



1) find transfer function  $H(s)$

$$I_1 = I_2 + I_3$$

$$\left( \frac{V_{in} - V_{out}}{R} \right) = \left( C \frac{dV_{out}}{dt} \right) + \left( \frac{1}{L} \int [V_{out} dt] \right)$$

$$RC \frac{dV_{out}}{dt} + V_{out} + \frac{R}{L} \int [V_{out} dt] = V_{in}$$

$$\mathcal{L} \left\{ RC \frac{dV_{out}}{dt} + V_{out} + \frac{R}{L} \int [V_{out} dt] \right\} = \mathcal{L} \{ V_{in} \}$$

$$RC(s \tilde{V}_{out} + V_{out}(0)) + (\tilde{V}_{out}) + \frac{R}{L} \left( \frac{1}{s} \tilde{V}_{out} \right) = \tilde{V}_{in}$$

lets  $V_{out}(0) = 0$  as the transfer function is defined under a zero state condition.

$$RCs \tilde{V}_{out} + \tilde{V}_{out} + \frac{R}{Ls} \tilde{V}_{out} = \tilde{V}_{in}$$

$$\left( RCs + 1 + \frac{R}{Ls} \right) \tilde{V}_{out} = \tilde{V}_{in}$$

$$\left( RCs + 1 + \frac{R}{Ls} \right) \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = 1$$

$$\frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{1}{\left( RCs + 1 + \frac{R}{Ls} \right)}$$

$$\frac{\widetilde{V_{out}}}{\widetilde{V_{in}}} = \frac{L s}{(R C L s^2 + L s + R)}$$

$$\boxed{H(s) = \frac{L s}{(R C L s^2 + L s + R)}}$$

2) find impulse response  $h(t)$

$$V_{in} = \delta(t) \rightarrow \widetilde{V_{in}} = 1 \rightarrow H(s) = \widetilde{V_{out}}$$

$$\mathcal{L}\{h(t)\} = \frac{\frac{L}{R C L} s}{\left(s^2 + \frac{L}{R C L} s + \frac{R}{R C L}\right)}$$

$$\mathcal{L}\{h(t)\} = \frac{\frac{1}{R C} s}{\left(s^2 + \frac{1}{R C} s + \frac{1}{C L}\right)}$$

$$\mathcal{L}\{h(t)\} = \frac{\frac{1}{R C} s}{\left(\left(s + \frac{1}{2 R C}\right)^2 - \left(\frac{1}{2 R C}\right)^2 + \frac{1}{C L}\right)}$$

$$\text{let } \omega^2 = \frac{1}{C L} - \left(\frac{1}{2 R C}\right)^2$$

$$\text{let } \alpha = \frac{1}{2 R C}$$

$$\mathcal{L}\{h(t)\} = \frac{2 \alpha s}{(s + \alpha)^2 + \omega^2}$$

$$\mathcal{L}\{h(t)\} = 2 \alpha \left( \frac{s}{(s + \alpha)^2 + \omega^2} \right)$$

$$\mathcal{L}\{h(t)\} = 2 \alpha \left( \frac{s + \alpha - \alpha}{(s + \alpha)^2 + \omega^2} \right)$$

$$\mathcal{L}\{h(t)\} = 2 \alpha \left( \frac{s + \alpha}{(s + \alpha)^2 + \omega^2} + \frac{-\alpha}{(s + \alpha)^2 + \omega^2} \right)$$

$$\mathcal{L}\{h(t)\} = 2\alpha \left( \frac{s+\alpha}{(s+\alpha)^2 + \omega^2} \right) + 2\alpha \left( \frac{-\alpha}{(s+\alpha)^2 + \omega^2} \right)$$

$$\mathcal{L}\{h(t)\} = 2\alpha \left( \frac{s+\alpha}{(s+\alpha)^2 + \omega^2} \right) - 2\alpha^2 \left( \frac{1}{(s+\alpha)^2 + \omega^2} \right)$$

$$\mathcal{L}\{h(t)\} = 2\alpha \left( \frac{s+\alpha}{(s+\alpha)^2 + \omega^2} \right) - 2\alpha^2 \left( \frac{\frac{\omega}{\omega}}{(s+\alpha)^2 + \omega^2} \right)$$

$$\mathcal{L}\{h(t)\} = 2\alpha \left( \frac{s+\alpha}{(s+\alpha)^2 + \omega^2} \right) - 2\frac{\alpha^2}{\omega} \left( \frac{\omega}{(s+\alpha)^2 + \omega^2} \right)$$

$$h(t) = 2\alpha e^{-\alpha t} \cos(\omega t) u(t) - 2\frac{\alpha^2}{\omega} e^{-\alpha t} \sin(\omega t) u(t)$$

$$\text{let } \omega^2 = \frac{1}{CL} - \left( \frac{1}{2RC} \right)^2$$

$$\text{let } \alpha = \frac{1}{2RC}$$

$$h(t) = 2\frac{1}{2RC} e^{\frac{-1}{2RC}t} \cos\left(\sqrt{\frac{1}{CL} - \left(\frac{1}{2RC}\right)^2}t\right) u(t) - 2\frac{\left(\frac{1}{2RC}\right)^2}{\sqrt{\frac{1}{CL} - \left(\frac{1}{2RC}\right)^2}} e^{\frac{-1}{2RC}t} \sin\left(\sqrt{\frac{1}{CL} - \left(\frac{1}{2RC}\right)^2}t\right) u(t)$$

$$h(t) = \frac{1}{RC} e^{\frac{-1}{2RC}t} \cos\left(\sqrt{\frac{1}{CL} - \left(\frac{1}{2RC}\right)^2}t\right) u(t) - \frac{2\left(\frac{1}{2RC}\right)^2}{\sqrt{\frac{1}{CL} - \left(\frac{1}{2RC}\right)^2}} e^{\frac{-1}{2RC}t} \sin\left(\sqrt{\frac{1}{CL} - \left(\frac{1}{2RC}\right)^2}t\right) u(t)$$

for the given values: R = 1000  $\Omega$ , L = 0.027 H, and C = 10E-9 F

$$h(t) = 100000 e^{-100000t} \cos\left(\frac{50000\sqrt{39}}{9}t\right) u(t) - \frac{300000\sqrt{39}}{13} e^{-100000t} \sin\left(\frac{50000\sqrt{39}}{9}t\right) u(t)$$

or approximately

$$h(t) = 100000 e^{-100000t} \cos(34694.4t) u(t) - 144115 e^{-100000t} \sin(34694.4t) u(t)$$

or

$$h(t) = 175412 e^{-100000t} \cos(34694.4t + 55.2437^\circ) u(t)$$