$HSGPA
data$HSGPA_M[selectht][runif(ns)<0.5] <- NA

# Missing Completely at Random (MCAR)
# A case where the probability of being missing is the same for all cases
data$GPA_M <- ifelse(runif(n)<0.5, NA, data$GPA)

# plot missing data
data_model <- data[, c("GPA_M", "HSGPA_M", "SATV", "SATM", "White", "FirstGen")]
md.pattern(data_model)

```



```

##      SATV SATM White FirstGen HSGPA_M GPA_M
## 83      1      1      1          1          1      1      0
## 81      1      1      1          1          1      0      1
## 33      1      1      1          1          0      1      1
## 22      1      1      1          1          0      0      2
##              0      0      0          0          55     103     158

```



```

# model on no missing data
print(summary(lm(GPA ~ HSGPA + SATV + SATM + White + FirstGen, data=data)))

##
## Call:
## lm(formula = GPA ~ HSGPA + SATV + SATM + White + FirstGen, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.0583 -0.2763  0.0307  0.2831  0.8767
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.45e-01   3.21e-01   2.32   0.0214 *
## HSGPA        5.22e-01   7.38e-02   7.07  2.2e-11 ***
## SATV         6.56e-04   4.05e-04   1.62   0.1062
## SATM        -7.60e-06   4.21e-04  -0.02   0.9856
## White        2.21e-01   7.17e-02   3.08   0.0023 **
## FirstGen    -1.57e-01   8.88e-02  -1.77   0.0786 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.395 on 213 degrees of freedom
## Multiple R-squared:  0.295, Adjusted R-squared:  0.278
## F-statistic: 17.8 on 5 and 213 DF, p-value: 9.7e-15

# model with MAR
print(summary(lm(GPA ~ HSGPA_M + SATV + SATM + White + FirstGen, data=data)))

##
## Call:
## lm(formula = GPA ~ HSGPA_M + SATV + SATM + White + FirstGen,
##     data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.1207 -0.2478  0.0547  0.2652  0.7019
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.62e-01   3.60e-01   2.39   0.018 *
## HSGPA_M      5.29e-01   8.23e-02   6.43  1.5e-09 ***
## SATV        -8.73e-07   4.88e-04   0.00   0.999
## SATM         5.01e-04   4.77e-04   1.05   0.295
## White        2.03e-01   8.21e-02   2.47   0.015 *
## FirstGen    -1.25e-01   1.13e-01  -1.11   0.269
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.388 on 158 degrees of freedom
## (55 observations deleted due to missingness)
## Multiple R-squared:  0.277, Adjusted R-squared:  0.255
## F-statistic: 12.1 on 5 and 158 DF, p-value: 5.88e-10

```

```
# model with MAR + MCAR
```

```
print(summary(lm(GPA_M ~ HSGPA_M + SATV + SATM + White + FirstGen, data=data)))
```

```
##
## Call:
## lm(formula = GPA_M ~ HSGPA_M + SATV + SATM + White + FirstGen,
##     data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.1088 -0.2777  0.0234  0.2504  0.7685
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.06e-01   5.46e-01   1.48    0.144
## HSGPA_M      5.70e-01   1.12e-01   5.10 2.4e-06 ***
## SATV        -9.13e-05   7.05e-04  -0.13   0.897
## SATM         2.90e-04   8.01e-04   0.36   0.718
## White        2.89e-01   1.43e-01   2.01   0.048 *
## FirstGen    -2.22e-02   2.01e-01  -0.11   0.913
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.417 on 77 degrees of freedom
## (136 observations deleted due to missingness)
## Multiple R-squared:  0.304, Adjusted R-squared:  0.258
## F-statistic: 6.71 on 5 and 77 DF, p-value: 3.07e-05
```

Fit the models with and without multiple imputation and compare the results. Also suggest how to create (not missing at random) NMAR missingness, and outline how this might be implemented. This need not be programmed.

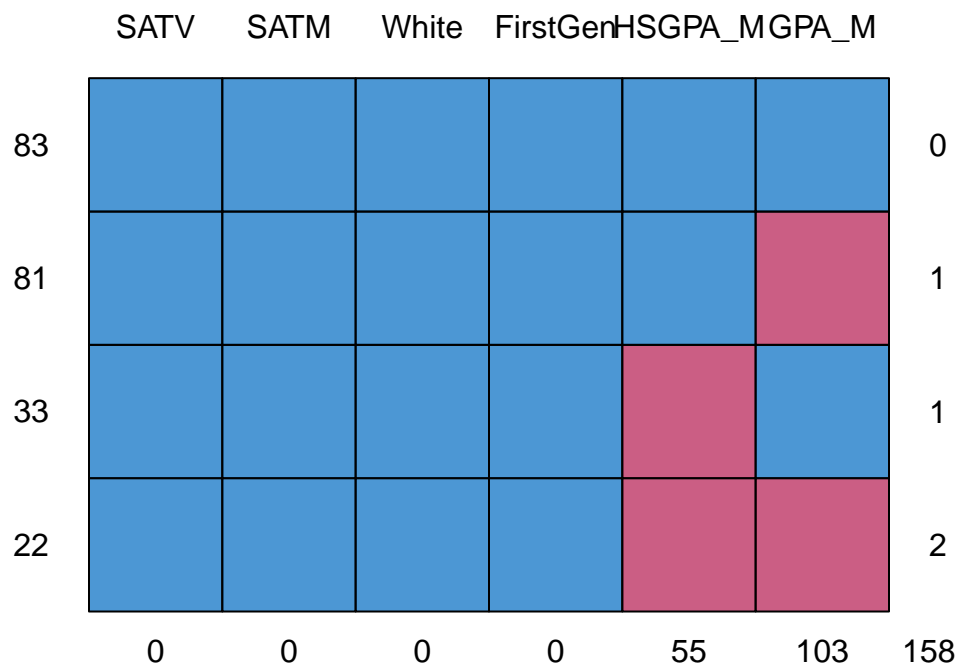
```
#data_raw
```

```
print(summary(lm(GPA_M ~ HSGPA_M + SATV + SATM + White + FirstGen, data=data)))
```

```
##
## Call:
## lm(formula = GPA_M ~ HSGPA_M + SATV + SATM + White + FirstGen,
##     data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.1088 -0.2777  0.0234  0.2504  0.7685
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.06e-01   5.46e-01   1.48    0.144
## HSGPA_M      5.70e-01   1.12e-01   5.10 2.4e-06 ***
## SATV        -9.13e-05   7.05e-04  -0.13   0.897
## SATM         2.90e-04   8.01e-04   0.36   0.718
## White        2.89e-01   1.43e-01   2.01   0.048 *
## FirstGen    -2.22e-02   2.01e-01  -0.11   0.913
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.417 on 77 degrees of freedom
## (136 observations deleted due to missingness)
## Multiple R-squared:  0.304, Adjusted R-squared:  0.258
## F-statistic: 6.71 on 5 and 77 DF, p-value: 3.07e-05
```

```
# plot missing data
md.pattern(data_model)
```



```
##      SATV SATM White FirstGen HSGPA_M GPA_M
## 83      1      1      1          1          1      1      0
## 81      1      1      1          1          1      0      1
## 33      1      1      1          1          0      1      1
## 22      1      1      1          1          0      0      2
##          0      0      0          0          55     103    158
```

```
# MICE method
imput_mice <- mice(data_model, m=5)
```

```
##
## iter imp variable
##   1   1   GPA_M   HSGPA_M
```

```
## 1 2 GPA_M HSGPA_M
## 1 3 GPA_M HSGPA_M
## 1 4 GPA_M HSGPA_M
## 1 5 GPA_M HSGPA_M
## 2 1 GPA_M HSGPA_M
## 2 2 GPA_M HSGPA_M
## 2 3 GPA_M HSGPA_M
## 2 4 GPA_M HSGPA_M
## 2 5 GPA_M HSGPA_M
## 3 1 GPA_M HSGPA_M
## 3 2 GPA_M HSGPA_M
## 3 3 GPA_M HSGPA_M
## 3 4 GPA_M HSGPA_M
## 3 5 GPA_M HSGPA_M
## 4 1 GPA_M HSGPA_M
## 4 2 GPA_M HSGPA_M
## 4 3 GPA_M HSGPA_M
## 4 4 GPA_M HSGPA_M
## 4 5 GPA_M HSGPA_M
## 5 1 GPA_M HSGPA_M
## 5 2 GPA_M HSGPA_M
## 5 3 GPA_M HSGPA_M
## 5 4 GPA_M HSGPA_M
## 5 5 GPA_M HSGPA_M
```

```
imput_mice_fit <- with(data=imput_mice, exp=lm(GPA_M ~ HSGPA_M + SATV + SATM +
                                                White + FirstGen))
summary(pool(imput_mice_fit))
```

##	term	estimate	std.error	statistic	df	p.value
## 1	(Intercept)	1.214636	0.522214	2.326	9.83	0.04275
## 2	HSGPA_M	0.551571	0.127962	4.310	8.60	0.00218
## 3	SATV	0.000334	0.000665	0.502	9.61	0.62676
## 4	SATM	-0.000664	0.000504	-1.317	46.23	0.19444
## 5	White	0.242913	0.095033	2.556	21.83	0.01807
## 6	FirstGen	-0.149140	0.188495	-0.791	6.04	0.45875

MNAR means that the probability of being missing varies for reasons that are unknown to us. To create Not Missing at Random (NMAR) missing values, we should do the following steps: 1. Choose the pattern of missingness 2. Choose the variable(s) to be applied the missingness pattern on 3. Choose the base distribution to be applied to the variable(s) with the chosen pattern of missingness. Ideally this should not be a uniform distribution, because we want the probability of missingness on each row to be different with each other (e.g. log-normal distribution)

Covariate Measurement Error

2.c In the same models, add substantial additive measurement error to a continuous predictor variable and see how estimates are affected for the variable of interest and for the other variables in the model. This can be done using the `rnorm()` statement in R.

```
# add measurement error on continuous variable
data$HSGPA_error_noise <- data$HSGPA + rnorm(219, 0.1, 1)

model <- lm(GPA ~ HSGPA + SATV + SATM + White + FirstGen, data=data)
```

```
model_noise <- lm(GPA ~ HSGPA_error_noise + SATV + SATM + White + FirstGen,
                  data=data)
summary(model)
```

```
##
## Call:
## lm(formula = GPA ~ HSGPA + SATV + SATM + White + FirstGen, data = data)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-1.0583	-0.2763	0.0307	0.2831	0.8767

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	7.45e-01	3.21e-01	2.32	0.0214 *
HSGPA	5.22e-01	7.38e-02	7.07	2.2e-11 ***
SATV	6.56e-04	4.05e-04	1.62	0.1062
SATM	-7.60e-06	4.21e-04	-0.02	0.9856
White	2.21e-01	7.17e-02	3.08	0.0023 **
FirstGen	-1.57e-01	8.88e-02	-1.77	0.0786 .

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.395 on 213 degrees of freedom
## Multiple R-squared:  0.295, Adjusted R-squared:  0.278
## F-statistic: 17.8 on 5 and 213 DF,  p-value: 9.7e-15
```

```
summary(model_noise)
```

```
##
## Call:
## lm(formula = GPA ~ HSGPA_error_noise + SATV + SATM + White +
##     FirstGen, data = data)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-1.0000	-0.3117	0.0416	0.3289	1.2204

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.98e+00	2.86e-01	6.93	4.9e-11 ***
HSGPA_error_noise	6.72e-02	2.74e-02	2.45	0.0149 *
SATV	1.22e-03	4.37e-04	2.78	0.0059 **
SATM	-2.39e-05	4.67e-04	-0.05	0.9593
White	2.09e-01	7.86e-02	2.66	0.0085 **
FirstGen	-7.19e-02	9.66e-02	-0.74	0.4578

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.433 on 213 degrees of freedom
## Multiple R-squared:  0.153, Adjusted R-squared:  0.133
## F-statistic: 7.71 on 5 and 213 DF,  p-value: 1.11e-06
```