Geometric Models of Meaning
Lecture 2

Daniel Edmiston

October 4, 2019

Geometric Metaphor of Meaning

- Embedding words/documents/morphemes as vectors imparts a spatial representation.

Geometric Metaphor of Meaning

- Embedding words/documents/morphemes as vectors imparts a spatial representation.

- Upshot: (i) We can use geometric methods to reason about these spatial representations (ii) Psychologically plausible?

Geometric Metaphor of Meaning

• Embedding words/documents/morphemes as vectors imparts a spatial representation.

• Upshot: (i) We can use geometric methods to reason about these spatial representations (ii) Psychologically plausible?

• The geometric metaphor of meaning rests on two basic, smaller metaphors.

- **Metaphor 1**: *Similarity is proximity*

- **Metaphor 1**: *Similarity is proximity*

- **Metaphor 2**: *Entities are locations*

- **Metaphor 1**: *Similarity is proximity*

- **Metaphor 2**: *Entities are locations*

- **The geometric metaphor of meaning**: Meanings are locations in a semantic space, and semantic similarity is proximity between the locations.

- This metaphor has limits!

- This metaphor has limits!

- Usual space of embedding good for proximity, but no obvious way of dealing with, e.g. hierarchical relationships.

- This metaphor has limits!

- Usual space of embedding good for proximity, but no obvious way of dealing with, e.g. hierarchical relationships.

- Dimensions don't necessarily have interpretable meanings (though sometimes they do, or can be deciphered).

- Core idea for our purposes: Semantic similarity can be represented as proximity in embedding space, usually $\mathbb{R}^n$.

- Core idea for our purposes: Semantic similarity can be represented as proximity in embedding space, usually $\mathbb{R}^n$.

- High values for n impossible to visualize, but we can reason about them with the same tools from low-dimensional geometry

- Core idea for our purposes: Semantic similarity can be represented as proximity in embedding space, usually $\mathbb{R}^n$.

- High values for n impossible to visualize, but we can reason about them with the same tools from low-dimensional geometry

- We can build (some) intuitions for high-dimensional reasoning from examining low-dimensional embeddings.

- *Board*: Let's embed some words into $1D$ space.

- *Board*: Let's embed some words into $1D$ space.

- What about this matches our intuitions? What doesn't?

- *Board*: Let's embed some words into $1D$ space.

- What about this matches our intuitions? What doesn't?

- What might a (partial) solution to the counterintuitive portions of our model look like?

- *Board*: Add another dimension, i.e. embed words into $2D$ space.

- *Board*: Add another dimension, i.e. embed words into $2D$ space.

- What does this improve?

- *Board*: Add another dimension, i.e. embed words into $2D$ space.

- What does this improve?

- What problem remains? What's a potential solution?

- Till now, we've been discussing proximity intutively with pictures. How can speak of proximity formally?

- Till now, we've been discussing proximity intutively with pictures. How can speak of proximity formally?

- Distance function on a set A:

$$d : A \times A \to \mathbb{R}$$

- Till now, we've been discussing proximity intutively with pictures. How can speak of proximity formally?

- Distance function on a set A:

$$d : A \times A \to \mathbb{R}$$

- Distance functions must satisfy certain requirements:

(i) $\forall x_{\in A}.d(x,x) = 0$
(ii) $\forall x, y_{\in A}.d(x,y) = d(y,x)$
(iii) $\forall x, y_{\in A}.d(x,y) \geq 0$
(iv) $\forall x, y, z_{\in A}.d(a,c) \leq d(x,y) + d(y,z)$

- Euclidean distance is the standard choice for $\mathbb{R}^n$.

- Euclidean distance is the standard choice for $\mathbb{R}^n$.

- $d_{Euclidean} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$

$$d_{Euclidean}(x, y) := \sqrt{\Sigma_{i=1}^{n}(x_i - y_i)^2}$$

- Euclidean distance is the standard choice for $\mathbb{R}^n$.

- $d_{Euclidean} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$

$$d_{Euclidean}(x, y) := \sqrt{\Sigma_{i=1}^{n}(x_i - y_i)^2}$$

- *Board*: Let's do some examples in $\mathbb{R}^2$.

- Euclidean distance can be calculated by the Pythagorean theorem.

- Euclidean distance can be calculated by the Pythagorean theorem.

- This translates to higher dimensions.

- Euclidean distance can be calculated by the Pythagorean theorem.

- This translates to higher dimensions.

- *Board*: Let's try an example in $\mathbb{R}^5$ (we can't visualize this, but the math still works), then let's try in very high-dimensions