MSCF Finance
Homework Set 2

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1 Question 1: ***Consumption and Savings***

1.1 Part (a)

	Today (t=0)	One Year (t=1)
Income	1000	0
Trade	(237.5)	250
Consume	762.5	250

1.2 Part (b)

Allow b to represent the number of bonds invested in. At $t = 0$, we see the consumption $c_0 = 1000 - 95b$, and at $t = 1$, the consumption $c_1 = 100b$. Following the Utility formula presented in the handout, we would like to maximize the following function.

$$U(c) = log(c_0) + 0.98log(c_1)$$

Plugging in c_0 and c_1, we must optimize

$$U(b) = log(1000 - 95b) + 0.98log(100b)$$

We set the derivative of $U(b)$ equal to zero, and solve for b.

$$0 = \frac{dU}{db} = \frac{98}{100b} - \frac{95}{1000 - 95b}$$

We find that $b = 5.21$.

1.3 Part (c)

	Today (t=0)	One Year (t=1)
Income	0	1052.6
Trade	95b	-100b
Consume	95b	1052.6 - 100b

In the same fashion, we would like to find the b that maximizes the utility function but with $c_0 = 95b$ and $c_1 = 1052.6 - 100b$.

$$0 = \frac{dU}{db} = \frac{1}{b} - \frac{98}{1052.6 - 100b}$$

We find that $b = 5.31616$.

1.4 Part (d)

As of 9/15/2017, the current US 1 year T-Bill rate is 1.30%Ṫherefore, assuming we buy 2.5 bonds again, we resolve Part (a) using the current price of the bond as $P_0 = \frac{100}{1.0130} = 98.72\$$. We then have the following,

	Today (t=0)	One Year (t=1)
Income	1000	0
Trade	(246.8)	250
Consume	753.2	250

1.5 Part (e)

Something about a convex function here........ write later...

2 Question 2: *Means and Variances*

2.1 Part (a)

We use the following equations, and the information provided, to determine the expected return, standard devation, Sharpe Ratio, and Beta of the portfolio.

$$\mathbb{E}[r_p] = \sum_i w_i r_i$$

$$Var(r_p) = w_A Var(r_A) + w_B Var(r_B) + 2 w_A w_B Cov(r_A, r_B)$$

$$SharpeRatio = \frac{\mathbb{E}[r_p] - r_f}{\sigma_p}$$

$$\beta_p = \sum_i w_i \beta_i$$

We summarize the findings in the table below.

Metric	Numerical Value
$\mathbb{E}[r_p]$	0.0728
$Var(r_p)$	0.17088
$SharpeRatio$	fdsaf
β_p	fdsaf

2.2 Part (b)

CAPM establishes a linear relationships between the expected return on the asset and the expected excess return of the market as follows

$$\mathbb{E}[r_i] = r_f + \beta_i(\mathbb{E}[r_m] - r_f)$$

Let's check that CAPM holds for the assets A and B.

$$\text{Stock A: } 0.0680 \stackrel{?}{=} 0.02 + 0.80(0.08 - 0.02) = 0.0680$$
$$\text{Stock B: } 0.1040 \stackrel{?}{=} 0.02 + 1.40(0.08 - 0.02) = 0.1040$$

So as we can see, the CAPM paradigm holds for these assets under the information provided.

3 Question 3: *CAPM*

3.1 Part (a)

For each of the industries

$$[Aero, Guns, Steel, Ships, Beer, Toys, Fin, Rtail]$$

We must run the following regression,

$$r_{i,t} - r_f = \alpha_i + \beta_i(r_{m,t} - r_f) + \epsilon_i$$

We use the data analysis feature in Microsoft Excel to run the regressions, and we summarize our findings in the following table. For convenience, allow $y = r_{i,t} - r_f$ and $x = r_{m,t} - r_f$.

Industry	Regressed Equation
Aero	$y = 1.11339631x + 0.002110029$
Guns	$y = 0.761863487x + 0.004183162$
Steel	$y = 1.406022569x - 0.006847625$
Ships	$y = 1.1128971x - 0.001077632$
Beer	$y = 0.70326213x + 0.004168411$
Toys	$y = 1.117458206 - 0.002335538$
Fin	$y = 1.25111787x - 0.000151558$
Rtail	$y = 0.981050605x + 0.001276015$

3.2 Part(b)

If CAPM is correct, the α_i term should be zero for each of the industries, because if there is alpha then, by the equilibrium argument, the demand would increase, the supply would therefore drop, and the price would rise such that it eliminates the excess return or the alpha. In each of the regressed equations in the table above, the alpha values are all on the order of magnitude 10^{-3} or 10^{-4}, approximately zero. Therefore, the CAPM paradigm fits the data well.

3.3 Part (c)

The following table ranks the industry be descending β.

Industry	Regressed Equation
Steel	1.406022569
Fin	1.25111787
Toys	1.117458206
Aero	1.11339631
Ships	1.1128971
Rtail	0.981050605
Guns	0.761863487
Beer	0.70326213

3.4 Question 4: Fama-French Three Factor Model

3.5 Part (a)

For each of the industries we've been working with, and the data provided, we perform the following regression and summarize the results in the following table.

$$r_{i,t} - r_f = \alpha_i + \beta_{i,m}(r_{m,t} - r_f) + \beta_{i,smb}(r_{smb,t} - r_f) + \beta_{i,hml}(r_{hml,t} - r_f) + \epsilon_i$$

Industry	Regressed Equation
Aero	$y = 0.001946387 + 0.507397279\beta_m + 0.142068084\beta_{smb} - 0.276827754\beta_{hml}$
Guns	$y = 0.003255019 + 0.365494066\beta_m + 0.209652324\beta_{smb} - 0.359685719\beta_{hml}$
Steel	$y = 0.006117535 + 0.450168847\beta_m - 0.08808312\beta_{smb} - 0.253781436\beta_{hml}$
Ships	$y = 0.004508555 + 0.390859388\beta_m + 0.095163034\beta_{smb} - 0.353268272\beta_{hml}$
Beer	$y = 0.001532498 + 0.516389875\beta_m + 0.30683985\beta_{smb} - 0.214640562\beta_{hml}$
Toys	$y = 0.004604494 + 0.465072179\beta_m - 0.001811019\beta_{smb} - 0.253540929\beta_{hml}$
Fin	$y = 0.001966042 + 0.621533385\beta_m + 0.002698543\beta_{smb} - 0.208226521\beta_{hml}$
Rtail	$y = 0.001538837 + 0.644125411\beta_m + 0.089195594\beta_{smb} - 0.141109902\beta_{hml}$

3.6 Part (b)

Again, for similar reasons previously discussed, the α values should be zero, which is quite consistent with our multivariate regression results. Nearly all of the α values that we see here are on the order of 10^{-3}, which is approximately zero.