Measuring the Wavelength of Laser

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Abstract

In this experiment the wavelength of a laser is measured using a metallic ruler. The reflection pattern off the ruler exhibits diffraction which can characterized the wavelength. By finding the angles where constructive interference occurs the wavelength is calculated to be $539nm \pm uncertainty$. The experiment is repeated twice to improve initial results and uncertainties.

Introduction

In this experiment the wavelength of a green laser is measured using the diffraction pattern created by the mm markings of a ruler. The black ticks absorb the incident light while the metallic surface between reflects. We know from Huygen's Principle that each point on the wavefront is a source of new secondary spherical wavelets and the wavefront at a later time is the envelope of all the wavelets. This is the underlying explanation for why we observe interference patterns. The wavelength, λ, can be derived by measuring the spacing between constructive inference points resulting from diffraction. First, the setup and geometry is explained, then the relevant theory is introduced, followed by results and uncertainties.

Experimental Setup

The equipment used include a standard metallic ruler, flexible measuring tape $[cm]$ and $532nm$ laser pointer. Shining the laser pointer at a grazing incidence at the tip of the ruler creates a diffraction pattern as in figure. Figure shows the location of the ruler with respect to the image and relevant measurements. The quality of the diffraction pattern on the wall varied depending on the arrangement of ruler and laser. I decided to aim the pointer at the tip of the ruler so that the initial and reflected beam locations are always known. Tape is used to fixed the ruler to the wall while making measurements, reducing chances of systematic error.

1 Theory

Deriving the wavelength requires applying Huygen's Principle, wave interference and geometry. Interference is the result of multiple interacting waves, the resulting wave is the sum of amplitudes. The waves add either constructively or destructively. The bright spots on the wall are consequences of constructive interference. This occurs when the two wavelets travel integer differences in wavelength, refer to equation 1. The same is true for destructive interference, except the path length difference must be half integer values.

$$P_1 - P_2 = n\lambda \quad n = 1, 2, 3... \tag{1}$$

$$d(\cos(\alpha) - \cos(\beta)) = n\lambda \quad n = 1, 2, 3... \tag{2}$$

$$\beta_i = \tan^{-1} \frac{S_i}{L} \tag{3}$$

$$\alpha = \tan^{-1} \frac{S_0}{L} \tag{4}$$

The path length difference is determined by the geometry in figure . Comparing two incident paths, P_1 and P_2, at angles α and β we can calculate the interference spots at some distance L. We can see from figure something, the path difference can be expressed as in equation 2. The wavelength is dependent on the spacing of ruler ticks, d, and the angles, β and α. Equations 3 & 4 give us the necessary angles. Finally the wavelength can be inferred from each bright spot, S_i, from the reflection, S_0, using equation 5. Up to 10 bright spots are measured, the final wavelength is taken to be the mean, $\bar{\lambda}$.

$$\lambda = \frac{d}{n}[\cos(\alpha) - \cos(\beta)] \tag{5}$$

2 Results

The experiment is conducted twice. The first time, 4 consecutive interference patterns are measured to yield $\bar{\lambda}_1 = 553nm\pm$. Better results are obtained by measuring 10 interference points, $\bar{\lambda}_2 = 539nm\pm$. A python script is used to efficiently calculate all the angles and respective wavelengths. The result is within a reasonable deviation of the specified wavelength of $\lambda = 534nm$. Next, a more detailed explanation of the uncertainty is provided.

2.1 Uncertainties

The uncertainty is assumed to be completely random. There are two sources of uncertainty, the distance measurements and width or spread of the observed spots. First, the general formula for error propagation, equation 6 is used. The result is compared to the standard deviation of the mean (SDOM) method, equation 9. Again, python is used to propagate the error.

$$\delta\lambda = \sqrt{\left(\frac{\partial\lambda}{\partial\alpha}\delta\alpha\right)^2 + \left(\frac{\partial\lambda}{\partial\beta}\delta\beta\right)^2} \tag{6}$$

$$\sigma_\lambda = \sqrt{\frac{1}{N-1}\sum(\lambda_i - \bar{\lambda})^2} \tag{7}$$

$$\sigma_{\bar{\lambda}} = \frac{\sigma_x}{\sqrt{N}} \tag{8}$$

$$\delta\lambda = \sigma_{\bar{\lambda}} \tag{9}$$

2.1.1 Variations in Measurements

Individual measurements have independent error. The error measuring distance from reflection to bright spots, S_i is given by equation 10. The spread of light, ΔS_i is the width of a given bright spot. All uncertainties taken with the tape measure (δL) are assumed to be $\pm 0.2cm$.The uncertainty in measuring the length is given to be a result of the tape measure, δT. Each data point carries its own error which is taken into consideration when applying the general formula.

$$\delta S_i = \frac{1}{2}(\Delta S_0 + \Delta S_i) + 2\delta T \tag{10}$$

2.1.2 Uncertainties from General Formula

The the uncertainty calculated using equation 6 can be seen in figure 2.1.2. The spread of wavelength measurements suggest a smaller uncertainty than calculated. . In fact the smallest uncertainty, $\pm 32nm$ is more than $4\times$ greater than the standard deviation of the first data set. Perhaps the uncertainties were overestimated? A second estimate is made using uncertainties that are two orders of magnitude greater. The results are presented in figure 2.1.2. The data represents a reasonable amount of error, $\pm 20nm$, and all data points are within uncertainty. This is a construct but signifies the possibility of systematic error in the calculations.

$$\delta\lambda(n) = \sqrt{\left[\delta d \frac{1}{n}(\cos\alpha - \cos\beta_n)\right]^2 + \left]\delta\phi \frac{-d}{n}\frac{1}{(1+\phi^2)}\sin(\arctan\phi)\right]^2 + \left]\delta\Omega \frac{-d}{n}\frac{1}{(1+\Omega^2)}\sin(\arctan\Omega)\right]^2} \tag{11}$$
$$\phi = \frac{S_0}{L}\ , \quad \Omega = \frac{S_i}{L}$$

Before moving on,

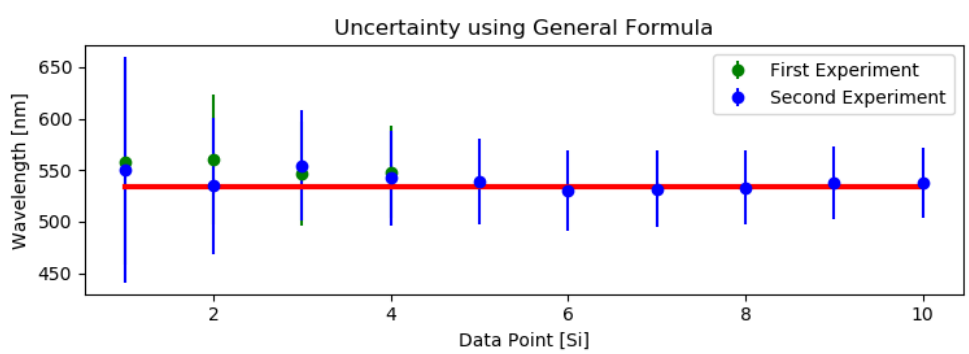

Figure 1: The error calculated with equation 6 ranges from $\pm 112nm$ to $\pm 32nm$. Of course this does not suggest precise data. The narrow spread of calculated wavelengths infer small random errors.The following figures are used for δq_i: $\delta S_i = 12.9mm$, $\delta S_0 = 13.0mm$ $\delta L = 10.0mm$ $\delta_d = 0.05mm$

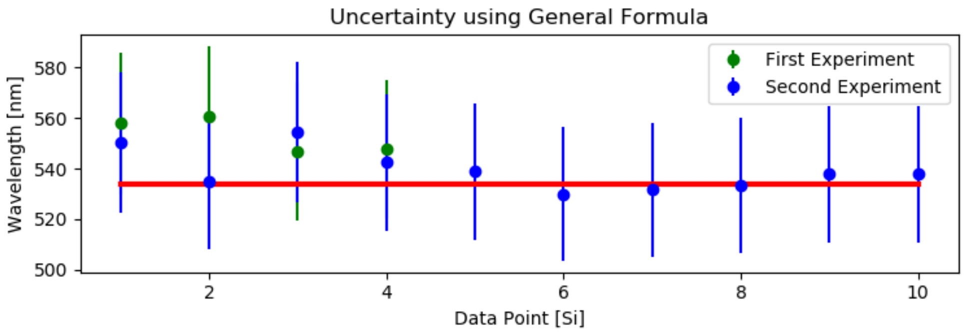

Figure 2: Changing the calculated uncertainties of the measured distances reveals a more reasonable solution. The following changes were made: $\delta S_i = 0.1mm$, $\delta S_0 = 0.1mm$ $\delta L = 0.2mm$ $\delta_d = 0.05mm$. Now the

3 Conclusion