Smart Routing v2

Giddy & Dave

4/5/2022

1 Problem Description

Ref Finance is the first and premier DEX on the Near protocol. The Ref team is continually working to add features and to improve the User Experience for Ref users. [1]

One key feature of Ref Finance has been the ability to create multiple liquidity pools for any given token pair. This feature brings a lot of opportunities as well as some interesting challenges. For example, suppose you want to swap Token A for Token B, and that there are two A-B pools available for the swap. That is, there are two pools, each of which has reserves of Token A and Token B to facilitate trades at a price determined by the constant-product market maker (CPMM) formula. Depending on the composition of each pool and the amount of the swap, you may find a better price in one pool or another, or by splitting the swap between the pools. The Ref UI has recently implemented an optimal "parallel swap" that determines the best allocation between parallel pools. [2] This parallel swap is applied automatically where it is applicable, in order to give the user the best value for the transaction (that is - to maximize the amount of Token B to be received). However, the current state of the Ref UI requires at least one pool to exist with reserves of Token A and Token B in order to facilitate direct trades between the tokens.

This leads us to define the two primary problems we address in this white paper, along with steps towards solving them:

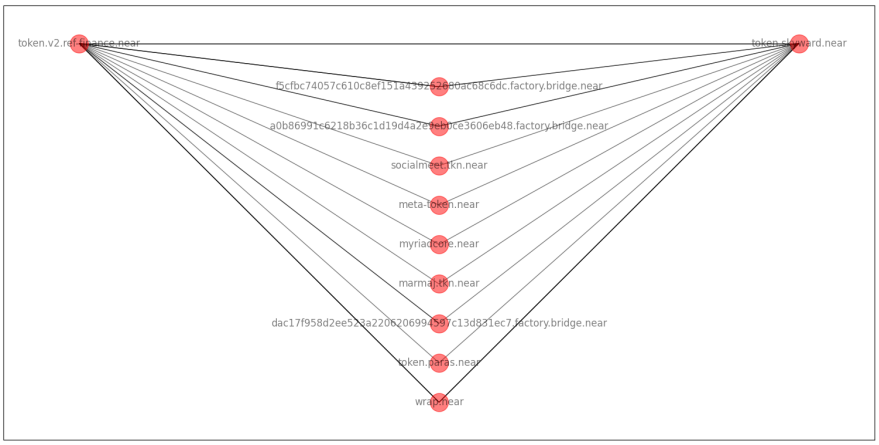
1. Enable users to trade directly among more token assets in the Ref network.
2. Ensure a good price for the trades performed.

While multihop routing was rolled out as a preliminary capability in Ref Finance with the moniker Smart Routing V1, the more advanced method we propose here will be an extension, and can be defined as Smart Routing V2.

1.1 Motivation

At present, the only way to trade a given Token for another token is if a particular pool exists with reserves of each token. However, in the absence of a pool to facilitate a direct trade between a given pair of tokens, there might be a series of trades (multiple swaps through the pool network) to allow an effective trade between the two desired tokens.

It is possible, through a series of manual trades, to perform multiple swaps, beginning with one pool, then taking the resulting output from the first pool to trade into the second pool, and so on, until a pool is reached with the desired output token. However, this process adds friction to the user experience, and thus could be taking away from the potential trades.

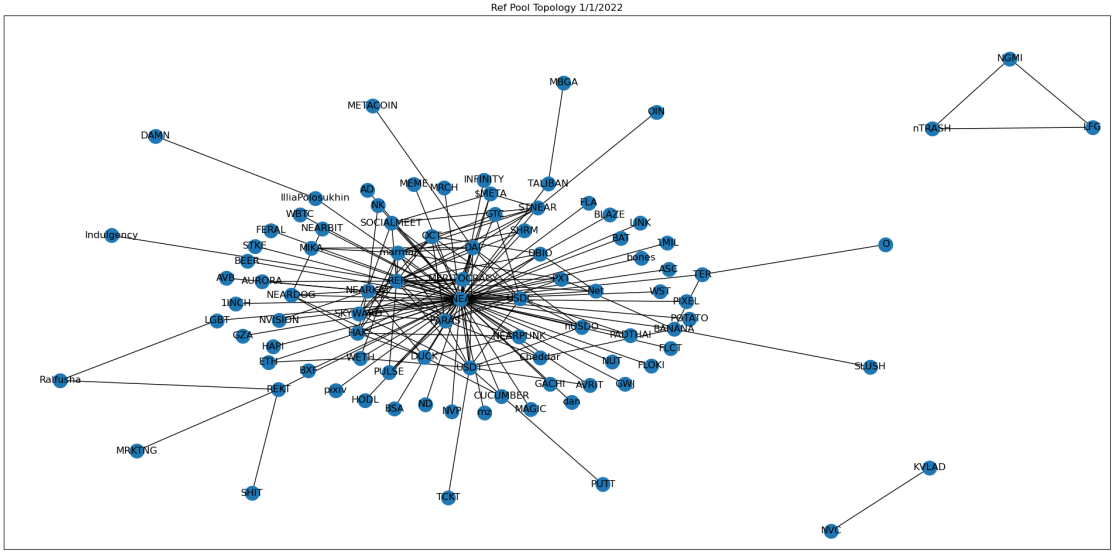


A more streamlined approach would allow the user to trade between any two tokens on the Ref Finance network, without the user having to worry about which pools to use or the underlying implementation of the series of transactions. Under the hood, this would transform the problem of having a pool for a token pair into a network-connectedness problem, which, given the network topology of the Ref Finance pools we will show here, allows for a much simpler and reliable experience for the user.

In the figure above, we display an example set of smart routes going from token.v2.ref-finance.near to token.skyward.near, both directly and via intermediary hops.

1.2 The Ref Finance Network as a Graph

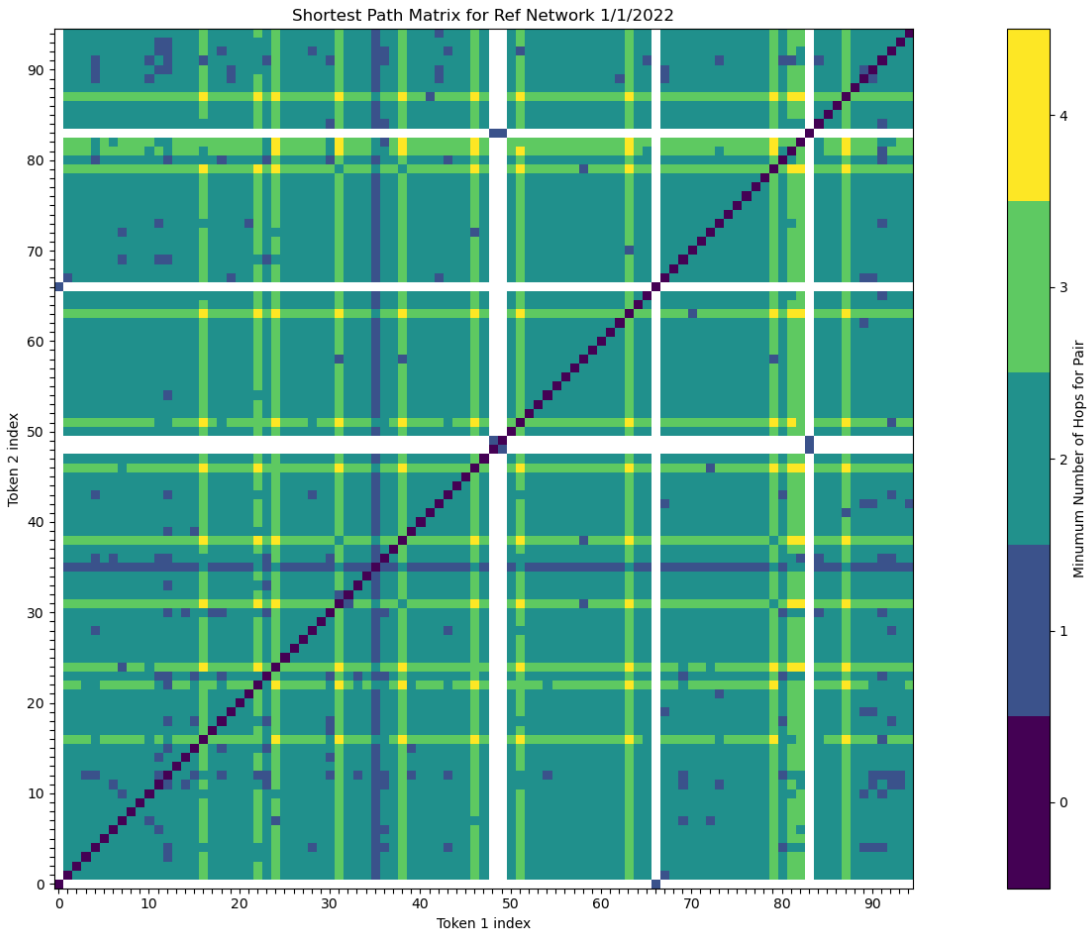
One way to model the collection of Ref Finance liquidity pools is as a mathematical graph. Graphs in mathematics consist of nodes and edges, and can be used to represent networks ranging from internet traffic and routing to supply chain logistics.



Above is a depiction of the topology of the non-zero-liquidity swap pools for Ref Finance, as of 1/1/2022. Each node (circle) in the graph represents a token, and each edge (line) in the graph represents one or more pools that facilitate swapping between the connected pair of tokens. That is, if a line exists between two tokens, there is a direct swapping pool between them. If there is not a line between the two tokens, then there is currently no pool to facilitate a direct swap between the two assets. Note that this visualization could provide an incentive for liquidity providers to instantiate their own pools between pairs.

By performing multiple swaps (or multiple hops in the graph network), we can facilitate trades between a much large collection of tokens programatically, without requiring the user to manually perform a series of swaps themselves. Note, on the right side of the image are a few tokens that form their own sub-graphs, but are disconnected from the main graph. For these tokens, unless a new connector pool is added, there is no way to perform multi-hops to facilitate trades with the rest of the network. They are essentially isolated from the rest of the network and therefore unreachable.

One way to think about the network graph is in terms of the minimum "distance", or minimum number of network hops, required to trade an arbitrary pair of assets. We capture this data below in a matrix of the minimum number of hops between a given pair of tokens:



The matrix is symmetric about the diagonal (the dark blue diagonal going from bottom left to top

right in the figure), which is to be expected, since the ability to swap a between a pair of tokens in a pool is bi-directional. That is, if there exists a series of swaps among, say, pools $1 \to 2 \to 3$, then there also must exist a possible series of swaps going the other way (from pools $3 \to 2 \to 1$). The blue cross at token index 35 near the center of the matrix (which implies a certain token has 1-hop connection to most other tokens in the network) represents the *wrap.near* token. The series of white crosses on the figure represent the sub-networks mentioned above. These tokens are not connected by any number of hops to the main network nodes, and are therefore only reachable from within their own sub-network nodes.

The predominance of the dark-green color (corresponding to connectedness of 2 hops) implies qualitatively that most of the network is reachable through only 2 hops. We will show the exact statistics below.

The following is a table of summary statistsics for how many pairs in the network are connected by a certain minimum number of hops. Other than the few tokens mentioned above that are disconnected from the primary connected network of pools, which we will ignore for now, all other tokens are reachable by 4 swaps (network hops) or less.

Minimum # Hops	Frequency in Network	Percentage of Pairs in Network
1	177	4.4 %
2	2916	72.7 %
3	862	21.5 %
4	54	1.3 %

Currently, there are only 177 single-hop pairs in the Ref Finance network. This represents the current state of the Ref network in terms of possible direct swaps the user can currently make. However, note the substantial number (2916) of token pairs connected by 2 hops.

> Stated another way, by introducing only a double-hop, we will increase the number of direct swaps accessible by the Ref users by more than 1600 %!

From the above table, we can see that the connectedness of the network is such that about 77% of the network pairs are connected by 2 hops or less.

If you extend the consideration to allow for 3 hops, then that covers almost 99% of the network pairs.

Looking at the connectivity using a different metric, (that of connected TVL rather than pure connectivity), we found that the vast majority of 99.7% of the Ref Finance network's TVL is covered by one hop from wnear.

With more hops comes more algorithmic complexity and more computation time. However, substantial progress towards solving problem (1) of the paper can be achieved with only slightly more complexity than the parallel swap algorithm.

As such, this case (2 hops or less) will be the primary focus of this white paper.

1.3 Graph Topology for Single-Hop and Double-hops

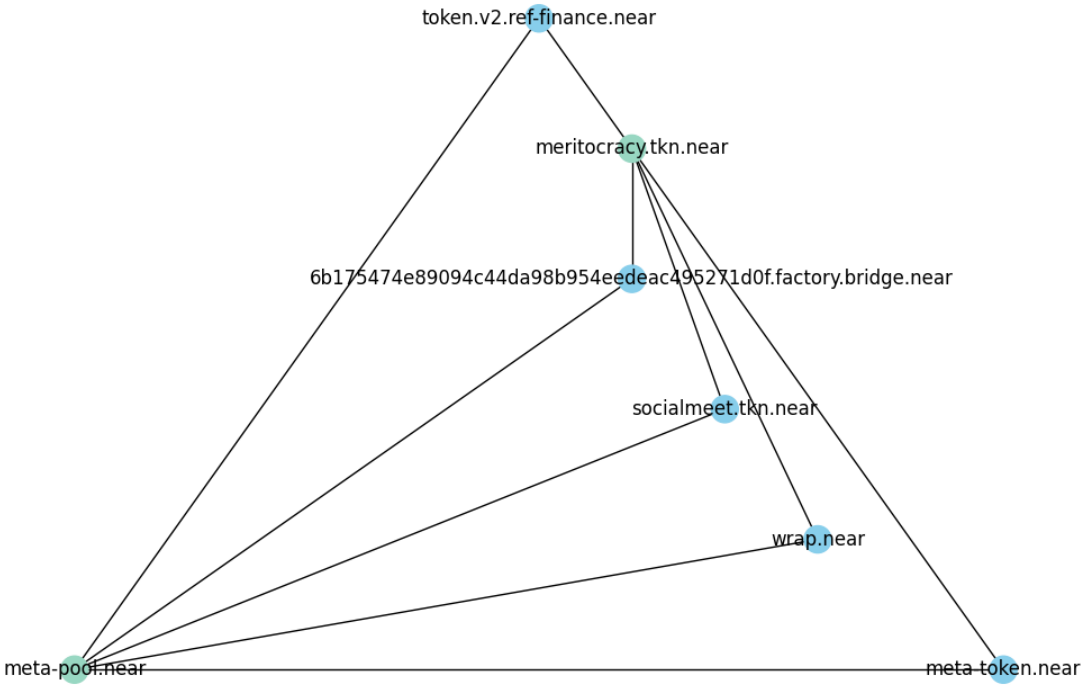
Consider the set of tokens taken from the Ref Finance network of pools:

* `meta-pool.near`

* `meritocracty.tkn.near`

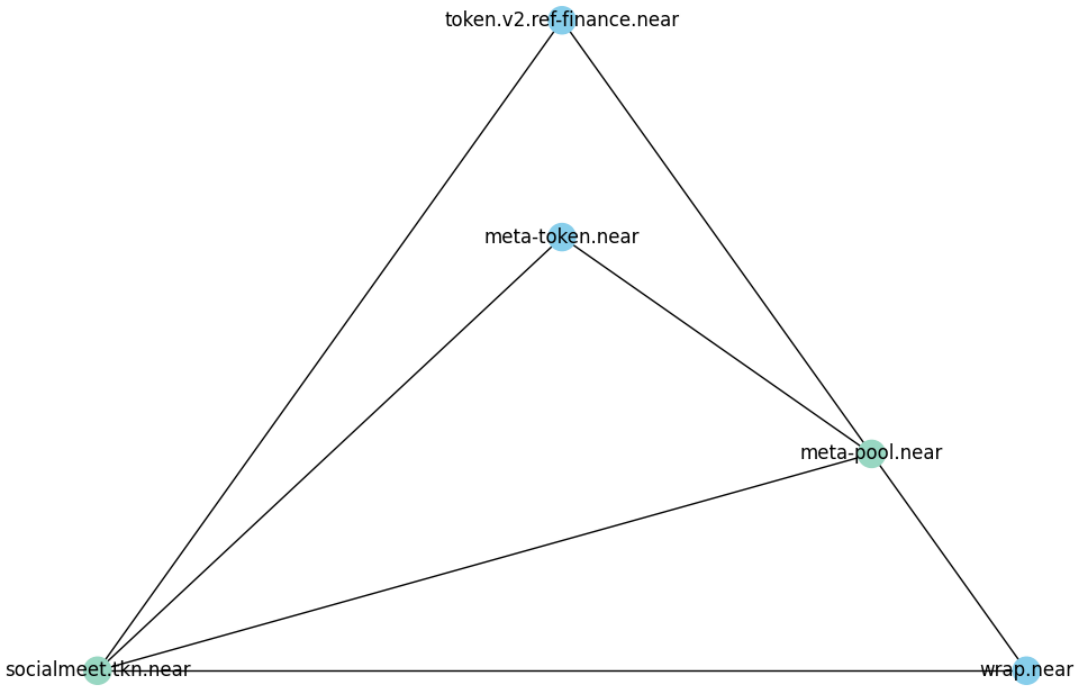
* `socialmeet.tkn.near`

If we consider two tokens from this set, we can determine all of the 1-hop and 2-hop path connections between them. For example, in the figure below, we consider the pair of `meta-pool.near` and `meritocracy.tkn.near`, which have double-hops, but no single-hops (that is, no direct-swap pools):



Here, there are 5 intermediary tokens shared as common nearest-neighbors to both `meta-pool.near` and `meritocracy.tkn.near`.

Next, we consider below the pair of tokens `meta-pool.near` and `socialmeet.tkn.near`. Here, there are both single-hop paths and double-hop paths between the tokens.



But for all of these cases, what routes give the user better value? At first, it would seem that a single-hop would possibly provide better value, since there is only one fee application in this case (rather than doubling up on fees if two hops are taken). But, there can be cases where the value mismatch between the pools is such that a double-hop could provide better value than a single-hop. Interestingly, this value mismatch primarily occurs when the prices are "out of equilibrium," and the value mismatch is the natural territory of arbitrageurs, who extract value from the system in return for bringing the system closer to equilibrium in the form of market efficiency and price consistency across the market.

> A benefit of Smart Routing V2 is that latent value in the market inefficiency, which had previously only been within the purview of arbitrageurs, is now capturable by all Ref users as well.

The Smart Routing V2 algorithm adapts dynamically to the changing market conditions, in order to provide the best value to Ref Users.

To help in solving problem (2) of the paper, we propose two candidate algorithms. The first is an extension of the parallel swap algorithm, which makes use of the mathematical technique of Lagrange Multipliers to determine the optimal allocation of trades among these paths. The second is a method that uses linearization. But first, we'd like to give an overview of the concept of optimization in general, and lay out a few of the possible methods of tackling the problem of giving value to users through a good token price.

2 Introduction to Optimization

2.1 Calculus

The mathematical field of optimization is dominated by calculus. Students who have completed Calculus 1 have received the training needed for most basic optimization problems. The goal is to take the derivative of the desired function (which we call the objective function) and set it to zero. Everywhere the derivative is zero represents a maximum or a minimum of the function. We use the term extreme in place of maximum or minimum, and it is usually straightforward to determine if an extreme is a maximum or a minimum.

2.2 Constraints and Lagrange Multipliers

Sometimes there are constrained optimization problems. The calculus of taking the derivative and setting it to zero still applies, but with a notable modification. Since the constraint must be accounted for, and it must be a part of the derivative, it is necessary to write the constraint as an expression that should equal zero. We can add the expression multiplied by a term λ known as a Lagrange Multiplier and then take our derivative, setting it to zero. Almost magically, the optimization is solved, including correct values of all the constraints. The Lagrange Multiplier is used to enable the Parallel Swap feature on Ref Finance. [2]

2.3 Convex Optimization

In general, optimization can become very complicated for a large number of constraints and for complicated objective functions. If constraints or the objective function are complicated, then the path to the solution will be complicated as well. In that case, the time and resources required to solve the problem grow rapidly as the number of variables increases. It is better for hand calculations and for computers to solve a special type of optimization, namely a convex optimization problem. This type of problem requires certain restrictions on the objective function and the constraints. Convex problems can be solved quickly and easily by a computer, even with many variables and constraints. Optimizing DeFi seems to be a convex problem; it certainly is for the task at hand. Therefore, we can confidently develop an algorithm and trust that the optimal solution can be found quickly and efficiently. Recently, Angeris *et al*. at Stanford wrote a paper that uses a standard Python convex optimization library, `cvxpy`, to find an optimal net set of trades among a family of liquidity pools of various types. [3] Because their code is open source, we were able to adapt it to model the same parallel swap problem we solved before, and found that their method gives identical results to ours.

2.4 Linear Programming

If the objective function and the constraints are all linear functions, then the optimization is linear. Linear optimization is a special case of convex optimization, for a faster solution. Constant Product Market Makers are definitely not linear; however, users generally want to trade within an approximately linear regime (as little slippage as possible), and it may be possible to use linear approximations within limitations to get a "good enough" solution in a much more reasonable amount of time. As our algorithms are implemented, it may be desirable to employ linear approximations. This will help the Ref team include reasonable constraints directly into the optimization, such as

- relative liquidity
- number of intermediate swaps (currently limited to 1, but could potentially be higher)
- maximum total number of transactions in the overall swap
- single algorithm to handle parallel swap and multi-swap
- single algorithm capable of CPMM and stable swap

Constraints such as these can be input with hard cutoffs, or as a secondary optimization problem.

2.4.1 But what is a "good enough" solution?

Some optimization problems have an obvious solution. For example, in a Decentralized Exchange with parallel swaps, one pool might have the best price and most liquidity, and the solution will be 100% of the swap in that pool. However, we rely on the computer to find the optimum for the cases that are not obvious. Even the computer might take some amount of time to crunch the numbers and find a solution. In that period of time, the prices will potentially change due to other investors trading. We recognize for any trade that a slippage tolerance is allowed, of 0.1%, 0.5%, or even 1%. This betrays our knowledge that even after we initiate a trade, the prices can change by that much in essentially no time.

Any trade may fail due to slippage, as discussed above. As the number of trades increases, the probability of failure also increases. Spending more time to get a tiny increase in the outcome might be the factor of whether the entire trade fails or is completed. This factor is not based on any kind of profit percentage, but it should be noted that the amount of time spent to solve the problem should be proportional to the desired outcome.

Gas fees are not included in the optimization algorithm. While gas fees are small on the Near protocol, they are nonzero, and will start to add up, especially for a trade split into many (say, 20-30) swaps. These gas fees comprise a small but nonzero percentage of the swaps.

Consider the complex problem of numerous liquidity pools. Some may have an unfavorable price and/or very little liquidity. The algorithm must still consider all these pools. Imagine that using the single largest pool would yield a return of $1000, but that a second transaction with a smaller pool could yield $1000.01 and be selected as the best solution. This would actually be a worse solution, since the gas fees would be higher, and the risk of a failed transaction would be much higher.

The need for a "good enough" solution considers that the true optimum will always have error bars of around 1%. Therefore, if we can reach a solution within 1% below the maximum but in less time (really, 99% of the time would work, but we might be significantly faster) then we can have the best overall outcome and better user experience.

In the field of regression, more variables always gives a better pure result. However, this better numerical result is not always better in real life, and is not always helpful. Several techniques employed in regression are used to get the best result considering the number of variables. The ideas behind these techniques translate into optimization, such that it will be possible to get a good enough result using the smallest number of trades practical, and to improve the outcomes and the experience of Ref users.

2.5 Examples

2.5.1 Optimizing the number of trades

The idea of smart routing (including or in addition to parallel swap) is to allow trading from multiple pools simultaneously to get the best overall price for a trade. This is obviously good if I can swap equal amounts in two pools to reduce the slippage. However, if I have pools with very different liquidity, I might optimize the swap and get only 0.1% better outcome. This is a tiny improvement, especially considering gas fees, increased failure risk due to slippage, and the potential of a transaction succeeding with a slippage tolerance of 0.5% or 1% which might actually be less optimal.

Besides including a constraint for the maximum number of simultaneous swaps, it is very helpful to get the best outcome with the fewest simultaneous swaps. Linear optimization allows for regularization, which is similar to Lagrange Multipliers. Regularization allows for additional optimization by applying a penalty for the number of swaps, meaning a swap would only be added if it contributes meaningfully to the outcome.

2.5.2 Optimizing the serial and parallel swapping

Similarly, a parallel swap might give a pretty good price while trading in 5 pools. However, a "serial" smart routing swap might actually yield a better price, even going through 2 or more intermediate pools. This would be very tedious to determine manually, but if the equations are set up correctly, it would be a straightforward outcome of the optimization algorithm.

3 Optimizing DeFi

3.1 General Equations

The task is to maximize ΔB_{total} as a function of ΔA_{total}, subject to the constraints,

$$\Delta A_{total} = \sum_i \Delta a_i$$
$$\Delta a_i \geq 0$$

A single pool is characterized by three terms:

- x: the amount of Token X in the pool
- y: the amount of Token Y in the pool
- ρ: the fee for trades in the pool (ie. 0.003 for a 0.3% fee)

Defining $\gamma \equiv 1 - \rho$, we see that

$$\Delta y = \frac{\Delta x \cdot y \cdot \gamma}{x + \Delta x \cdot \gamma}$$

Now we assume pool 1 contains Tokens A and C, and pool 2 contains Tokens B and C. I want to trade Token A for Token B, but I'll have to go indirectly from A to C in pool 1, and then from C to B in pool 2.

First, I'll swap from A to C. I'll define Δc as the amount of C I gain in this first swap, and the same amount I swap into pool 2 afterwards.

$$\Delta c = \frac{\Delta a \cdot c_1 \cdot \gamma_1}{a + \Delta a \cdot \gamma_1}$$

$$\Delta b = \frac{\Delta c \cdot b_2 \cdot \gamma_2}{c_2 + \Delta c \cdot \gamma_2}$$

$$\Delta b = \frac{\Delta a \cdot c_1 \cdot \gamma_1 \cdot b_2 \cdot \gamma_2}{c_2 \cdot (a + \Delta a \cdot \gamma_1) + \Delta a \cdot c_1 \cdot \gamma_1 \cdot \gamma_2}$$

$$\Delta b = \frac{\Delta a \cdot c_1 \cdot \gamma_1 \cdot b_2 \cdot \gamma_2}{a_1 \cdot c_2 + \Delta a \cdot (c_2 \cdot \gamma_1 + c_1 \cdot \gamma_1 \cdot \gamma_2)}$$

This is simple enough if there is exactly one path from A to C, and one from C to B. However, if there are multiple pools from A to C, then Δc will be a separate optimal solution with some parallel combination of the pools. Likewise, multiple pools from C to B will require a parallel solution there.

Below, we will introduce one of the possible methods of achieving a smart route swap, which not only allows for double-hops and thereby opens up a significant amount of the Ref network to direct swaps, but also delivers an excellent price.

This method uses the method using Lagrange Multipliers, and is a more advanced and generalized version of the parallel swap algorithm.

3.2 Solution Algorithm 1: Lagrange Multipliers

We can extend the parallel swap expression to a more general one that captures both single hops and double-hops, all in terms of the initial amount of Token A traded in, and of the reserve and fee parameters of each pool.

We seek to optimize the allocations of Δa_i across all *routes* i, rather than over all parallel pools, as in the case of the parallel swap algorithm .[2] The key difference between this solution and the parallel swap solution is that more information about the network topology must be calculated in order to determine all relevant routes between token A and token B. This adds complexity when determining the

Here, we define a route as an ordered list of pools, where all of allocation Δa_i is assumed to be traded into the first first pool of the list of pools for route i. If further pools remain in the route, then all of the output from the first trade is traded into the next pool, and so on.

For now, we seek the shortest routes between each token pair by considering only single-hops and double-hops.

The total amount of ΔB, the amount of Token B that we want to maximize, is given by:

$$\Delta B_{total} = \sum_i \Delta b_i = \sum_{singlehops} \Delta b_i + \sum_{doublehops} \Delta b_i$$

The benefit of this solution is that it would not require the heavy machinery of a convex optimization solver. These solvers are plentiful in python, but are much more scarce in Javascript.

Using Lagrange Multipliers, we can maximize ΔB, given the constraints:

$$g_T(a_i) = \sum_i \Delta a_i - A_T = 0$$

$$g_i(a_i, v_i) = \Delta a_i - \frac{1}{2} v_i^2$$

Here, i is an index corresponding to a specific route, and Δa_i is the amount of input token A allocated to route i. The value v_i is a slack variable for route i, used to enforce the non-negativity of Δa_i.

The g_T constraint ensures that the sum of all the inputs (that is, the total over all the allocations) is equal to the total amount of input token A.

The g_i constraints are a collection of N equations, one for each route, in order to enforce the non-negative nature of the allocation value a_i.

We set up a gradient defined over all variables, assuming there are N routes:

$$\nabla \equiv < \partial_{\Delta a1}, \partial_{\Delta a2}, \partial_{\Delta a3}, ..., \partial_{\Delta aN}, \partial_{v1}, \partial_{v2}, \partial_{v3}, ..., \partial_{vN} >$$

$$\text{Maximize } B_T(\Delta b_1, \Delta b_2, ..., \Delta b_N)$$

$$\text{subject to the constraints:} g_T, g_1, g_2, g_3, ..., g_N$$

We can solve this by introducing (N+1) Lagrange multipliers, λ_T and $\lambda_1, \lambda_2, \ldots, \lambda_N$:

$$\nabla B_T = \lambda_T \nabla g_T + \sum_i \lambda_i \nabla g_i$$

We note that the i^{th} element of ∇B_T will have a different value, depending if route i is a single-hop or a double-hop. If route i is a single-hop, the value is:

$$\nabla_i B_T = \frac{a_1 \cdot b_1 \cdot \gamma_1}{(a_1 + \Delta a_i \gamma_1)^2}$$

However, if route i is a double-hop, then the i^{th} element of the gradient of B_T is given by:

$$\nabla_i B_T = \frac{a_1 \cdot c_1 \cdot c_2 \cdot b_2 \cdot \gamma_1 \cdot \gamma_2}{[a_1 \cdot c_2 + \Delta a_i (c_2 \cdot \gamma_1 + c_1 \cdot \gamma_1 \cdot \gamma_2)]^2}$$

In both of these cases, a letter (a_j, b_j, c_j) corresponds to the reserves of that token amount for pool j. Recall that for a single-hop, the only pool will contain reserves of a and b. For the double-hop, the first pool in the route will contain reserves of a and c, while the second pool in the route will contain reserves of c and b.

Using these equations, we can simplify and solve for λ_T in terms of the pool parameters and the total input A_T. Then, we can plug the value of λ_T into each expression in order to calculate the value of the allocation Δa_i for each route i.

The N multipliers λ_i can be used to enforce non-negativity of a_i in the following manner. If a particular route i is found to have a negative allocation (that is, if $\Delta a_i < 0$), then the allocation for that route is set to zero, and that route is removed from consideration. The calculation must be performed another time using the remaining routes with non-negative allocations. This will give new values of λ_T, λ_j and Δa_j for the remaining routes.

Happily, ∇B_T, whether single-hop or double-hop, is of the form:

$$\frac{\alpha}{(\beta + \epsilon \cdot \Delta a_i)^2}$$

Where α, β, and ϵ are constants that depend only on route and constituent pool parameters. For a single hop, assuming the trade goes from Token A to token B, the values of the constants are as follows:

SINGLE-HOP:

$$\alpha_{SH} = a \cdot b \cdot \gamma$$

$$\beta_{SH} = a$$

$$\epsilon_{SH} = \gamma$$

And, for a double-hop:

DOUBLE-HOP:

$$\alpha_{DH} = a_1 \cdot c_1 \cdot c_2 \cdot b_2$$

$$\beta_{DH} = a_1 \cdot c_1$$

$$\epsilon_{DH} = c_2 \cdot \gamma_1 + c_1 \cdot \gamma_1 \cdot \gamma_2$$

Therefore, if we solve the expression using Lagrange Multipliers, we end up with a series of equations (1 equation for each route i, of the form:

$$\lambda_T = \frac{\alpha_i}{(\beta_i + \epsilon_i \cdot \Delta a_i)^2}$$

We can solve each of these N equations for Δa_i:

$$\Delta a_i = \frac{\sqrt{\frac{\alpha_i}{\lambda_T}} - \beta_i}{\epsilon_i} \equiv \frac{\phi \cdot \sqrt{\alpha_i} - \beta_i}{\epsilon_i}$$

Here, we have introduced a variable $\phi \equiv \frac{1}{\sqrt{\lambda_T}}$ to simplify the expression.

And, since the sum over all routes of the allocations Δa_i should sum to A_T, we can now solve for ϕ in terms of all route pool parameters and A_T:

$$A_T = \sum_i a_i = \sum_i \frac{\phi\sqrt{\alpha_i} - \beta_i}{\epsilon_i}$$

$$\phi = \frac{A_T + \sum_i \frac{\beta_i}{\epsilon_i}}{\sum_i \frac{\sqrt{\alpha_i}}{\epsilon_i}}$$

So solving for the optimal allocation of token inputs per route amounts to solving for ϕ, and then plugging ϕ back into the expression for a_i in order to calculate the allocation for route i. Then, if any of the routes allocations are negative, those route allocations should be set to zero, and the ϕ should be re-calculated for the remaining routes.

Solving for ϕ becomes a simple matter of calculating the values of α_i, β_i, and ϵ_i for each route i, and then summing the expressions above to get ϕ.

3.2.1 Benefits and Challenges of Method

Benefits:

This method – that of using Lagrange Multipliers, is fairly straightforward to implement. Note, our method reveals not only the best single route, but also the top N routes. Further still, the solution naturally yields the best allocation among these top routes, as an optimal superposition.

Challenges:

We have an implementation in python currently, and the only 3rd-party algorithm we make use of is one from the networkx module, which finds all paths between two nodes in a network. We can either try to implement this algorithm in Javascript, or we could look into wrapping the python to call from Javascript.

Note, depending on the topology of the network, this algorithm might not give the absolute optimal answer. That is because the method assumes all routes are independent. However, when multiple routes share a certain leg (that is, if the paths split or merge, there is some interdependence, becuase the reserves in the pool they share will not be the same for each. This will mainly be an issue if there are two more or less equally allocated routes, and if the allocations are large enough to cause significant slippage in the shared pool.

The algorithm will identify the best allocations among the top N independent routes, but gas limitations could inhibit the ability to take advantage of all of these routes at once. Therefore, for extremely large value trades, it might be advantageous to break the transaction into a few smaller transactions. In this way, the dynamic algorithm will automatically choose the best routes each time, yielding more value to the large trader.

4 Final Algorithm Implementation

We ended up choosing to do an implementation of the Lagrange Multiplier solution in javascript. Our original solution was in python, but the final version entailed porting some of the code from the python library networkx. In particular, we implemented Yen's shortest path algorithm, Dijkstra's algorithm, and a basic graph class in javascript in which to store the pool transition data.

Further, we found that there were additional constraints for the smart routing solution, including a limit on the number of batch transactions allowable due to gas limits. This introduced an interesting balance of priorities, since the original algorithm finds the best allocations among independent routes. Depending on network topology and the mismatch of value among pools, an optimal solution may involve allocations among dozens of double-hop routes. However, with the constraint of only 4 actions (up to two independent 2-hop routes, or up to 4 single-hop parallel pools), we want to ensure that Ref Finance users can get the best price, given these extra constraints. However, due to the highly nonlinear nature of the problem, there are various ways of tackling this problem. We implemented a solution to find the optimal allocation across all pairs of independent routes.

Additionally, while the Lagrange multiplier solution only takes onto account simple (CPMM) pools, we also wanted to leverage the recent Stable Pools, which have enhanced bonding curves to allow for much lower-slippage trades among three stablecoins.

To include the stable pools in the algorithm, we checked whether the input token or the output token, or both input and output tokens were stablecoins. If both were stablecoins, we took the best price between the Lagrange routes and the stableswap.

If the input token is a stablecoin, but the output token is not, then we consider a first hop route using the stable pool, followed by a direct hop from a stable middle token to the output token.

If the output token is a stablecoin, but the input token is not, then we consider a first hop to a stable middle token, followed by a second hop using the stable pool. Finally, the best price is chosen, whether using the stable swap, the hybrid stable - direct hop, or the two-route Lagrange solution.

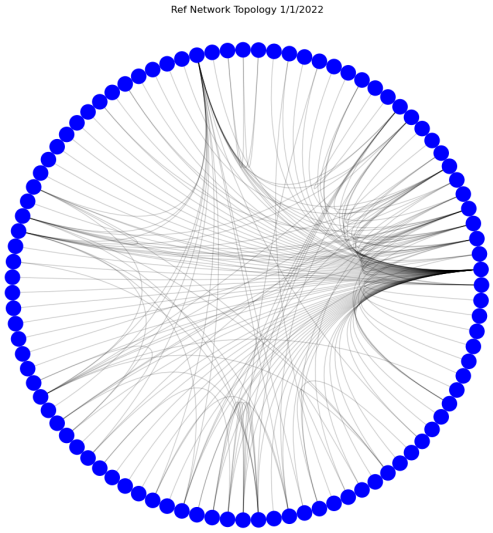
5 References

1. https://app.ref.finance
2. P. Gartland & J.D. Jackson. “Parallel Swap Optimization”. Sept. 2021.
3. G. Angeris, T. Chitra, A. Evans, & S. Boyd. “Optimal Routing for Constant Function Market Makers”. Dec. 2021.

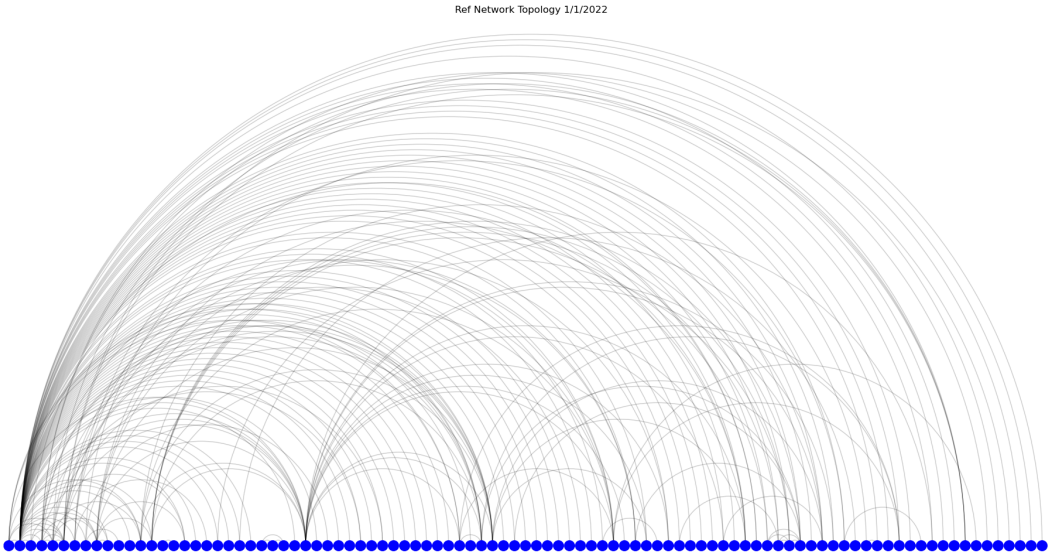
5.1 BACKUP: Extra Plots for Ref Finance Token/Pool Network Topology

Here are a few other graph representations of the network, to give a visual feel for how the nodes are connected and therefore what pools are available.

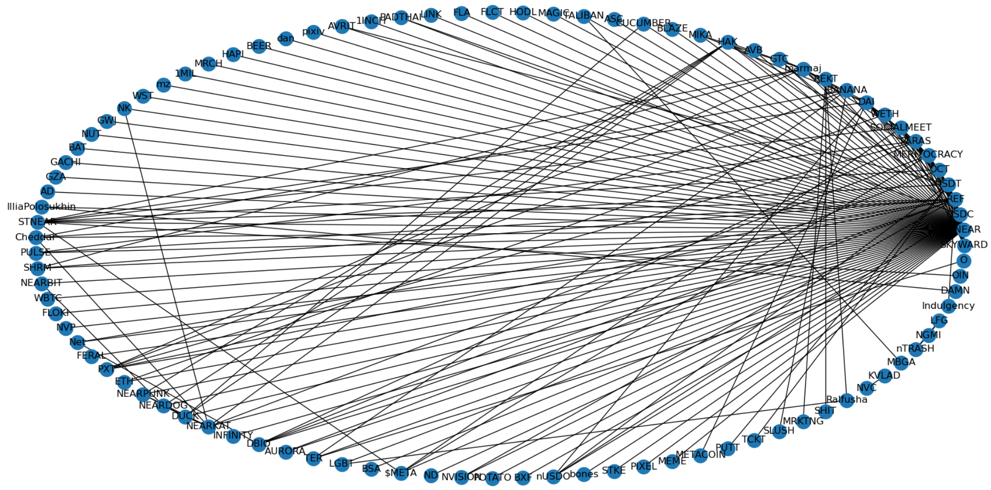
First, a Circos Plot displays a non-labeled picture of the connectedness of the network. The dense series of branches coming from the right side of the plot stem from the *wrap.near* token.



Next, we display an Arc Plot. This is just another way to see how connected the nodes of the network are.



And finally, we display a circular graph with token symbol labels to further plot the connectedness of the network.


1INCH
PADTHAI
LINK
FLA
FLCT
HODL
MAGIC
TALIBAN
ASC
CUCUMBER
BLAZE
MIKA
HAK
AVB
GTC
BANANA
DAI
WETH
SOCIALMEET
OCT
USDT
REF
USDC
NEAR
SKYWARD
DAMN
Indulgency
LFG
NGMI
nTRASH
MBGA
KVLAD
NVC
Balrusha
SHIT
MRKTNG
SLUSH
TCKT
PUTT
METACOIN
MEME
PIXEL
STKE
bones
nUSDO
BXF
POTATO
NVISION
NI
$META
BSA
LGBT
TEK
AURORA
DBIO
INFINITY
NEARKAT
DUCK
NEARDOG
NEARPUNK
ETH
PXT
FERAL
Ner
NVP
FLOKI
WBTC
NEARBIT
SHRM
PULSE
Cheddar
STNEAR
IlliaPolosukhin
AC
GZA
GACHI
BAT
NUT
GWL
NK
WST
mz
1MIL
MRCH
HAPI
BEER
dan
pixiv
AVRIT