Lab6 Prelab

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$$y''(t) + 10y'(t) + 24y(t) = x''(t) + 6x'(t) + 12x(t)$$

1 task 1

By hand, find the transfer function. Assume all initial conditions are zero.

$$y''(t) + 10y'(t) + 24y(t) = x''(t) + 6x'(t) + 12x(t)$$

$$\left(s^2Y(s) - sy(0^-) - y^{(1)}(0^-)\right) + 10\left(sY(s) - y(0^-)\right) + 24\left(Y(s)\right) =$$
$$\left(s^2X(s) - sx(0^-) - x^{(1)}(0^-)\right) + 6\left(sX(s) - x(0^-)\right) + 12\left(X(s)\right)$$

$$\left(s^2Y(s)\right) + 10\left(sY(s)\right) + 24\left(Y(s)\right) = \left(s^2X(s)\right) + 6\left(sX(s)\right) + 12\left(X(s)\right)$$

$$Y(s)\left(s^2 + 10s + 24\right) = X(s)\left(s^2 + 6s + 12\right)$$

$$Y(s) = X(s)\frac{\left(s^2 + 6s + 12\right)}{\left(s^2 + 10s + 24\right)}$$

$$Y(s) = X(s)\left(\frac{-6}{s+6} + \frac{2}{s+4} + 1\right)$$

$$\frac{Y(s)}{X(s)} = \left(\frac{-6}{s+6} + \frac{2}{s+4} + 1\right)$$

$$H(s) = \left(\frac{-6}{s+6} + \frac{2}{s+4} + 1\right)$$

$$h(t) = -6e^{-6t}u(t) + 2e^{-4t}u(t) + \delta(t)$$

2 task 2

For the system *H(s)*, find *y(t)* for a step input using partial fraction expansion and inverse Laplace transforms. Perform the calculation by hand and type your final answer and other significant results.

$$H(s) = \left(\frac{-6}{s+6} + \frac{2}{s+4} + 1 \right)$$

$$Y(s) = \frac{1}{s} \left(\frac{-6}{s+6} + \frac{2}{s+4} + 1 \right)$$

$$Y(s) = \left(\frac{1}{s+6} + \frac{-1/2}{s+4} + \frac{1/2}{s} \right)$$

$$y(t) = e^{-6t}u(t) - \frac{1}{2}e^{-4t}u(t) + \frac{1}{2}u(t)$$