Project Report for ECE 351

Lab 4: System Step Response Using Convolution

Andrew Gibson

7 February 2023

https://github.com/gibs0630/ECE351_Code

https://github.com/gibs0630/ECE351_Reports

Contents

1 Introduction

There is a special case what computing convolutions. If one of them is the unit step function, then the answer will be the integral of the other.

2 Equations

Formula's used unit step function

$$u(x) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

ramp function

$$r(x) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

discrete convolution with uniform time interval at 1

$$h(t) = \sum_{k=-\infty}^{\infty} \left[\sum_{n=0}^{k} [f_n * g_{k-n} * \Delta t] \right]$$

discrete convolution with uniform time intervals

$$h(t) = \sum_{k=-\infty}^{\infty} \left[\sum_{n=0}^{k} [f_n * g_{k-n}] \right]$$

functions from lab

$$h_1(t) = e^{-2t}(u(t) - u(t-3))$$
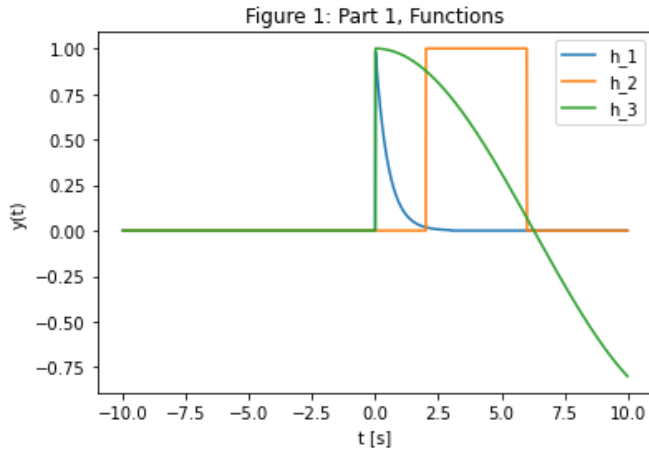$$h_2(t) = u(t-2) - u(t-6)$$
$$h_3(t) = cos(\omega_0 t)u(t)$$

3 Methodology

This lab had us create some functions and plot them, discrete convolve them with the unit step function, and then compare those with the mathamatical answer.

4 Results

Part 1

Figure 1: Part 1, Functions

The purpose producing Figure 1 was to show the functions as they are.
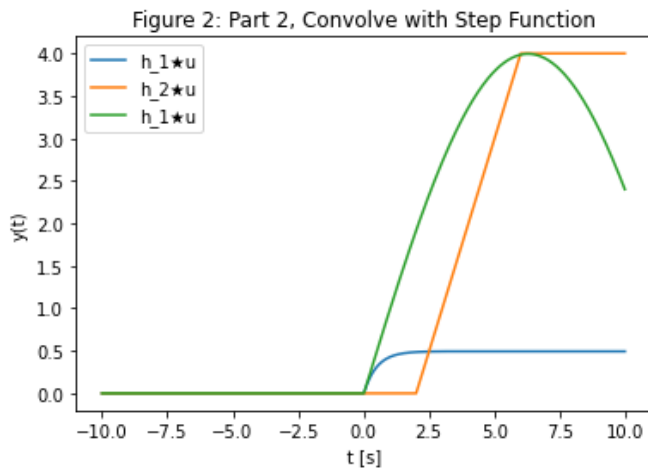
Part 2

Figure 2: Part 2, Convolve with Step Function

Figure 2 shows the functions convolved with the unit step function.

The math for the convolution are as follows: for h_1

$$h_1(t) \star u(t) = \int_{-\infty}^{\infty} \left[\left(e^{-2\tau} (u(\tau) - u(\tau - 3)) \right) (u(t - \tau)) \, d\tau \right]$$

$$= \int_{-\infty}^{\infty} \left[\left(e^{-2\tau} u(\tau) - e^{-2\tau} u(\tau - 3) \right) \right) (u(t - \tau)) \, d\tau \right]$$

$$= \int_{-\infty}^{\infty} \left[e^{-2\tau} u(\tau) u(t - \tau) d\tau \right] - \int_{-\infty}^{\infty} \left[e^{-2\tau} u(\tau - 3) u(t - \tau) d\tau \right]$$

2

$$= \int_0^t \left[e^{-2\tau} d\tau \right] u(t) - \int_{-\infty}^{\infty} \left[e^{-2\tau} u(\tau - 3) u(t - \tau) d\tau \right]$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - \int_{-\infty}^{\infty} \left[e^{-2\tau} u(\tau - 3) u(t - \tau) d\tau \right]$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - \int_{-\infty}^{\infty} \left[e^{-2(\tau - 3 + 3)} u(\tau - 3) u(t - \tau) d\tau \right]$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - \int_{-\infty}^{\infty} \left[e^{-2(\tau - 3)} e^{-6} u(\tau - 3) u(t - \tau) d\tau \right]$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - e^{-6} \int_{-\infty}^{\infty} \left[e^{-2(\tau - 3)} u(\tau - 3) u(t - \tau) d\tau \right]$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - e^{-6} \left[\left(e^{-2(t-3)} u(t - 3) \right) \star u(t) \right]$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - e^{-6} \left[\left(e^{-2(t)} u(t) \right) \star u(t) \right]_{t-3}$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - e^{-6} \left[\left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) \right]_{t-3}$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - e^{-6} \left[\left[\frac{1}{-2} e^{-2(t-3)} - \frac{1}{-2} \right] u(t - 3) \right]$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - \left[\frac{e^{-6}}{-2} e^{-2(t-3)} - \frac{1}{-2} e^{-6} \right] u(t - 3)$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - \left[\frac{1}{-2} e^{-2(t-3)-6} - \frac{1}{-2} e^{-6} \right] u(t - 3)$$

$$= \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} \right] u(t) - \left[\frac{1}{-2} e^{-2t} - \frac{1}{-2} e^{-6} \right] u(t - 3)$$

3

$$= \frac{1}{2} \left(\left[1 - e^{-2t} \right] u(t) - \left[e^{-6} - e^{-2t} \right] u(t-3) \right)$$

for h$_2$

$$h_2(t) \star u(t-2) = (u(t-2) - u(t-6)) \star u(t-2)$$

$$= (u(t-2) \star u(t)) - (u(t-6) \star u(t))$$

$$= [u(t) \star u(t)]_{t-2} - [u(t) \star u(t)]_{t-6}$$

$$= \left[\int_{-\infty}^{\infty} [(u(\tau)) (u(t-\tau)) \, d\tau] \right]_{t-2} - [u(t) \star u(t)]_{t-6}$$

$$= \left[\int_{0}^{t} [d\tau] \, u(t) \right]_{t-2} - [u(t) \star u(t)]_{t-6}$$

$$= [[(t) - (0)] \, u(t)]_{t-2} - [u(t) \star u(t)]_{t-6}$$

$$= (t-2)u(t-2) - [u(t) \star u(t)]_{t-6}$$

$$= (t-2)u(t-2) - (t-6)u(t-6)$$

for h$_3$

$$h_3(t) \star u(t) = \int_{-\infty}^{\infty} [(cos(\omega_0\tau)u(\tau)) (u(t-\tau)) \, d\tau]$$

$$= \int_{0}^{t} [cos(\omega_0\tau) d\tau] \, u(t)$$

$$= \left[\frac{1}{\omega_0} sin(\omega_0(t)) - \frac{1}{\omega_0} sin(\omega_0(0)) \right] u(t)$$

$$= \frac{1}{\omega_0} sin(\omega_0(t))u(t)$$
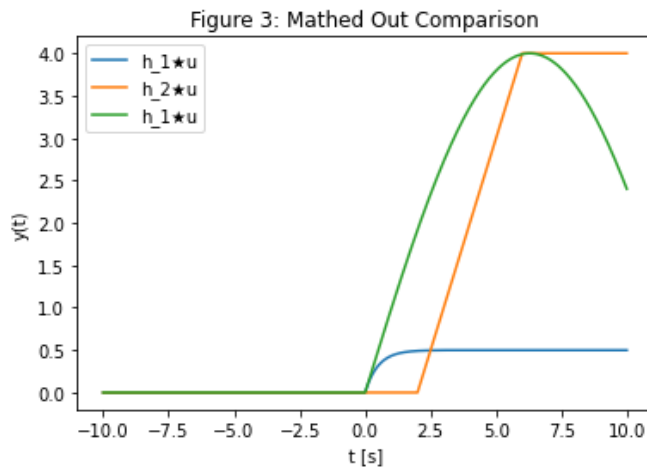
Figure 3: Mathed Out Comparison

Figure 3 shows the functions that were generated after the math was done, and they match up very closely. These functions just so happen to be integrals of the original functions, as attributed to the special case where a function convolved with the unit step function is the integral of said function.

5   Questions

Q: 1. Leave any feedback on the clarity of lab tasks, expectations, and deliverables.

A: 1. Because my convolution assumed that the index started at zero, I had to rewrite my function entirely in order to accommodate that. while I was fixing that I also corrected for the change in amplitude based on the step size.