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CSE331

HW 1

Problem 1

- Submitted to handin

Problem 2

- Submitted to handin

Problem 3

Use proof by induction to prove

$$\frac{d}{dx}(x^n) = nx^{(n-1)}$$

Proof:

Prove the base case

Base: n = 1

$$\frac{d}{dx}(x^1) = (1)x^0 = 1, \qquad \textit{Base Case Holds}$$

Assume for n = k, prove for n = k+1 (I used product rule)

Want to show:

$$\frac{d}{dx}(x^{k+1}) = (k+1)x^{k+1}$$

$$\frac{d}{dx}(x^{k+1}) = \frac{d}{dx}(x^k * x)$$

$$\frac{d}{dx}(x^k * x) = x * \frac{d}{dx}(x^k) + x^k \frac{d}{dx}(x) \ (\textit{product rule})$$

$$(x * kx^{k-1}) + (x^k * 1) = (k+1)x^k$$

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Problem 4

Use proof by induction to prove

$$\ln(n!) \leq n * ln(n) \;\forall\, n \in \mathbb{N}, n \geq 1$$

Proof:

Before beginning the induction proof I show the equivalency between eq. 1 and eq. 2:

$$\ln(n!) \leq n * ln(n) \qquad (1)$$
$$\ln(n!) \leq \ln(n^n)$$
$$n! \leq n^n \qquad (2)$$

So by proving eq. 2, I implicitly prove eq. 1 as well.

Base: $n = 1$,

$$1! = 1 \leq 1 = 1^1$$

So this holds for the base case.

Assume for n = k, prove for n = k+1

$$(k+1)! \leq (k+1)^{(k+1)}$$

$$\therefore (k+1)\, k! \leq (k+1)^{(k+1)}$$

$$\therefore k! \leq (k+1)^k$$

$$\therefore (By\ induction\ hypothesis):$$
$$k! \leq k^k < (k+1)^k$$

$$Q.E.D. \blacksquare$$