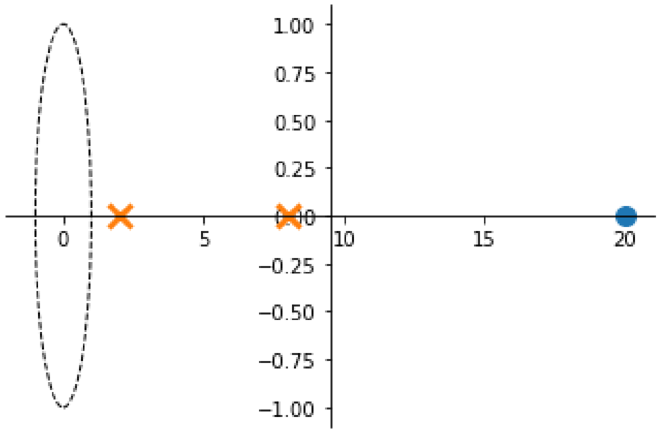

Project Report for ECE 351

Lab 11: Z - Transform Operations

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https://github.com/gibs0630/ECE351_Code

https://github.com/gibs0630/ECE351_Reports

Contents

1 Introduction

With Python, it is possible to easily compute properties of a z-transformed function.

2 Equations

equations from the lab

$$y[k] = 2x[k] - 40x[k-1] + 10y[k-1] - 16y[k-2]$$

3 Methodology

This lab had us compute the transfer function in the time domain and z domain. Python was then used to extrapolate additional information, such as stability, or gain for a given frequency.

4 Results

task 1

$$y[k] = 2x[k] - 40x[k-1] + 10y[k-1] - 16y[k-2]$$

y[k] is causal, so both the left hand side and right hand side are multiplied by u[k]

$$Y[z] = 2X[z] - 40(z^{-1}X[k] + x[-1]) + 10(z^{-1}Y[k] + y[-1]) - 16(z^{-2}Y[k] + z^-1y[-1] + y[-2])$$

$$Y[z] = 2X[z] - 40(z^{-1}X[k] + (0)) + 10(z^{-1}Y[k] + (0)) - 16(z^{-2}Y[k] + z^{-1}(0) + (0))$$

$$Y[z] = 2X[z] - 40(z^{-1}X[k] + 10z^{-1}Y[k] - 16z^{-2}Y[k]$$

$$16z^{-2}Y[k] - 10z^{-1}Y[k] + Y[z] = 2X[z] - 40z^{-1}X[k]$$

$$(16z^{-2} - 10z^{-1} + 1)Y[z] = (2 - 40z^{-1})X[k]$$

$$\frac{Y[z]}{X[k]} = \frac{2 - 40z^{-1}}{16z^{-2} - 10z^{-1} + 1}$$

$$H[k] = \frac{2 - 40z^{-1}}{16z^{-2} - 10z^{-}1 + 1}$$

$$H[k] = \frac{2z^2 - 40z^1}{16 - 10z^1 + 1z^2}$$

$$H[k] = \frac{2z^2 - 40z}{z^2 - 10z^1 + 16}$$

task 2

$$\frac{H[k]}{z} = \frac{2z - 40}{z^2 - 10z^1 + 16}$$

$$\frac{2z - 40}{(z-8)(z-2)} = \frac{A}{(z-8)} + \frac{B}{(z-2)}$$

$$A = \left.\frac{2z - 40}{z - 2}\right|_{z=8}$$

$$A = -4$$

$$B = \left.\frac{2z - 40}{z - 8}\right|_{z=2}$$

$$B = 6$$

$$\frac{H[k]}{z} = \frac{-4}{z-8} + \frac{6}{z-2}$$

$$H[k] = \frac{-4z}{z-8} + \frac{6z}{z-2}$$

$$H[k] = \frac{-4z}{z-8} + \frac{6z}{z-2}$$

$$h[k] = (-4)8^k u[k] + (6)2^k u[k]$$

task 3

Figure 1: residuez

```
task 3
r:  [ 6. -4.] , p:  [2. 8.] , k:  []
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Figure 1 shows the residue (aka zeroes), poles, and stand alone coefficients using the scipy.signal.residuez function

task 4

Figure 2: zplane

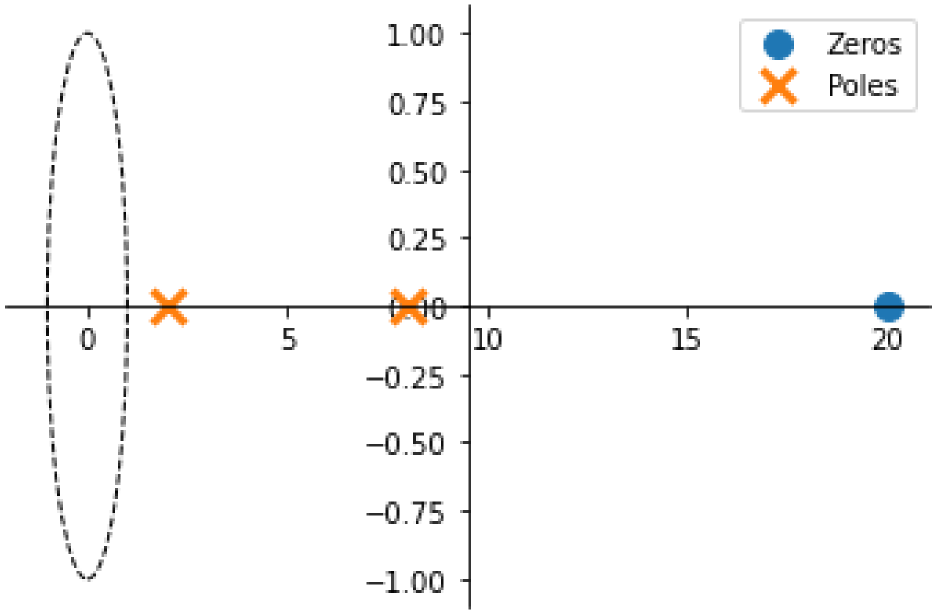

Figure 2 shows the plot of complex z - plane given a transfer function. If the poles are within the unit circle (dashed oval) then the function is stable. if there is a mix of poles inside and outside the unit circle, then L'Hospital rule may have to be used.

task 5

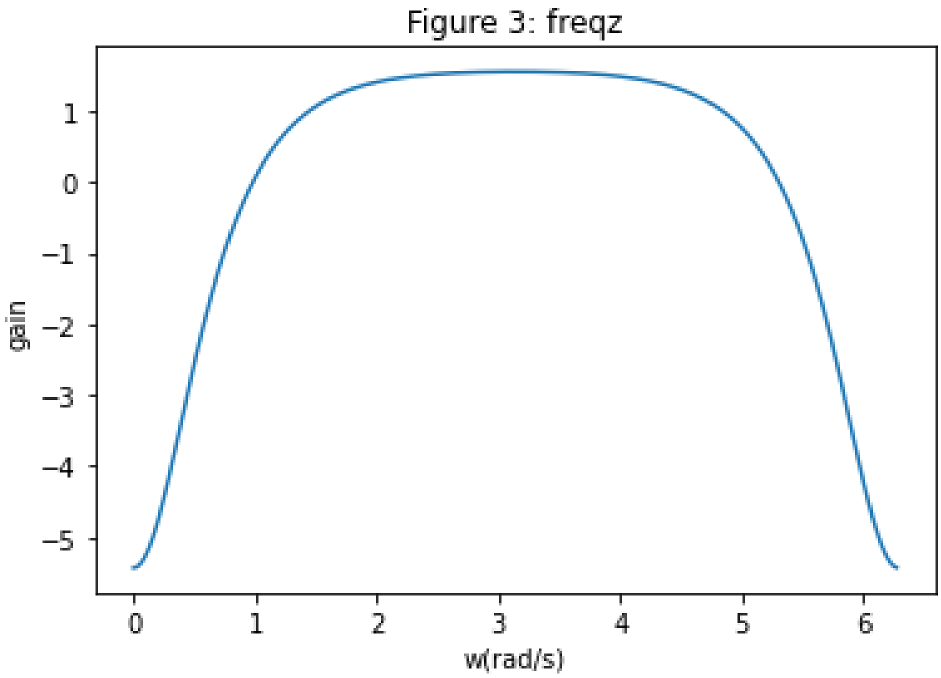

Figure 3 the gain for a given frequency using the scipy.signal.freqz function.

5 Questions

Q:Looking at the plot generated in Task 4, is H(z) stable? Explain why or why not.

A: zeroes have no effect on stability, so we can ignore the zero at 20 Sue to the relation $\mathcal{Z}\{a^k u[k]\} = \frac{z}{(z-a)}$, then if a is is less then one, then it is stable. looking at the plot, there are two poles, and both are outside the unit circle, thus this . This would be causal function would be unstable.

Q:Leave any feedback on the clarity of lab tasks, expectations, and deliverables

A: the poles and zeroes plot was new, perhaps an explanation of what it is prior to the lab would be helpful for future classes.