# MSCF Finance
# Homework Set 2

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## 1 Question 1: *Consumption and Savings*

### 1.1 Part (a)

| | Today (t=0) | One Year (t=1) |
|---|---|---|
| Income | 1000 | 0 |
| Trade | (237.5) | 250 |
| Consume | 762.5 | 250 |

### 1.2 Part (b)

Allow $b$ to represent the number of bonds invested in. At $t = 0$, we see the consumption $c_0 = 1000 - 95b$, and at $t = 1$, the consumption $c_1 = 100b$. Following the Utility formula presented in the handout, we would like to maximize the following function.

$$U(c) = log(c_0) + 0.98log(c_1)$$

Plugging in $c_0$ and $c_1$, we must optimize

$$U(b) = log(1000 - 95b) + 0.98log(100b)$$

We set the derivative of $U(b)$ equal to zero, and solve for $b$.

$$0 = \frac{dU}{db} = \frac{98}{100b} - \frac{95}{1000 - 95b}$$

We find that $b = 5.21$.

## 1.3 Part (c)

| | Today (t=0) | One Year (t=1) |
|---|---|---|
| Income | 0 | 1052.6 |
| Trade | $95b$ | -100$b$ |
| Consume | $95b$ | 1052.6 - 100$b$ |

In the same fashion, we would like to find the $b$ that maximizes the utility function but with $c_0 = 95b$ and $c_1 = 1052.6 - 100b$.

$$0 = \frac{dU}{db} = \frac{1}{b} - \frac{98}{1052.6 - 100b}$$

We find that $b = 5.31616$.

## 1.4 Part (d)

As of 9/15/2017, the current US 1 year T-Bill rate is 1.30%$\dot{}$Therefore, assuming we buy 2.5 bonds again, we resolve Part (a) using the current price of the bond as $P_0 = \frac{100}{1.0130} = 98.72\$$. We then have the following,

| | Today (t=0) | One Year (t=1) |
|---|---|---|
| Income | 1000 | 0 |
| Trade | (246.8) | 250 |
| Consume | 753.2 | 250 |

## 1.5 Part (e)

Assume an individual would like to save some fixed amount of money $x$. If the rate were lower, the individual would need to save more at the current time in order to generate $x$ in the future.. On the contrary, if the rate were higher, the individual wouldn't need to save as much in order to generate $x$ in the future.

# 2 Question 2: *Means and Variances*

## 2.1 Part (a)

We use the following equations, and the information provided, to determine the expected return, standard devation, Sharpe Ratio, and Beta of the portfolio.

$$\mathbb{E}[r_p] = \sum_i w_i r_i$$

$$Var(r_p) = w_A Var(r_A) + w_B Var(r_B) + 2w_A w_B Cov(r_A, r_B)$$

$$SharpeRatio = \frac{\mathbb{E}[r_p] - r_f}{\sigma_p}$$

$$\beta_p = \sum_i w_i \beta_i$$

We summarize the findings in the table below.

| Metric | Numerical Value |
|---|---|
| $\mathbb{E}[r_p]$ | 0.0728 |
| $Var(r_p)$ | 0.34176 |
| $SharpeRatio$ | 0.1534494 |
| $\beta_p$ | 0.88 |

## 2.2 Part (b)

CAPM establishes a linear relationships between the expected return on the asset and the expected excess return of the market as follows

$$\mathbb{E}[r_i] = r_f + \beta_i(\mathbb{E}[r_m] - r_f)$$

Let's check that CAPM holds for the assets A and B.

$$\text{Stock A: } 0.0680 \stackrel{?}{=} 0.02 + 0.80(0.08 - 0.02) = 0.0680$$
$$\text{Stock B: } 0.1040 \stackrel{?}{=} 0.02 + 1.40(0.08 - 0.02) = 0.1040$$

So as we can see, the CAPM paradigm holds for these assets under the information provided.

## 2.3 Part (c)

Following a similar procedure from Homework 1, we determined the weights for the optimal market portolio. We then establish the Capital Market Line. Using the CML, we then solve for the weights by calibrating to the expected return of the current portfolio. The optimal weights and the metrics of interest (Expected Return, Standard Deviation, Sharpe Ratio, and Beta) are summarized in the tables below.

| | |
|---|---|
| $w_A$ | 0.768 |
| $w_B$ | 0.189 |
| $w_{r_f}$ | 0.041 |

| Metric | Numerical Value |
|---|---|
| $\mathbb{E}[r_p]$ | 0.0728 |
| $Var(r_p)$ | 0.2759 |
| $SharpeRatio$ | 0.1914 |
| $\beta_p$ | 0.88 |

## 2.4 Part (d)

We would suggest the client to buy at least some of the asset if it is uncorrelated with the market, since the diversification will probably increase the risk adjusted returns. The maximum price should be a price that have the returns higher than the risk free rate. Since it will be above the risk free rate and its returns are uncorrelated with the other assets adding some of this asset will improve the efficient frontier. Specifically,

$$\frac{157.5}{P_0} = (1 + 0.02) \Rightarrow P_0 = 154.41$$

## 2.5 Part (e)

Using the definition of the Sharpe Ratio, this is straightforward with $P_0 = 150$

$$SharpeRatio = \frac{\mathbb{E}[r_p] - r_f}{\sigma_p} = \frac{(\frac{157.5}{150}) - 1}{23.848} = 0.001257967$$

## 2.6 Part (f)

Since the Lumber prices are correlated this means that we can generate the returns combining market assets. Therefore, we would at least need the return defined by the Efficient Frontier of the portfolio of assets A and B. We can use the solver to find the weights of assets that give the $\sigma = 23.848$ with the weights we can find the return needed. We found the expected return on the Efficient Frontier corresponding to when $\sigma = 23.848$ to be $\mu = 1.0772$. With this return in mind, much like in Part (d), we determine the initial price that will generate this return. We find that $P_0 = \$75.823$.

# 3 Question 3: *CAPM*

## 3.1 Part (a)

For each of the industries

$$[Aero, Guns, Steel, Ships, Beer, Toys, Fin, Rtail]$$

We must run the following regression,

$$r_{i,t} - r_f = \alpha_i + \beta_i(r_{m,t} - r_f) + \epsilon_i$$

We use the data analysis feature in Microsoft Excel to run the regressions, and we summarize our findings in the following table.

| Industry | $\beta_{i,m}$ | $\alpha_i$ |
|---|---|---|
| *Aero* | 1.11339631 | 0.002110029 |
| *Guns* | 0.761863487 | 0.004183162 |
| *Steel* | 1.406022569 | −0.006847625 |
| *Ships* | 1.1128971 | −0.001077632 |
| *Beer* | 0.70326213 | 0.004168411 |
| *Toys* | 1.117458206 | −0.002335538 |
| *Fin* | 1.25111787 | −0.000151558 |
| *Rtail* | 0.981050605 | 0.001276015 |

## 3.2 Part(b)

If CAPM is correct, the $\alpha_i$ term should be zero for each of the industries, because if there is alpha then, by the equilibrium argument, the demand would increase, the supply would therefore drop, and the price would rise such that it eliminates the excess return or the alpha. In each of the regressed equations in the table above, the alpha values are all on the order of magnitude $10^{-3}$ or $10^{-4}$, approximately zero. Therefore, the CAPM paradigm fits the data well.

### 3.3 Part (c)

The following table ranks the industry be descending $\beta_{i,m}$.

| Industry | $\beta_{i,m}$ |
|---|---|
| *Steel* | 1.406022569 |
| *Fin* | 1.25111787 |
| *Toys* | 1.117458206 |
| *Aero* | 1.11339631 |
| *Ships* | 1.1128971 |
| *Rtail* | 0.981050605 |
| *Guns* | 0.761863487 |
| *Beer* | 0.70326213 |

## 4 Question 4: Fama-French Three Factor Model

### 4.1 Part (a)

For each of the industries we've been working with, and the data provided, we perform the following regression and summarize the results in the following table.

$$r_{i,t} - r_f = \alpha_i + \beta_{i,m}(r_{m,t} - r_f) + \beta_{i,smb}(r_{smb,t} - r_f) + \beta_{i,hml}(r_{hml,t} - r_f) + \epsilon_i$$

| | $\alpha$ | $\beta_{Market}$ | $\beta_{SMB}$ | $\beta_{HML}$ |
|---|---|---|---|---|
| Aero | 0.001 | 1.164 | -0.018 | 0.307 |
| Guns | 0.003 | 0.818 | 0.050 | 0.417 |
| Steel | -0.008 | 1.372 | 0.472 | 0.313 |
| Ships | -0.003 | 1.151 | 0.214 | 0.488 |
| Beer | 0.004 | 0.750 | -0.209 | 0.065 |
| Toys | -0.004 | 1.092 | 0.348 | 0.233 |
| Finance | -0.001 | 1.269 | 0.106 | 0.234 |
| Retail | 0.001 | 0.980 | 0.047 | 0.050 |

## 4.2 Part (b)

Again, for similar reasons previously discussed, the $\alpha$ values should be zero, which is quite consistent with our multivariate regression results. Nearly all of the $\alpha$ values that we see here are on the order of $10^{-3}$, which is approximately zero.

## 4.3 Part (c)

Although there are multiple $\beta$ factors here, to establish an ordering of riskiness, we summed the coefficients of all the factors for each of the industry and then reported them in descending order. From this, we determine that *Beer* is the most risky, but *Rtail* is not far behind. The following table summarizes these findings.

| Industry | $\sum_k \beta_{i,k}$ |
|---|---|
| *Beer* | 0.6085 |
| *Rtail* | 0.5922 |
| *Fin* | 0.4160 |
| *Aero* | 0.3726 |
| *Guns* | 0.2154 |
| *Toys* | 0.2097 |
| *Ships* | 0.1327 |
| *Steel* | 0.1083 |