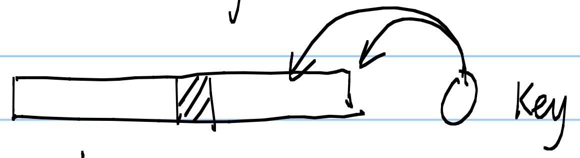
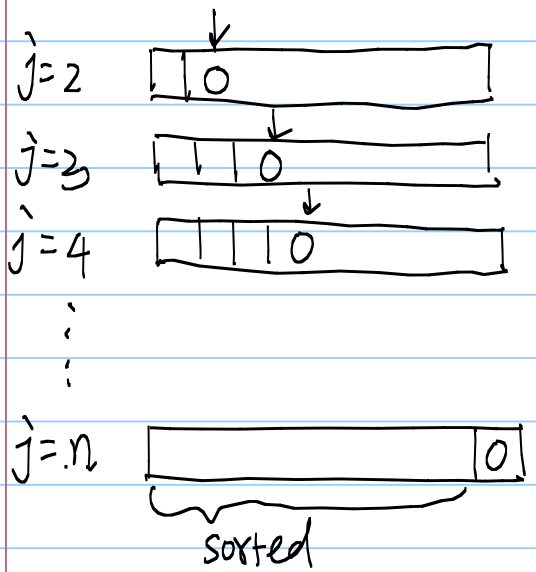
Note Title 1/13/2016

Insertion sort, running time analysis

either 3rd or 4th edition is fine

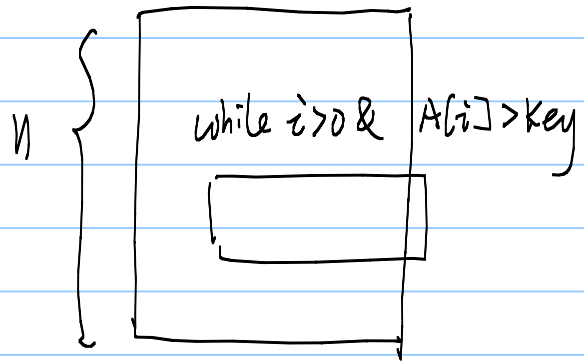
Basic operation: insert an element into a sorted list s.t. the final list is sorted.





inner-loop: comparison

for $j=2$ to n



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INSERTION_SORT (A) // length[A] = n

	cost	times
for $j = 2$ to length[A]	c_1	n
do key ← A[j]	c_2	$n-1$
//insert A[j] to sorted list A[1..j-1]	0	
$i \leftarrow j-1$	c_4	$n-1$
while $i > 0$ and $A[i] > key$	c_5	$\sum_{j=2 \text{ to } n} t_j$
do $A[i+1] \leftarrow A[i]$	c_6	$\sum_{j=2 \text{ to } n} (t_j - 1)$
$i \leftarrow i-1$	c_7	$\sum_{j=2 \text{ to } n} (t_j - 1)$
$A[i+1] \leftarrow key$	c_8	$n-1$

* The loop head is executed one more time than the loop body

Total running time:

$$T(n) = c_1 \cdot n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} t_j + c_6 \sum_{j=2}^{n} (t_j - 1) + c_7 \sum_{j=2}^{n} (t_j - 1) + c_8(n-1)$$

input size

- minimum value of $T(n)$

best case: $t_j = 1$

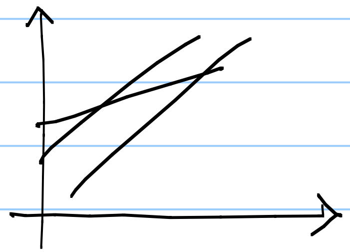
$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5 \sum_{j=2}^{n} 1 + c_8(n-1)$$

$$= \underline{c_1 n} + \underline{c_2 n} - c_2 + \underline{c_4 n} - c_4 + \underline{c_5(n-1)} + \underline{c_8 n} - c_8$$

$$= (c_1 + c_2 + c_4 + c_5 + c_8)n - (c_2 + c_4 + c_5 + c_8)$$

$$= an + b$$

best case input: 1 3 6 100



$$a = c_1 + c_2 + c_4 + c_5 + c_8$$

$$b = -(c_2 + c_4 + c_5 + c_8)$$

— maximum value of $T(n)$

worst case: $t_j = j$

$$\sum_{j=2}^{n} t_j = \sum_{j=2}^{n} j = 2+3+4+5+\cdots+n = \frac{n(n+1)}{2} - 1$$

worst case input: 5 4 3 2 1

$$+\begin{cases} S = 1+2+3+4+\cdots+n \\ S = n+(n-1)+(n-2)+\cdots+1 \end{cases}$$

$$2S = (n+1)+(n+1)+(n+1)+\cdots+(n+1) = n\cdot(n+1) \Rightarrow S = \frac{n(n+1)}{2}$$

$$T(n) = c_1 n + c_2(n-1) + c_4(n-1) + c_5\left(\frac{n(n+1)}{2} - 1\right) + c_6\left(\frac{n(n-1)}{2}\right) + c_7\left(\frac{n(n-1)}{2}\right) + c_8(n-1)$$

$$= \left(\frac{c_5}{2} + \frac{c_6}{2} + \frac{c_7}{2}\right)n^2 + \left(c_1 + c_2 + c_4 + \frac{c_5}{2} - \frac{c_6}{2} - \frac{c_7}{2} + c_8\right)n - (c_2 + c_4 + c_5 + c_8)$$

$$= a'n^2 + b'n + c'$$

Average case analysis: $t_j = \frac{j}{2}$

$$c_5 \sum_{j=2}^{n} t_j = c_5 \cdot \sum_{j=2}^{n} \frac{j}{2} = \frac{c_5}{2} \sum_{j=2}^{n} j = \frac{c_5}{2}\left(\frac{n(n+1)}{2} - 1\right)$$

$$T(n) = a_1 n^2 + b_1 n + c_1$$

- Running time analysis using random-access machine (RAM) model

  RAM: - a generic one-processor

  instructions are executed one after another; no concurrent operations

  - Each "simple" operation ($+$, $*$, $-$, $==$, if, else, $=(\leftarrow)$) takes exactly 1 step

  - Loops and subroutines are not simple operations, but depend on the size of input data & the contents of the subroutine

"Sort" "matrix multiplication" "length of an array"

— Each memory access takes 1 step

RAM: $C_i = 1$

Exercises:

①

1.	for $i=1$ to n	$n+1$, n	
2.	$T \leftarrow i$	n	$3n$
3.	print T	n	

②

1.	for $i=1$ to n	n	
2.	for $j=1$ to n	$n \cdot n$	$2n^2+n$
3.	print $i+j$	n^2	

③

1.	for $i=1$ to n	n
2.	$T \leftarrow i$	n
3.	for $j=1$ to i	$\sum_{i=1}^{n} i = 1+2+3+\cdots+n$
4.	$T \leftarrow T+j$	$\sum_{i=1}^{n} i$
5.	print T	n

$$T(n) = n + n + \frac{n(n+1)}{2} + \frac{n(n+1)}{2} + n$$

$$= \underline{a}n^2 + \underline{b}n + \underline{c}$$