Carnegie Mellon Tepper SCHOOL OF BUSINESS

46-972
MSCF Finance
Mini 1 – 2017

Bryan Routledge (routledge@cmu.edu)

Assignment 2
Due: 9.20.2017 – 5.30pm

Notes –

- If you are having difficulty with the analysis or just have questions that pop up, feel free to contact me (email is easy). In particular, if you are spinning and uncertain as to where to start, contact me. The assignments cover material we have done and foreshadow things we will do. They are designed to help us learn and are not meant as "review" or "testing." Part of the learning includes discussions with your colleagues and me.
- Assignment is due: 9.20.2017 – 5.30pm, but feel free to aim for completing things for Monday's class on 9.18.2017

Question 1: [*Consumption / Savings*]

In our first meeting, we talked about the general consumption / savings / portfolio problem. We then focused in on the risk allocation component that lead us into mean-variance portfolio theory. Here, let's look a the consumption-savings component. The consumption-savings problem is how to use financial markets to make your consumption "smooth" (i.e. the same or similar each period) given an income profile that is variable. Income profiles are typically start small (or zero when you are picking up some human capital), grow as you work, and then fall to zero when you retire.

In this question, we will push into the background the "risk aspect." For now, we will look at saving (and lending) with a risk-free bond. We can also focus on just a two period problem so you can solve this with some calculus or numerically (Excel, R, etc.).

Your income is \$1000 now (t=0) and \$0 one year from now (t=1). You can invest in a risk-free bond. This will allow you to have something to spend now and in one year. The one asset we will consider is a US Treasury Bill that pays \$100 one year from now for \$95.00 (r=5.26%).

(a) To help set the notation. Fill in the table assuming you buy \$250 worth of bonds (i.e. 2.5 bonds with a payoff cash flow of \$100 each). Note when you buy bonds, this is a negative cash flow at date 0 and positive cash flow at date 1.

	Today ($t=0$)	One Year ($t=1$)
Income	1000	0
Trade		
Consume		

(b) To represent a desire for "smooth" consumption we need to assume that the marginal utility from an additional dollar of consumption is decreasing. For example, imagine you are hungry and visit Joe?s hot dog. The first hot dog makes you happy. The second hot dog makes you happier, but the additional happiness is smaller, the third hotdog increases your happiness ? but not by much. The fourth hot dog?

One way to capture this is to assume your utility is a concave function of consumption. For example, utility is $\log(c)$ is concave in c (the log here is base e; natural logarithms).

The second component of preference that matters is your "patience". A hot dog now is worth more than a hot dog later. This is a big component of the "time vale of money" and the risk-free rate. Usually this is a geometric decay with a rate of $\delta = 0.98$. Meaning a hot-dog one year from now nets you 98% of the happiness.

Choose an investment in the bonds, b, that maximizes:

$$\log(c_0) + 0.98\log(c_1)$$

c_0 is consumption today; c_1 is consumption in one year.

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(c) How would your choice of b change if you have \$0 now and \$1052.6 at $t = 1$? Would your consumption be different? [Hint: Think of two bond trades. Can you make an initial trade of bonds that makes this problem look like (a)?]

(d) The 5.2% rate of return on a risk-free US Treasury Bond assumed in this question is not consistent with current rates. What is the current one-year US T-bill rate? Resolve (a) with that rate (bond price).

(e) The answer you get in (c) is specific to the $\log$ utility function. In general, if the rate of return is smaller, why might you want to save more,.... why might you want to save less? [short one or two sentences is all that is needed.]

Question 2: [*Means and Variances*]

A variation of this question was on a past exam. Hence, it gives you a sense of how this might pop up in an exam setting.

Monica was a good salesperson. A great salesperson. A legend. As a result, she is wealthy. To help others, she is settling up a foundation to help others learn to communicate. The Closing Organization runs seminars and outreach programs to help kids, particularly in disadvantageous settings, learn how to communicate and to communicate effectively. The organization is supported by a \$25 million endowment. The endowment earnings (the "draw") are used to fund the activities.

The investments are currently \$10 million in Stock A, \$10 million in Stock B, and \$5 in Treasury bonds. Returns are:

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Asset	Beta	Return Standard Deviation	Expected Return	Current Portfolio (Millions $$)
A	0.800	0.300	0.0680	10
B	1.400	0.800	0.1040	10
Market	1.000	0.200	0.0800	0
T-Bill	0.000	0.000	0.0200	5
				25

The table reads: The expected return of Company A is 6.8% per year with a standard deviation of 30% per year. Company A and B are uncorrelated with each other (a simplifying assumption). The "Market" is the portfolio of all stocks on US exchanges. It has an expected return of 8% (0.08) with a standard deviation of 20% (0.20). The T-Bill, the one-year risk-free rate, is 2%.

(a) For the current portfolio, calculate the following for the current portfolio: (1) Expected return, (2) Standard deviation of the portfolio return, (3) Sharpe ratio, (4) Beta.

(b) Are the returns of A and B consistent with the CAPM? Yes? No? Maybe? Calculate/Explain

(c) Find the optimal (best) portfolio for The Closing Organization that has the same expected return as the current portfolio. (Use any of the assets listed in the table)

For this portfolio calculate: (1) Expected return, (2) Standard deviation of the portfolio return, (3) Sharpe ratio, (4) Beta.

In your role as a portfolio adviser, you often review investment opportunities that are not available to the general public. These transactions, called "private equity" or "alternative assets," can sometimes be good deals and sometimes bad. The current deal being evaluated is as follows:

Tree Farms Inc. is a private lumber company. They are issuing a security backed by the commodity price of lumber. The security is called a "Tree

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Farm Backed Security." They are available for purchase (you cannot "short" or sell them). The cash flows for this security all happen at year one and depend on whether the price of lumber in year one is "low," "medium" or "high" (all terms that are carefully defined in the contract). Probability and cash flows (for one "unit") as well as the mean and standard deviation of the cash flows are as follows:

Tree Farm Backed Security

State	Low	Med	High	Mean	St.Dev
Probabilty	5%	75%	20%		
Cash Flow ($)	**100**	**150**	**200**	157.5	23.848

Recall from above: The Market is the portfolio of all stocks on US exchanges. It has an expected return of 8% per year (0.08) with a standard deviation of 20% (0.20). The T-Bill, the one-year risk-free rate, is 2% per year. (And you can treat these returns as compounded once per year. I think that is bit easier to work with here.)

(d) If the price of Lumber is uncorrelated with the economy (and the market), what is the maximum price for a Tree Farm Backed Security such that you would recommend clients purchase (at least a small amount)?

(e) If the price if the of a Tree Farm Backed Security was $150. What is the Sharpe Ratio on this security?

(f) But, of course the price of lumber is correlated with the economy (and the market). Without knowing exactly what this correlation is, what is the maximum price of a Tree Farm Backed Security such that you would definitely view this investment as a good deal for all your clients? [This is a harder question. Assume the CAPM holds. Show your logic, explanations, calculations.]

Question 3: [*CAPM*]

Using the same data from Assignment 1, let's look at the CAPM.

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(a) For each of the eight industries,

$$\{Aero, Guns, Steel, Ships, Beer, Toys, Fin, Rtail\}$$

Run the following regression

$$r_{i,t} - r_f = \alpha_i + \beta_i(r_{m,t} - r_f) + \epsilon_i$$

To do this, use the two columns for excess returns, one for industry i and one for the market.

(You can put all the results in one table if that is easier. Or each regression individually.)

(b) If the CAPM is correct, what value for α_i should you see? Look at the α_i's you have found and comment on how well the CAPM fits the data.

(c) Looking at the CAPM beta's order the industries from least to most risky? Is this ordering consistent with the ordering you would get if you ordered based on return variance? (Recall you calculate the return variances in Assignment 1)

Question 4: [*Fama-French Three Factor Model*]

Using the same data from Assignment 1 [See the link on CANVAS for the spreadsheet that adds the two extra bits of detail], let's look at the Fama-French Three Factor Model. The model is the same idea as the CAPM. There are just three betas instead of the one.

(a) For each of the eight industries,

$$\{Aero, Guns, Steel, Ships, Beer, Toys, Fin, Rtail\}$$

Run the following regression

$$r_{i,t} - r_f = \alpha_i + \beta_{i,m}(r_{m,t} - r_f) + \beta_{i,smb}(r_{smb,t} - r_f) + \beta_{i,hml}(r_{hml,t} - r_f) + \epsilon_i$$

The excess returns for the portfolios SMB and HML are in the data. Like the market portfolio they are portfolios of assets. The reason people use these

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is they tend to work well (folks had looked for a long time before settling on these ones). The SMB portfolio is weighted towards small stocks i.e., stocks with a small market capitalization. The HML portfolio is similar. It is weighted towards stocks that happen to have a high "book value" relative to market capitalization. (We will come back and re-visit these portfolios and these terms soon).

(You can put all the results in one table if that is easier. Or each regression individually.)

(b) If the Fama-French (FF) Factor Model is correct, what value for α_i should you see? Look at the α_i's you have found and comment on how well the CAPM fits the data.

(c) Here it is harder to rank the portfolios as "riskier" since the measure of risk is described by three betas. Is there any of these industries that strike you as "most" or "least" risky? [Hint: The Factor Model tells you how to aggregate these betas!]

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