



$$x_0(t) = 2P_{T/2}(t - T/4) - 1$$

$$x(t) = \frac{1}{2}a_0 + \sum_{k=1}^{\infty} \left[ a_k \cos\left(k \frac{2\pi}{T} t\right) + b_k \sin\left(k \frac{2\pi}{T} t\right) \right]$$

$$a_k = \frac{2}{T} \int_0^T x(t) \cos\left(k \frac{2\pi}{T} t\right) dt$$

$$b_k = \frac{2}{T} \int_0^T x(t) \sin\left(k \frac{2\pi}{T} t\right) dt$$

$$a_0 = \frac{2}{T} \int_0^T [x(t) dt]$$

finding  $a_0$

$$a_0 = \frac{2}{T} \int_0^T [x(t) dt]$$

$$a_0 = \frac{2}{T}(0)$$

$$a_0 = 0$$

finding  $a_k$

$$a_k = \frac{2}{T} \int_0^T \left[ (2P_{T/2}(t - T/4) - 1) \cos\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$a_k = \frac{2}{T} \int_0^T \left[ 2P_{T/2}(t - T/4) \cos\left(k \frac{2\pi}{T} t\right) dt \right] - \frac{2}{T} \int_0^T \left[ \cos\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$a_k = \frac{2}{T} \int_0^T \left[ 2u(t) \cos\left(k \frac{2\pi}{T} t\right) dt \right] - \frac{2}{T} \int_0^T \left[ 2u\left(t - \frac{T}{2}\right) \cos\left(k \frac{2\pi}{T} t\right) dt \right] - \frac{2}{T} \int_0^T \left[ \cos\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$a_k = \frac{2}{T} \int_0^T \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_0^T \left[ 2u \left( t - \frac{t}{2} \right) \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_0^T \left[ \cos \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$a_k = \frac{2}{T} \int_0^T \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_{T/2}^T \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_0^T \left[ \cos \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_0^T \left[ \cos \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \left[ \frac{1}{k \frac{2\pi}{T}} \sin \left( k \frac{2\pi}{T} t \right) \right] \Bigg|_0^T$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \frac{1}{k \frac{2\pi}{T}} \left[ \sin \left( k \frac{2\pi}{T} t \right) \right] \Bigg|_0^T$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \frac{T}{k 2\pi} \left[ \sin \left( k \frac{2\pi}{T} t \right) \right] \Bigg|_0^T$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{1}{k \pi} \left[ \sin \left( k \frac{2\pi}{T} t \right) \right] \Bigg|_0^T$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{1}{k \pi} \left[ \sin \left( k \frac{2\pi}{T} (T) \right) - \sin \left( k \frac{2\pi}{T} (0) \right) \right]$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{1}{k \pi} \left[ \sin \left( k \frac{2\pi}{T} (T) \right) - 0 \right]$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{1}{k \pi} [\sin(k 2\pi) - 0]$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{1}{k \pi} [0 - 0]$$

$$a_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \cos \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$a_k = \frac{4}{T} \int_0^{T/2} \left[ \cos \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$a_k = \frac{4}{T} \left[ \frac{1}{k \frac{2\pi}{T}} \sin \left( k \frac{2\pi}{T} t \right) \right]_0^{T/2}$$

$$a_k = \frac{4}{T} \frac{1}{k \frac{2\pi}{T}} \left[ \sin \left( k \frac{2\pi}{T} t \right) \right]_0^{T/2}$$

$$a_k = \frac{2}{k\pi} \left[ \sin \left( k \frac{2\pi}{T} t \right) \right]_0^{T/2}$$

$$a_k = \frac{2}{k\pi} \left[ \sin \left( k \frac{2\pi}{T} (T/2) \right) - \sin \left( k \frac{2\pi}{T} (0) \right) \right]$$

$$a_k = \frac{2}{k\pi} \left[ \sin \left( k \frac{2\pi}{T} (T/2) \right) - 0 \right]$$

$$a_k = \frac{2}{k\pi} [\sin(k\pi) - 0]$$

$$a_k = \frac{2}{k\pi} [0 - 0]$$

$$\boxed{a_k = 0}$$

finding  $b_k$

$$b_k = \frac{2}{T} \int_0^T \left[ (2P_{T/2}(t - T/4) - 1) \sin \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$b_k = \frac{2}{T} \int_0^T \left[ 2P_{T/2}(t - T/4) \sin \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_0^T \left[ \sin \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$b_k = \frac{2}{T} \int_0^T \left[ 2u(t) \sin \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_0^T \left[ 2u \left( t - \frac{T}{2} \right) \sin \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_0^T \left[ \sin \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$b_k = \frac{2}{T} \int_0^T \left[ 2u(t) \sin \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_{T/2}^T \left[ 2 \sin \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_0^T \left[ \sin \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$b_k = \frac{2}{T} \int_0^T \left[ 2 \sin \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_{T/2}^T \left[ 2 \sin \left( k \frac{2\pi}{T} t \right) dt \right] - \frac{2}{T} \int_0^T \left[ \sin \left( k \frac{2\pi}{T} t \right) dt \right]$$

$$b_k = \frac{2}{T} \int_0^{T/2} \left[ 2 \sin\left(k \frac{2\pi}{T} t\right) dt \right] - \frac{2}{T} \int_0^T \left[ \sin\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$b_k = \frac{2}{T} \left[ \frac{-2}{k \frac{2\pi}{T}} \cos\left(k \frac{2\pi}{T} t\right) \right]_0^{T/2} - \frac{2}{T} \int_0^T \left[ \sin\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$b_k = \frac{-2}{k\pi} \left[ \cos\left(k \frac{2\pi}{T} t\right) \right]_0^{T/2} - \frac{2}{T} \int_0^T \left[ \sin\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$b_k = \frac{-2}{k\pi} \left[ \cos\left(k \frac{2\pi}{T} (T/2)\right) - \cos\left(k \frac{2\pi}{T} (0)\right) \right] - \frac{2}{T} \int_0^T \left[ \sin\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$b_k = \frac{-2}{k\pi} \left[ \cos\left(k \frac{2\pi}{T} (T/2)\right) - 1 \right] - \frac{2}{T} \int_0^T \left[ \sin\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$b_k = \frac{-2}{k\pi} [\cos(k\pi) - 1] - \frac{2}{T} \int_0^T \left[ \sin\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$b_k = \frac{-2}{k\pi} [(-1)^k - 1] - \frac{2}{T} \int_0^T \left[ \sin\left(k \frac{2\pi}{T} t\right) dt \right]$$

$$b_k = \frac{-2}{k\pi} [(-1)^k - 1] - \frac{2}{T} \left[ \frac{-1}{k \frac{2\pi}{T}} \cos\left(k \frac{2\pi}{T} t\right) \right]_0^T$$

$$b_k = \frac{-2}{k\pi} [(-1)^k - 1] - \frac{-1}{k\pi} \left[ \cos\left(k \frac{2\pi}{T} t\right) \right]_0^T$$

$$b_k = \frac{-2}{k\pi} [(-1)^k - 1] - \frac{-1}{k\pi} \left[ \cos\left(k \frac{2\pi}{T} (T)\right) - \cos\left(k \frac{2\pi}{T} (0)\right) \right]$$

$$b_k = \frac{-2}{k\pi} [(-1)^k - 1] - \frac{-1}{k\pi} \left[ \cos\left(k \frac{2\pi}{T} (T)\right) - 1 \right]$$

$$b_k = \frac{-2}{k\pi} [(-1)^k - 1] - \frac{-1}{k\pi} [\cos(k2\pi) - 1]$$

$$b_k = \frac{-2}{k\pi} [(-1)^k - 1] - \frac{-1}{k\pi} [1 - 1]$$

$$b_k = \frac{-2}{k\pi} [(-1)^k - 1] - \frac{-1}{k\pi} [0]$$

$$\boxed{b_k = \frac{-2}{k\pi} [(-1)^k - 1]}$$

finding

$$x(t) = \frac{1}{2}(0) + \sum_{k=1}^{\infty} \left[ (0) \cos\left(k \frac{2\pi}{T} t\right) + \left(\frac{-2}{k\pi} [(-1)^k - 1]\right) \sin\left(k \frac{2\pi}{T} t\right) \right]$$

$$x(t) = \sum_{k=1}^{\infty} \left[ \left(\frac{-2}{k\pi} [(-1)^k - 1]\right) \sin\left(k \frac{2\pi}{T} t\right) \right]$$

$$x(t) = \frac{-2}{\pi} \sum_{k=1}^{\infty} \left[ \left(\frac{1}{k} [(-1)^k - 1]\right) \sin\left(k \frac{2\pi}{T} t\right) \right]$$

$$x(t) = \frac{-2}{\pi} \sum_{k=1}^{\infty} \left[ \left(\frac{1}{k} [(-1)^k - 1]\right) \sin\left(k \frac{2\pi}{T} t\right) \right]$$

$$x(t) = \frac{2}{\pi} \sum_{k=1}^{\infty} \left[ \left(\frac{1}{k} [1 - (-1)^k]\right) \sin\left(k \frac{2\pi}{T} t\right) \right]$$

let  $k = 2n - 1$

$$x(t) = \frac{2}{\pi} \sum_{2n-1=1}^{\infty} \left[ \left(\frac{1}{2n-1} [1 - (-1)^{(2n-1)}]\right) \sin\left((2n-1) \frac{2\pi}{T} t\right) \right]$$

$$x(t) = \frac{2}{\pi} \sum_{2n=2}^{\infty} \left[ \left(\frac{1}{2n-1} [1 - (-1)^{(2n-1)}]\right) \sin\left((2n-1) \frac{2\pi}{T} t\right) \right]$$

$$x(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \left[ \left(\frac{1}{2n-1} [1 - (-1)^{(2n-1)}]\right) \sin\left((2n-1) \frac{2\pi}{T} t\right) \right]$$

$$x(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \left[ \left(\frac{1}{2n-1} [1 - (-1)^{(2n)}(-1)]\right) \sin\left((2n-1) \frac{2\pi}{T} t\right) \right]$$

$$x(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \left[ \left(\frac{1}{2n-1} [1 + (-1)^{(2n)}]\right) \sin\left((2n-1) \frac{2\pi}{T} t\right) \right]$$

$$x(t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\left(\frac{1}{2n-1}\left[1+((-1)^2)^n\right]\right)\sin\left((2n-1)\frac{2\pi}{T}t\right)\right]$$

$$x(t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\left(\frac{1}{2n-1}\left[1+1^n\right]\right)\sin\left((2n-1)\frac{2\pi}{T}t\right)\right]$$

$$x(t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\left(\frac{1}{2n-1}\left[1+1\right]\right)\sin\left((2n-1)\frac{2\pi}{T}t\right)\right]$$

$$x(t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\left(\frac{1}{2n-1}\left[1\right]\right)\sin\left((2n-1)\frac{2\pi}{T}t\right)\right]$$

$$x(t)=\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\left(\frac{2}{2n-1}\right)\sin\left((2n-1)\frac{2\pi}{T}t\right)\right]$$

$$\boxed{x(t)=\frac{4}{\pi}\sum_{n=1}^{\infty}\left[\left(\frac{1}{2n-1}\right)\sin\left((2n-1)\frac{2\pi}{T}t\right)\right]}$$