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Distributed rotating formation control of second-order leader-following multi-agent systems with nonuniform delays☆

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Abstract

In this paper, the leader-following rotating formation control problem is investigated for second-order multi-agent systems with nonuniform time-delays. We propose a distributed algorithm to drive all agents to achieve a desired formation and orbit around a common point. By a frequency domain analysis method, the upper bound of the maximum time-delay is obtained. Finally, a numerical simulation is given to illustrate the obtained results.

1. Introduction

Recently, rotating formation control problems for multi-agent systems have received a considerable attention in various fields, such as unnamed aerial vehicles, satellite formation flight, the exploration of multiple underwater robots and so on [1–30]. For the ocean sampling,

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a collective circle motion that all particles do circular motion with fixed relative spacing was studied in [6]. Articles [7] and [8] investigated circle formation motion of multi-vehicles in two-dimensional and three-dimensional space. However, the radius of the circular motion and the formation structure can not be set arbitrarily in [6–8]. To solve this problem, article [9] introduced a virtual leader and proposed control algorithms to make all agents orbit around a common center with a desired formation structure. Founded on [9], article [10] addressed the rotating formation motion problem in three-dimensional space, and article [11] studied the finite-time rotating encirclement motion.

However, articles [6–11] do not take the communication delays into consideration. In fact, the time-delays are quite prevalent in information communication because of the restriction of communication environment. It is well known that the communication delays degrade the performance of the controller and even lead to the instability of the closed-loop system. Furthermore, in practice, due to the different communication disturbance among agents, the communication delays between two different agents are usually different, such time-delays are called the nonuniform time-delays. Therefore, it is meaningful to consider the effects of nonuniform time-delays on rotating formation control. The main approaches to deal with the delays include frequency domain approach, Lyapunov function and nonnegative matrix, which were widely applied to consensus problem [12–17] and formation control problem [20]–[21]. For example, with the help of the frequency domain approach, articles [12] and [13] investigated the consensus problem of the single-integrator and second-order multi-agent system with time-delays, respectively, where only fixed topology is considered. For the cases of switching topology, articles [14] and [15] obtained the convergence conditions for multi-agent systems with arbitrary bounded communication delays by utilizing the property of the nonnegative matrix. Besides, the Lyapunov-based approach was employed in [16] to study the average consensus problem of multi-agent system with balanced digraphs and time-delays, and the results were extended to the cases with jointly-connected communication topologies and time-delays in [17]. In addition, a Lyapunov-Krasovskii functional was used in [20] and [21] to obtain some sufficient conditions, under which the formation control problem of multi-vehicles systems with communication delays can be solved. In the aforementioned works, however, the consensus problem aims to drive the positions of all agents converge to a common point [12], [14–17], while the flocking-like consensus problems or formation control problems aims to make all agents finally move together in a line [13], [18–21]. Such control objectives are relatively simple.

Motivated by the above discussions, we investigate a leader-following rotating formation control problem of second-order multi-agent systems with nonuniform communication time-delays. In contrast to the existing works, e.g., [12–17], where the desired relative positions between agents are time-invariant, the desired relative positions between agents in this paper are time varying due to the existence of the rotating motion, and thus the rotating formation control problem cannot be transformed into the corresponding consensus problem by introducing a desired relative separation vectors as in [12–17]. Moreover, the system in this paper is coupled with nonuniform delays, which render the analysis much more complicated. To overcome these difficulties, first of all, we introduce a distributed leader-following rotating formation control algorithm with nonuniform time-delays. Then we give the upper bound of maximum time-delay by utilizing the frequency domain approach. The algorithm designed in this paper guarantees that the desired rotating formation can be achieve even with nonuniform and upper bounded time-delays. Also, the analysis approach here is universal, which can be extended to deal with the rotating consensus problem of multi-agent systems with nonuniform time-delays (see Definition 1 of [9]).

The following notations will be used throughout this paper. We denote by $\mathbb{C}$ and $\mathbb{C}^n$ the set of one dimensional column vector and n dimensional column vector, respectively. Let j be the imaginary unit and I_n denotes the n dimensional unit matrix. We use $\mathbf{1_n}$ denote $\mathbf{1}$ with dimension n. For a vector x, x^T represents its transpose. The Kronecker product is denoted as $\otimes$. $\mathrm{diag}\{x_1, \ldots, x_n\}$ is used to represents a block diagonal matrix with its ith diagonal element being the matrices x_i for $i = 1, \ldots, n$. For matrix F, $[F]_{ij}$ denotes its ijth entry. $£[\psi(t)]$ denotes the Fourier transform of the function $\psi(t)$.

2. Preliminaries and problem formulation

2.1. Graph theory

Let $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ be an undirected graph of n nodes, where $\mathcal{V} = \{\mathrm{v}_1, \ldots, \mathrm{v}_n\}$ is the set of nodes, $\mathcal{E}$ is the set of edges, and $\mathcal{A} = [a_{ik}]$ is a weighted adjacency matrix. The indexes belong to a finite index set $\mathcal{I} = \{1, \ldots, n\}$. An edge of $\mathcal{G}$ is denoted by $e_{ik} = (\mathrm{v}_i, \mathrm{v}_k)$. If $e_{ik} \in \mathcal{E}$, then agent v_i can obtain the information from agent v_k, and in this case v_k is called the neighbor of agent v_i. In an undirected graph, if $e_{ik} \in \mathcal{E}$, then $e_{ki} \in \mathcal{E}$. For any $i, k \in \mathcal{I}$, $a_{ii} = 0$ and $a_{ik} = a_{ki} \geq 0$, $a_{ik} > 0$ if and only if $e_{ik} \in \mathcal{E}$. The set of agent v_i's neighbors is denoted by $\mathcal{N}_i = \{\mathrm{v}_k \in \mathcal{V} : (\mathrm{v}_i, \mathrm{v}_k) \in \mathcal{E}\}$. The Laplacian of graph $\mathcal{G}$ is defined as $L = [l_{ik}]_{n*n}$, where $l_{ii} = \sum_{k=1}^{n} a_{ik}$ and $l_{ik} = -a_{ik}$, $i \neq k$. A path is a sequence of ordered edges of the form $(\mathrm{v}_{i_1}, \mathrm{v}_{i_2}), (\mathrm{v}_{i_2}, \mathrm{v}_{i_3}), \ldots,$ where $(\mathrm{v}_{i_{k-1}}, \mathrm{v}_{i_k}) \in \mathcal{E}$. The graph is connected if there exits at least one path between any two agents. Noted that node $'0'$ denotes the leader. Let $B = \mathrm{diag}\{\mathrm{b}_{10}, \ldots, \mathrm{b}_{n0}\}$, where $b_{i0} > 0$ implies that agent i can receive the leader's information and $b_{i0} = 0$ otherwise. Please refer to [31] for more details.

Lemma 1 ([31]). *If the graph $\mathcal{G}$ is connected, then the corresponding Laplacian L has a simple zero eigenvalue associated with eigenvector $\mathbf{1}_n$ and all its other $n-1$ eigenvalues are positive and real.*

2.2. Problem formulation

Consider a multi-agent system with n agents. Let $r_i \in \mathbb{C}$ and $v_i \in \mathbb{C}$ be the position and velocity states of agent $i \in \mathcal{I}$, respectively. Suppose that each agent has the dynamics as follows:

$$\begin{aligned} \dot{r}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \end{aligned} \tag{1}$$

where $u_i(t) \in \mathbb{C}$ is the control input.

Noted that the leader's dynamic is given by

$$\begin{aligned} \dot{r}_0(t) &= v_0(t), \\ \dot{v}_0(t) &= \mathrm{j}\omega v_0(t), \end{aligned}$$

where, $\omega > 0$ denotes the angular velocity.

Definition 1 ([9]). Let $h=[h_1,h_2,\ldots,h_n]^T=[\rho_1 e^{\mathrm{j}\theta_1},\rho_2 e^{\mathrm{j}\theta_2},\ldots,\rho_n e^{\mathrm{j}\theta_n}]^T\in\mathbb{C}^n$, where $\theta_i\in[0,\ 2\pi)$ and $\rho_i>0$ denote the rotation angle and rotation radius of agent i respectively for $i\in\mathcal{I}$. The multi-agent system (1) is said to achieve the rotating formation if

$$\lim_{t\to+\infty}\left(\frac{v_i(t)}{\rho_i e^{\mathrm{j}\theta_i}}-\frac{v_k(t)}{\rho_k e^{\mathrm{j}\theta_k}}\right)=0, \tag{2}$$

$$\lim_{t\to+\infty}\left[(r_i(t)+\omega^{-1}\mathrm{j}v_i(t))-(r_k(t)+\omega^{-1}\mathrm{j}v_k(t))\right]=0, \tag{3}$$

$$\lim_{t\to+\infty}\left[\dot{v}_i(t)-\mathrm{j}\omega v_i(t)\right]=0, \tag{4}$$

for any $i,k\in\mathcal{I}$.

Remark 1. If $\theta_1=\theta_2=\cdots=\theta_n$ and $\rho_1=\rho_2=\cdots=\rho_n$, the rotating formation problem degenerates into the rotating consensus problem.

The objective of the paper is to develop the distributed control input for each agent, such that the rotating formation as given in Definition 1 can be achieved. It is easy to see that the circle center is $r_i(t)+\omega^{-1}\mathrm{j}v_i(t)$ as shown in Fig. 3 of [9]. In Definition 1, condition (3) guarantees all agents surround a common circle center. Accordingly, the vector from $r_i(t)+\omega^{-1}\mathrm{j}v_i(t)$ to $r_i(t)$, corresponds to the vector $h_i=\rho_i e^{j\theta_i}$. Conditions (2) and (4) mean all agents finally form the same shape formation as the desired formation h and rotate with angular velocity ω.

3. Main results

To solve the rotating formation control problem of the multi-agent system (1), we propose the following leader-following algorithm

$$\begin{aligned}
u_i(t)&=u_{i1}(t)+u_{i2}(t),\\
u_{i1}(t)&=\mathrm{j}\omega v_i(t),\\
u_{i2}(t)&=-\sum_{k\in\mathcal{N}_i}a_{ik}\rho_i^{-2}\left[v_i(t-\tau_{ik})-\frac{\rho_i}{\rho_k}e^{\mathrm{j}(\theta_i-\theta_k)}v_k(t-\tau_{ik})\right]\\
&\quad-\sum_{k\in\mathcal{N}_i}a_{ik}[c_i(t-\tau_{ik})-c_k(t-\tau_{ik})]\\
&\quad-b_{i0}\rho_i^{-2}[v_i(t-\tau_{i0})-\rho_i e^{\mathrm{j}\theta_i}v_0(t-\tau_{i0})]-b_{i0}[c_i(t-\tau_{i0})-c_0(t-\tau_{i0})],
\end{aligned}\tag{5}$$

for any $i,k\in\mathcal{I}$, where $c_i(t-\tau_{ik})=r_i(t-\tau_{ik})+\mathrm{j}\omega^{-1}v_i(t-\tau_{ik})$, $c_i(t-\tau_{i0})=r_i(t-\tau_{i0})+\mathrm{j}\omega^{-1}v_i(t-\tau_{i0})$ and $c_0(t-\tau_{i0})=r_0(t-\tau_{i0})+\mathrm{j}\omega^{-1}v_0(t-\tau_{i0})$.

Let $\gamma_i(t)=r_i(t)-c_0(t)-\rho_i e^{\mathrm{j}\theta_i}(r_0(t)-c_0(t))$ and $\zeta_i(t)=c_i(t)-c_0(t)$, $i\in\mathcal{I}$. It is easy to see that $\dot{c}_0(t)=\dot{r}_0+\mathrm{j}\omega^{-1}\dot{v}_0=v_0(t)+\mathrm{j}\omega^{-1}(\mathrm{j}\omega v_0(t))=0$ and $r_i(t)-c_i(t)=-\mathrm{j}\omega^{-1}v_i(t)$. Calculating $\dot{\gamma}_i(t)$ and $\dot{\zeta}_i(t)$, we have

$$\begin{aligned}
\dot{\gamma}_i(t)&=v_i(t)-\rho_i e^{\mathrm{j}\theta_i}v_0(t)\\
&=\mathrm{j}\omega(r_i(t)-c_0(t)-(c_i(t)-c_0(t)))
\end{aligned}$$

$$
\begin{aligned}
&-\mathrm{j}\omega\rho_i e^{\mathrm{j}\theta_i}(r_0(t)-c_0(t)) \\
&=\mathrm{j}\omega\gamma_i(t)-\mathrm{j}\omega\zeta_i(t),
\end{aligned} \tag{6}
$$

and

$$
\begin{aligned}
\dot{\zeta}_i(t) &= \dot{r}_i(t)+\mathrm{j}\omega^{-1}\dot{v}_i(t) \\
&= \mathrm{j}\omega^{-1}u_{i2}(t).
\end{aligned} \tag{7}
$$

From (6), it follows that

$$
\begin{aligned}
&\sum_{k\in\mathcal{N}_i} a_{ik}\rho_i^{-2}\Big(v_i(t-\tau_{ik})-\frac{\rho_i}{\rho_k}e^{\mathrm{j}(\theta_i-\theta_k)}v_k(t-\tau_{ik})\Big) \\
&\quad=\sum_{k\in\mathcal{N}_i}\mathrm{j}\omega a_{ik}\frac{1}{\rho_i\rho_i}(\gamma_i(t-\tau_{ik})-\zeta_i(t-\tau_{ik})) \\
&\quad+\sum_{k\in\mathcal{N}_i}\mathrm{j}\omega(-a_{ik})e^{\mathrm{j}(\theta_i-\theta_k)}\frac{1}{\rho_i\rho_k}(\gamma_k(t-\tau_{ik})-\zeta_k(t-\tau_{ik})),
\end{aligned} \tag{8}
$$

and

$$
\begin{aligned}
&\sum_{k\in\mathcal{N}_i} a_{ik}(c_i(t-\tau_{ik})-c_k(t-\tau_{ik})) \\
&\ =\sum_{k\in\mathcal{N}_i} a_{ik}(c_i(t-\tau_{ik})-c_0(t-\tau_{ik})) \\
&\quad-\sum_{k\in\mathcal{N}_i} a_{ik}(c_k(t-\tau_{ik})-c_0(t-\tau_{ik})) \\
&\ =\sum_{k\in\mathcal{N}_i} a_{ik}(\zeta_i(t-\tau_{ik})-\zeta_k(t-\tau_{ik})).
\end{aligned} \tag{9}
$$

Similarly, it can be obtained that $b_{i0}\rho_i^{-2}(v_i(t-\tau_{i0})-\rho_i e^{\mathrm{j}\theta_i}v_0(t-\tau_{i0}))=\mathrm{j}\omega\frac{1}{\rho_i}b_{i0}\frac{1}{\rho_i}(\gamma_i(t-\tau_{ik})-\zeta_i(t-\tau_{ik}))$ and $b_{i0}(c_i(t-\tau_{i0})-c_0(t-\tau_{i0}))=b_{i0}\zeta_i(t-\tau_{i0})$. Using (6)–(9), it follows that

$$
\begin{aligned}
\dot{\zeta}_i(t) &= \mathrm{j}\omega^{-1}u_{i2}(t) \\
&= \sum_{k\in\mathcal{N}_i}\frac{1}{\rho_i}(a_{ik}+b_{i0})\frac{1}{\rho_i}(\gamma_i(t-\tau_{ik})-\zeta_i(t-\tau_{ik})) \\
&\quad+\sum_{k\in\mathcal{N}_i}\frac{1}{\rho_i}e^{\mathrm{j}\theta_k}(-a_{ik})e^{-\mathrm{j}\theta_k}\frac{1}{\rho_k}(\gamma_k(t-\tau_{ik})-\zeta_i(t-\tau_{ik})) \\
&\quad-\sum_{k\in\mathcal{N}_i}\mathrm{j}\omega^{-1}(a_{ik}(\zeta_i(t-\tau_{ik})-\zeta_k(t-\tau_{ik}))+b_{i0}\zeta_i(t-\tau_{ik})).
\end{aligned} \tag{10}
$$

Suppose that there are M different time-delays, which are denoted by $\tau_m\in\{\tau_{ik}, i,k\in\mathcal{I}\cup\{0\}\}$, with $\tau_{ik}\geq 0, m=1,2,\ldots,M$. Let $B_m=\mathrm{diag}\{\mathrm{b}_{10}^{\mathrm{m}},\ldots,\mathrm{b}_{\mathrm{n}0}^{\mathrm{m}}\}$, where $b_{i0}^m=b_{i0}$ if $\tau_{i0}=\tau_m$ and $b_{i0}^m=0$ otherwise. Let L_m be a Laplacian such that for $i\neq j$, $[L_m]_{ij}=[L]_{ij}$ if $\tau_{ij}=\tau_m$ and $[L_m]_{ij}=0$ otherwise, and $[L_m]_{ii}=\sum_{i\neq j}|[L_m]_{ij}|$. Let $H=\mathrm{diag}\{e^{\mathrm{j}\theta_1},e^{\mathrm{j}\theta_2},\ldots,e^{\mathrm{j}\theta_n}\}$, $F=\mathrm{diag}\{\rho_1,\rho_2,\ldots,\rho_n\}$, $\tilde{L}_m=L_m+B_m$, $\tilde{L}=L+B$, $\hat{L}_m=F^{-1}H\tilde{L}_mH^*F^{-1}$ and $\hat{L}=$

$F^{-1}H\tilde{L}H^*F^{-1}$, where '*' denotes conjugate transpose. It is easy to see that $B = \sum_{m=1}^{M} B_m$, $L = \sum_{m=1}^{M} L_m$, $\tilde{L} = \sum_{m=1}^{M} \tilde{L}_m$ and $\hat{L} = \sum_{m=1}^{M} \hat{L}_m$.

Denote $\psi(t) = [\gamma(t), \zeta(t)]^T$, where $\gamma(t) = [r_1(t) - c_0(t) - \rho_1 e^{j\theta_1}(r_0(t) - c_0(t)), \dots, r_n(t) - c_0(t) - \rho_n e^{j\theta_n}(r_0(t) - c_0(t))]$, $\zeta(t) = [c_1(t) - c_0(t), \dots, c_n(t) - c_0(t)]$. Then, system (1) with (5) can be rewritten as

$$\dot{\psi}(t) = P\psi(t) + \sum_{m=1}^{M} Q_m \psi(t - \tau_m), \tag{11}$$

where $P = \begin{bmatrix} j\omega I_n & -j\omega I_n \\ 0 & 0 \end{bmatrix}$, $Q_m = \begin{bmatrix} 0 & 0 \\ \hat{L}_m & -\hat{L}_m - j\omega^{-1}\tilde{L}_m \end{bmatrix}$. It is clear that if all delays are equal to 0, the system (11) can be written as

$$\dot{\psi}(t) = (P + Q)\psi(t), \tag{12}$$

where $Q = \begin{bmatrix} 0 & 0 \\ \hat{L} & -\hat{L} - j\omega^{-1}\tilde{L} \end{bmatrix}$.

Lemma 2. *If the graph $\mathcal{G}$ is fixed and connected, then the matrices $\tilde{L}$ and $\hat{L}$ are both positive definite.*

Proof. From the definition of $\tilde{L} = L + B$, it is clear that L is a symmetric matrix. Due to the fact that there is at least one agent connected to the leader, it follows that $B \neq 0$. Since the graph $\mathcal{G}$ is connected, it can be proved that $\tilde{L}$ is positive definite according to Lemma 1. Note that $\hat{L} = F^{-1}H\tilde{L}H^*F^{-1}$. Let $L_s = H\tilde{L}H^*$. Clearly, L_s is similar to $\tilde{L}$ and then L_s is positive definite. Recall that F is a diagonal matrix, i.e., $F^{-1} = (F^{-1})^T$. Hence, $\hat{L} = (F^{-1})^T L_s F^{-1}$. It follows that L_s is contract to $\hat{L}$. As a result, $\hat{L}$ is positive definite. □

Lemma 3 (Lemma 5 in [9]). *If the graph $\mathcal{G}$ is fixed and connected, then all the eigenvalues of $P + Q$ have negative real parts.*

Proof. Suppose that λ_u is an eigenvalue of $P + Q$ and $[u_1^T, u_2^T]^T$ be the corresponding eigenvector, where $u_1, u_2 \in \mathbb{C}^n$. It follows from (12) that

$$j\omega u_1 - j\omega u_2 = \lambda_u u_1, \tag{13}$$

$$\hat{L}u_1 - \hat{L}u_2 - j\omega^{-1}\tilde{L}u_2 = \lambda_u u_2. \tag{14}$$

Then, from (13), it easy to obtained that

$$u_1 = \frac{j\omega}{j\omega - \lambda_u} u_2. \tag{15}$$

Substituting (15) into (14), we have

$$\hat{L}u_2 + \frac{1}{\lambda_u}\tilde{L}u_2 + j\omega^{-1}\tilde{L}u_2 = (j\omega - \lambda_u)u_2. \tag{16}$$

Now, we construct a system such that

$$\dot{x} = -\left(\hat{L} + \frac{1}{\lambda_u}\tilde{L} + j\omega^{-1}\tilde{L}\right)x, \tag{17}$$

where $x \in \mathbb{C}^n$. Then, consider the following Lyapunov function

$$V = x^* x. \tag{18}$$

Calculating $\dot{V}$, we have

$$\dot{V} = -x^* \left(2\hat{L} + \frac{\lambda_u^* + \lambda_u}{\|\lambda_u\|^2} \tilde{L} \right) x. \tag{19}$$

Since $\hat{L}$ is positive definite, if the real part of λ_u is positive, then $\dot{V} < 0$ and hence all the eigenvalue of the matrix $\hat{L} + \frac{1}{\lambda_u}\tilde{L} + \mathrm{j}\omega^{-1}\tilde{L}$ have positive real parts. According to (16), $\mathrm{j}\omega - \lambda_u$ is the eigenvalue of $\hat{L} + \frac{1}{\lambda_u}\tilde{L} + \mathrm{j}\omega^{-1}\tilde{L}$ associated with the eigenvector u_2. Thus, it can be conclude that $Re(\mathrm{j}\omega - \lambda_u) > 0$, namely, $Re(\lambda_u) < 0$. □

Theorem 1. *Consider a network of agents with nonuniform time-delays. Suppose that communication topology $\mathcal{G}$ is fixed and connected. With the algorithm (5), the rotating-formation control problem of multi-agent system (1) can be solved if the following condition holds*

$$\tau_m < \min\left\{ \frac{1}{\mu} \arccos\left(\frac{\omega^{-1}\tilde{\lambda}_{min}}{\mu} \right), \frac{\pi}{3\mu} \right\},$$

for all m, where $\mu = \sqrt{\hat{\lambda}_{max}^2 + \omega^{-2}\tilde{\lambda}_{max}^2}$, $\tilde{\lambda}_{max}$ and $\tilde{\lambda}_{min}$ denote the largest and the smallest eigenvalue of $\tilde{L}$ respectively and $\hat{\lambda}_{max}$, $\hat{\lambda}_{min}$ denote the largest and the smallest eigenvalue of $\hat{L}$ respectively.

Proof. Consider the system (11) in frequency domain, by letting $\tilde{\psi}(s) = \pounds[\psi(t)]$ and $G(s) = P + \sum_{m=1}^{M} Q_m e^{-\tau_m s}$. Then, $\tilde{\psi}(s) = (sI_{2n} - G(s))^{-1}\psi(0)$. In the following analysis, we first consider the case of the maximum time-delays such that $G(s)$ has the possibility of having any eigenvalues on imaginary axis. Then, the communication time-delay is less than the corresponding maximum delay.

Suppose that $s = \mathrm{j}\eta$ is an imaginary eigenvalue of $G(s)$ and $z \in \mathbb{C}^{2n}$ is its corresponding eigenvector, where $z = z_1 \otimes [1, 0]^T + z_2 \otimes [0, 1]^T$, $z_1, z_2 \in \mathbb{C}^n$. It follows that

$$[\mathrm{j}\eta I_{2n} - G(\mathrm{j}\eta)]z = 0. \tag{20}$$

Calculating the first n rows, we have

$$\mathrm{j}\eta z_1 - \mathrm{j}\omega z_1 + \mathrm{j}\omega z_2 = 0. \tag{21}$$

Clearly, $z_2 = \frac{\omega - \eta}{\omega} z_1$.

Premultiplying both sides of (20) with z^*, we have

$$z^*[\mathrm{j}\eta I_{2n} - P - \sum_{m=1}^{M} Q_m e^{-\tau_m \mathrm{j}\eta}]z = 0, \tag{22}$$

which implies

$$\mathrm{j}\eta z_1^* z_1 + \mathrm{j}\eta z_2^* z_2 - \mathrm{j}\omega z_1^* z_1 + \mathrm{j}\omega z_1^* z_2 - \sum_{m=1}^{M} [z_2^* \hat{L}_m z_1 \cos(\eta\tau_m) - \mathrm{j} z_2^* \hat{L}_m z_1 \sin(\eta\tau_m)]$$
$$- \sum_{m=1}^{M} [-z_2^* \hat{L}_m z_2 \cos(\eta\tau_m) + \mathrm{j} z_2^* \hat{L}_m z_2 \sin(\eta\tau_m) - \mathrm{j}\omega^{-1} z_2^* \tilde{L}_m z_2 \cos(\eta\tau_m)$$

$$-\omega^{-1}z_2^*\tilde{L}_m z_2 \sin(\eta\tau_m)] = 0. \tag{23}$$

Let $\alpha_m = \frac{z_2^*\tilde{L}_m z_2}{z_2^* z_2}$ and $\beta_m = \frac{z_2^*\hat{L}_m z_2}{z_2^* z_2}$. Note that

$$\begin{aligned}
&\sum_{m=1}^{M}\left[-\mathrm{j}z_2^*\hat{L}_m z_1 \sin(\eta\tau_m) + \mathrm{j}z_2^*\hat{L}_m z_2 \sin(\eta\tau_m) - \mathrm{j}\omega^{-1}z_2^*\tilde{L}_m z_2 \cos(\eta\tau_m)\right]\\
&\quad= \sum_{m=1}^{M}\left[-\frac{\mathrm{j}z_2^*\hat{L}_m(1-\frac{\eta}{\omega})z_1}{1-\frac{\eta}{\omega}}\sin(\eta\tau_m) + \mathrm{j}z_2^*\hat{L}_m z_2 \sin(\eta\tau_m) - \mathrm{j}\omega^{-1}z_2^*\tilde{L}_m z_2 \cos(\eta\tau_m)\right]\\
&\quad= \sum_{m=1}^{M}\left[\frac{-\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \sin(\eta\tau_m) - \omega^{-1}\alpha_m \cos(\eta\tau_m)\right]\mathrm{j}z_2^* z_2
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{m=1}^{M}\left[z_2^*\hat{L}_m z_1 \cos(\eta\tau_m) - z_2^*\hat{L}_m z_2 \cos(\eta\tau_m) - \omega^{-1}z_2^*\tilde{L}_m z_2 \sin(\eta\tau_m)\right]\\
&\quad= \sum_{m=1}^{M}\left[\frac{\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \cos(\eta\tau_m) - \omega^{-1}\alpha_m \sin(\eta\tau_m)\right]z_2^* z_2.
\end{aligned}$$

Hence, separating the imaginary and real parts of (22) yields

$$\begin{aligned}
&\mathrm{j}\eta z_1^* z_1 + \mathrm{j}\eta z_2^* z_2 - \mathrm{j}\omega z_1^* z_1 + \mathrm{j}\omega z_1^* z_2 - \sum_{m=1}^{M}\left[\frac{-\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \sin(\eta\tau_m) - \omega^{-1}\alpha_m \cos(\eta\tau_m)\right]\mathrm{j}z_2^* z_2 = 0,\\
&\sum_{m=1}^{M}\left[\frac{\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \cos(\eta\tau_m) - \omega^{-1}\alpha_m \sin(\eta\tau_m)\right]z_2^* z_2 = 0.
\end{aligned} \tag{24}$$

Substituting (21) into (24), we have

$$\begin{aligned}
&\sum_{m=1}^{M}\left[\frac{-\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \sin(\eta\tau_m) - \omega^{-1}\alpha_m \cos(\eta\tau_m)\right] = \eta,\\
&\sum_{m=1}^{M}\left[\frac{\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \cos(\eta\tau_m) - \omega^{-1}\alpha_m \sin(\eta\tau_m)\right] = 0.
\end{aligned} \tag{25}$$

To proceed, we first consider the case that all time-delays are equal. For this case, we have

$$\begin{aligned}
&\sum_{m=1}^{M}\left[\frac{-\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \sin(\eta\tau_{max}) - \omega^{-1}\alpha_m \cos(\eta\tau_{max})\right] = \eta,\\
&\sum_{m=1}^{M}\left[\frac{\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \cos(\eta\tau_{max}) - \omega^{-1}\alpha_m \sin(\eta\tau_{max})\right] = 0,
\end{aligned} \tag{26}$$

where τ_{max} denotes the maximum time-delay. □

Take τ_{max} to be sufficiently small such that $|\eta\tau_{max}| < \frac{\pi}{3}$. Suppose that $\eta > 0$, the first equation in (26) only holds if $\eta > \omega$, while the second equation does not hold any longer, i.e., $\frac{\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \cos(\eta\tau_{max}) < 0$ and $\omega^{-1}\alpha_m \sin(\eta\tau_{max}) > 0$. By simple analysis, we have that $\eta \in (-\infty, 0)$.

Let $\alpha = \sum_{m=1}^{M} \alpha_m = \frac{z_2^* \tilde{L} z_2}{z_2^* z_2}$, $\beta = \sum_{m=1}^{M} \beta_m = \frac{z_2^* \hat{L} z_2}{z_2^* z_2}$. From the second equation in (26), we have that

$$\frac{\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}} = \frac{\omega^{-1}\alpha \sin(\eta\tau_{max})}{\beta \cos(\eta\tau_{max})}. \tag{27}$$

Substituting (27) into the first equation of (26), yields that

$$-\frac{\omega^{-1}\alpha \sin^2(\eta\tau_{max})}{\cos(\eta\tau_{max})} - \omega^{-1}\alpha \cos(\eta\tau_{max}) - \eta = -\frac{\omega^{-1}\alpha}{\cos(\eta\tau_{max})} - \eta = 0. \tag{28}$$

Then, we have

$$\cos(\eta\tau_{max}) = -\frac{\omega^{-1}\alpha}{\eta}. \tag{29}$$

Before analyzing the upper bound of τ_{max}, we need to determine the bound of η first. Calculating the quadratic sum of two equations in (26), we have

$$\left(\frac{\eta}{\omega}\right)^2 \beta^2 + \left(1-\frac{\eta}{\omega}\right)^2 \omega^{-2}\alpha^2 = \left(1-\frac{\eta}{\omega}\right)^2 \eta^2. \tag{30}$$

It follows that

$$\left(1-\frac{\eta}{\omega}\right)^2 \beta^2 + \left(1-\frac{\eta}{\omega}\right)^2 \omega^{-2}\alpha^2 > \left(\frac{\eta}{\omega}\right)^2 \beta^2 + \left(1-\frac{\eta}{\omega}\right)^2 \omega^{-2}\alpha^2 = \left(1-\frac{\eta}{\omega}\right)^2 \eta^2.$$

Hence, it can be obtained that

$$-\sqrt{\hat{\lambda}_{max}^2 + \omega^{-2}\tilde{\lambda}_{max}^2} < \eta < 0. \tag{31}$$

Using (31), it follows from (29) that

$$\cos(\eta\tau_{max}) = -\frac{\omega^{-1}\alpha}{\eta} > \frac{\omega^{-1}\tilde{\lambda}_{min}}{\mu}. \tag{32}$$

Then, we have

$$\tau_{max} < \frac{1}{\mu}\arccos\left(\frac{\omega^{-1}\tilde{\lambda}_{min}}{\mu}\right). \tag{33}$$

Recall that $|\eta\tau_{max}| < \frac{\pi}{3}$, and from (33), we have

$$\tau_{max} < \min\left\{\frac{1}{\mu}\arccos\left(\frac{\omega^{-1}\tilde{\lambda}_{min}}{\mu}\right), \frac{\pi}{3\mu}\right\}. \tag{34}$$

From Lemma 3, if $\tau_{max} = 0$, system (11) is Hurwitz stable. From the continuity, when the upper bound of τ_{max} satisfies (34), the first equation in (26) does not hold and hence system (11) is also Hurwitz stable.

Now, we prove that if $\tau_m < \tau_{max}$ for all m, system (11) is also stable. By a similar way to the above analysis, from (30) and (31), it can be obtained that $|\eta|$ is bounded.

Clearly, $|\eta\tau_m| < \frac{\pi}{3}$ for all m. From (26), $\sum_{m=1}^{M}[\frac{\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \cos(\eta\tau_m) - \omega^{-1}\alpha_m \sin(\eta\tau_m)] \leq \sum_{m=1}^{M}[\frac{\frac{\eta}{\omega}}{1-\frac{\eta}{\omega}}\beta_m \cos(\eta\tau_{max}) - \omega^{-1}\alpha_m \sin(\eta\tau_{max})]$. Then, if

$$\tau_m < \min\left\{\frac{1}{\mu}\arccos\left(\frac{\omega^{-1}\tilde{\lambda}_{min}}{\mu}\right), \frac{\pi}{3\mu}\right\},$$

for all m, the first equation in (26) does not hold, hence, system (11) is also stable. That is, $\lim_{t\to+\infty}[r_i(t) - c_0(t) - \rho_i e^{\mathrm{j}\theta_i}(r_0(t) - c_0(t))] = \lim_{t\to+\infty}[c_i(t) - c_0(t)] = 0$ for all i. By simple calculations, we have $\lim_{t\to+\infty}[\frac{v_i(t)}{\rho_i e^{\mathrm{j}\theta_i}} - \frac{v_k(t)}{\rho_k e^{\mathrm{j}\theta_k}}] = 0$, $\lim_{t\to+\infty}[(r_i(t) + \omega^{-1}\mathrm{j}v_i(t)) - (r_k(t) + \omega^{-1}\mathrm{j}v_k(t))] = 0$, and $\lim_{t\to+\infty}[\dot{v}_i(t) - \mathrm{j}\omega v_i(t)] = 0$ for all $i, k \in \mathcal{I}$. Therefore, with the algorithm (5), the rotating formation control problem of multi-agent with nonuniform time-delays is solved.

Remark 2. In Theorem 1, we have $\tau_m < \min\{\frac{1}{\mu}\ \arccos(\frac{\omega^{-1}\tilde{\lambda}_{min}}{\mu}), \frac{\pi}{3\mu}\}$, where $\mu = \sqrt{\hat{\lambda}_{max}^2 + \omega^{-2}\tilde{\lambda}_{max}^2}$. Clearly, the upper bound of delay is related to the parameters $\hat{\lambda}_{max}$, $\tilde{\lambda}_{max}$ and $\hat{\lambda}_{min}$ and the angular velocity ω. In particular, when ω is too small or too large, the upper bound of delay tends to zero. Note that when ω is too small or too large, the term $\sum_{k\in\mathcal{N}_i} a_{ik}[c_i(t-\tau_{ik}) - c_k(t-\tau_{ik})]$ or the term $v_i = \omega\rho_i$ will tend to infinity, which makes both cases sensitive to the delay.

Remark 3. In our previous study, we found that if we only consider the transmission delay between agents, there always exists an error between the position of agents due to the coupling of rotating motion and delays, which makes the rotating formation impossible. Therefore in (5), each agent delays its own measurement of state in the same number as the transmission delay. It is noted that $v_k(t-\tau_{ik})$ (also $c_k(t-\tau_{ik})$) is used due to the consideration of the transmission delay between agent i and agent k, while $v_i(t-\tau_{ik})$ (also $c_i(t-\tau_{ik})$) is used as we mentioned before.

4. One numerical simulation example

In this section, one numerical simulation example is given to illustrate the theoretical results. Fig. 1 shows one communication graph with 5 nodes, and the $'0'$ represents the leader.

Let $H = \mathrm{diag}\{1, e^{\mathrm{j}\pi/6}, 1, e^{-\mathrm{j}\pi/6}\}$, $F = \mathrm{diag}\{1, 2, 3, 2\}$, $\rho_0 = 2$, $\theta_0 = 0$, $a_{10} = 0.8$, $a_{12} = a_{14} = 0.7$, $a_{21} = a_{23} = 0.5$, $a_{32} = a_{34} = 0.5$, and $a_{41} = a_{43} = 0.7$. According to Theorem 1, when $\omega = 0.9$, $\tau_m < 0.107$. Fig. 2 shows four agents form a formation, rotate around a common circle center with their corresponding rotating radius and track the leader agent, which is consistent with Theorem 1.

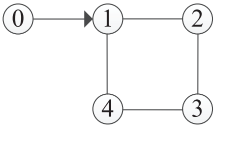

Fig. 1. One communication graph

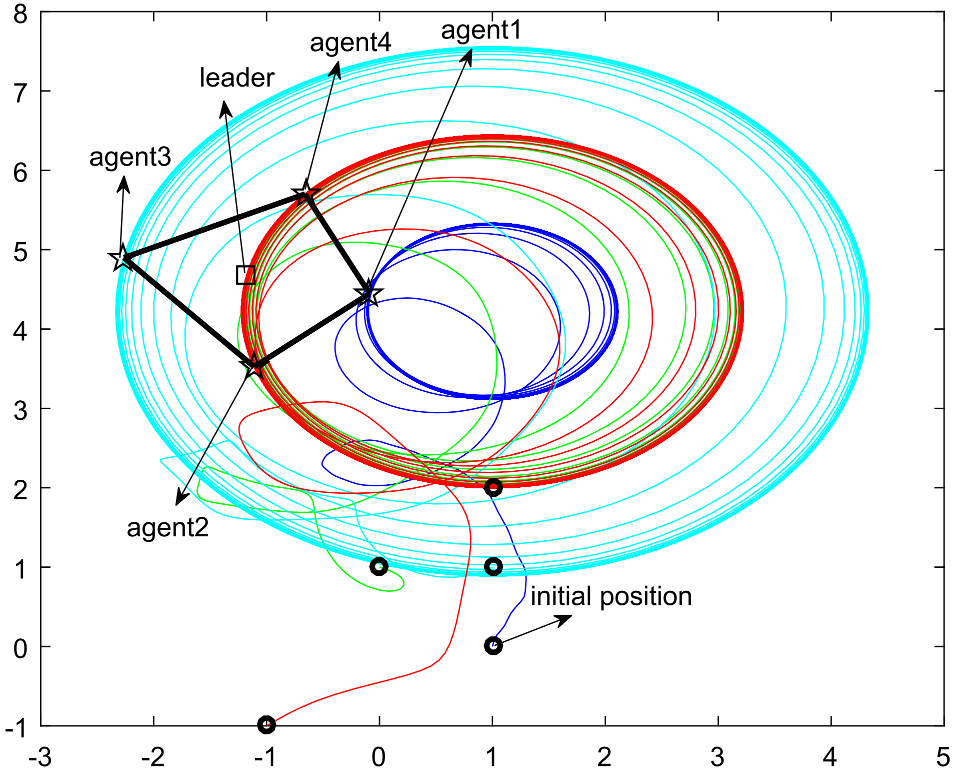

Fig. 2. Position trajectories of the multi-agent system ($[r_0(0), v_0(0), \ldots, r_4(0), v_4(0)] = [1+2j, 2, 1, 0, 1j, 0, 1+1j, 0, -1-1j, 0]$, $\omega = 0.9$, $\tau_{10} = \tau_{23} = \tau_{32} = \tau_{14} = \tau_{41} = 0.09$, $\tau_{12} = \tau_{21} = \tau_{34} = \tau_{43} = 0.1$).

5. Conclusions

In this paper, the leader-following rotating formation problem was studied for second-order multi-agent systems with nonuniform time-delays. A distributed leader-following rotating formation algorithm was proposed to drive all agents to form a formation while moving around a common point. The upper bound of maximum time-delay was deduced by the frequency domain method. Finally, a numerical simulation was given to illustrate the obtained results.

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