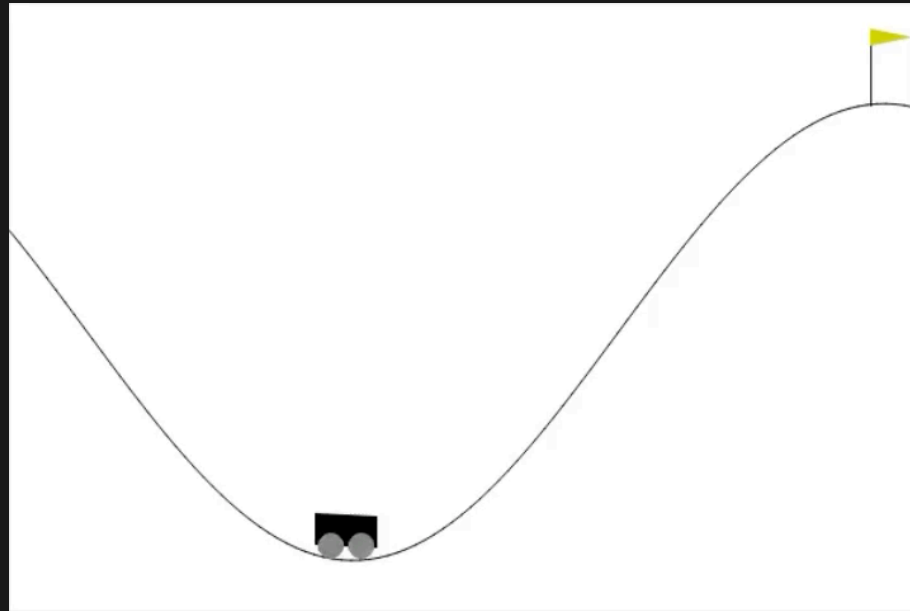
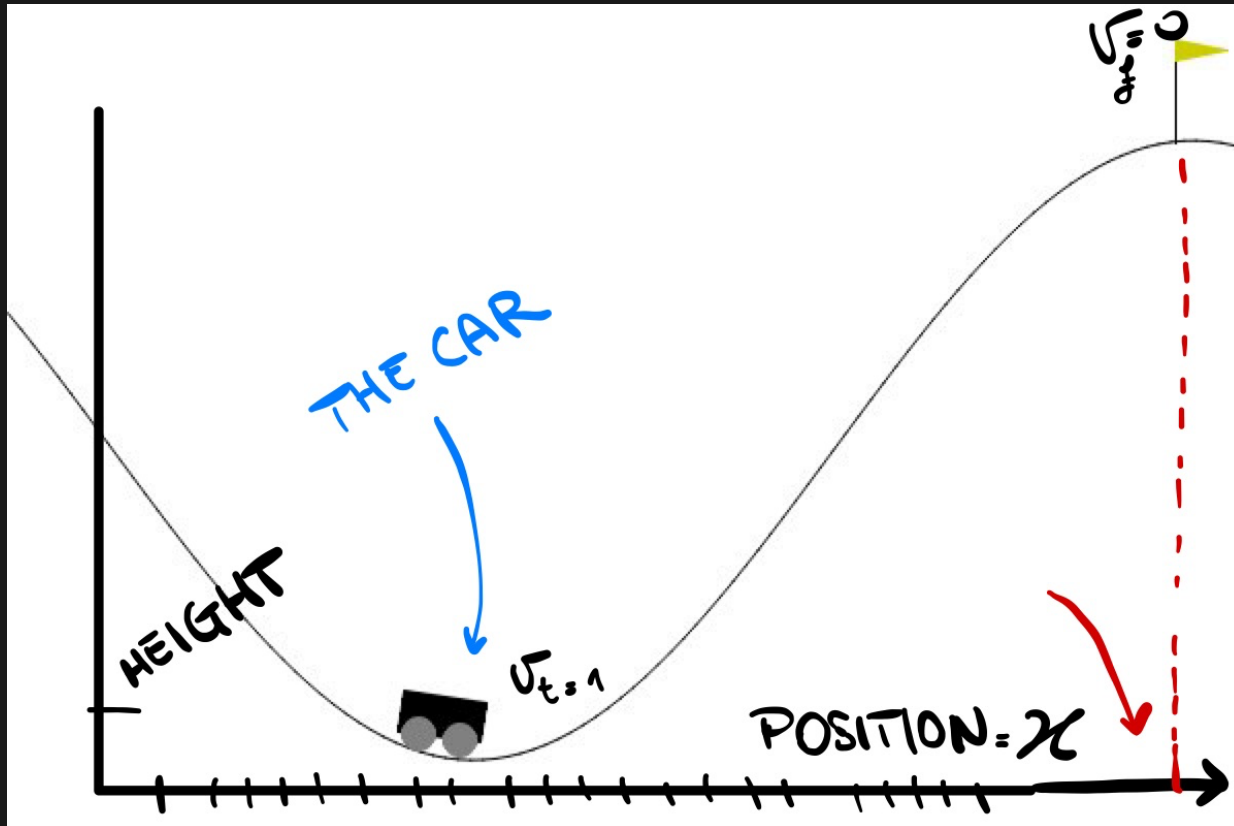


# THE MOUNTAIN CAR PROBLEM

Alessio G. & Campagnolo A.



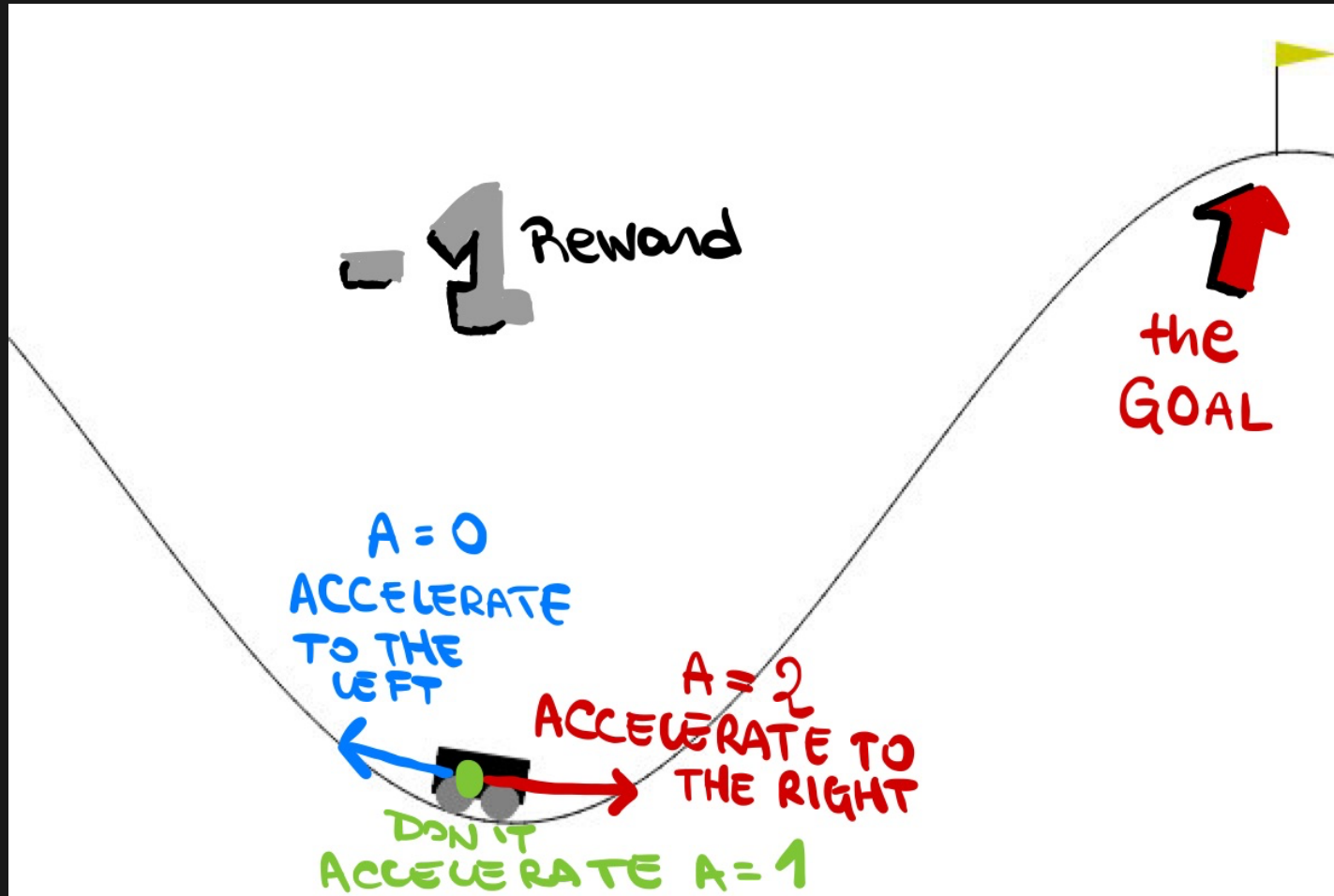
# OBSERVATION SPACE



Num	Observation
0	Position of the car along the x-axis
1	Velocity of the car



# ACTION SPACE



Num	Observation
0	Accelerate to the left
1	Don't accelerate
2	Accelerate to the right



# THE AGENT-ENVIRONMENT INTERFACE

```
env = gym.make("MountainCar-v0")
```

```
1
2 class MCE(gym.Env):
3     metadata = {
4         "render_modes": ["human", "rgb_array"],
5         "render_fps": 30,
6     }
7
8     def __init__(self, render_mode: Optional[str] = None, goal_velocity=0):
9         self.min_position = -1.2
10        self.max_position = 0.6
11        self.max_speed = 0.07
12        self.goal_position = 0.5
13        self.goal_velocity = goal_velocity
14
15        self.force = 0.001
```

Property

---

Action Space

---

Observation  
Shape

---

Observation High

---

Observation Low

# THE EPISODE

The car moves along the x-axis. The episode ends if either of the following happens:

- Termination: The position of the car is greater than or equal to 0.5 (the goal position on the right hill)
- Truncation: The length of the episode is 200.

# STEP FUNCTION

Given an action, the mountain car follows the following transition dynamics:

$$\text{Velocity}_{t+1} = \text{Velocity}_t + (\text{action} - 1) \cdot \text{force} - \cos(3 \cdot \text{Position}_t) \cdot g$$

$$\text{Position}_{t+1} = \text{Position}_t + \text{Velocity}_{t+1}$$

```
1 def step(self, action: int):
2     assert self.action_space.contains(
3         action
4     ), f"{action!r} ({type(action)}) invalid"
5
6     position, velocity = self.state
7     velocity += (action - 1) * self.force + math.cos(3 * position) * (-self.gravity)
8     velocity = np.clip(velocity, -self.max_speed, self.max_speed)
9     position += velocity
10    position = np.clip(position, self.min_position, self.max_position)
11    if position == self.min_position and velocity < 0:
12        velocity = 0
13
14    terminated = bool(
15        position >= self.goal_position and velocity >= self.goal_velocity
```





# STARTING STATE

The position of the car is assigned a uniform random value in  $[-0.6, -0.4]$ . The starting velocity of the car is always assigned to 0.

```
1 def reset(self, *, seed: Optional[int] = None, options: Optional[dict] = None):
2     super().reset(seed=seed)
3     # Note that if you use custom reset bounds, it may lead to out-of-bound
4     # state/observations.
5     low, high = utils.maybe_parse_reset_bounds(options, -0.6, -0.4)
6     self.state = np.array([self.np_random.uniform(low=low, high=high), 0])
7
8     if self.render_mode == "human":
9         self.render()
10    return np.array(self.state, dtype=np.float32), {}
```

# THE SARSA ALGORITHM

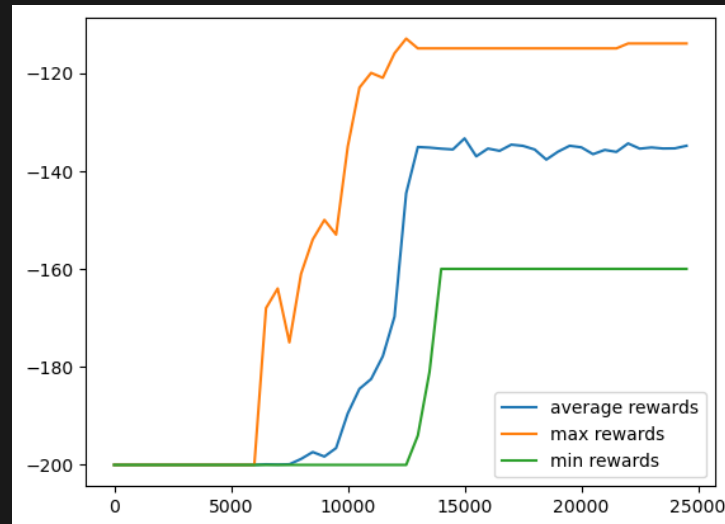
$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + \alpha [r + \gamma Q(s', a')]$$

```
1
2 for episode in range(EPIISODES):
3     # initialize
4     state, _ = env.reset()
5     discrete_state = get_discrete_state(state)
6     done = False
7     episode_reward = 0
8
9     # epsilon greedy
10    action = epsilon_greedy(q_table, discrete_state, epsilon)
11
12    # episode loop
13    while not done:
14        # new_state and reward
15        new_state, reward, done, _ = env.step(action)
```

# RESULTS SARSA

Parameters used - Average reward per episode - Rendering

```
# parameter setting  
LEARNING_RATE = 0.2  
DISCOUNT = 0.95  
EPISODES = 25000  
SHOW_EVERY = 500  
FRAMES_EVERY = 5000  
epsilon = 1
```



# THE Q LEARNING ALGORITHM

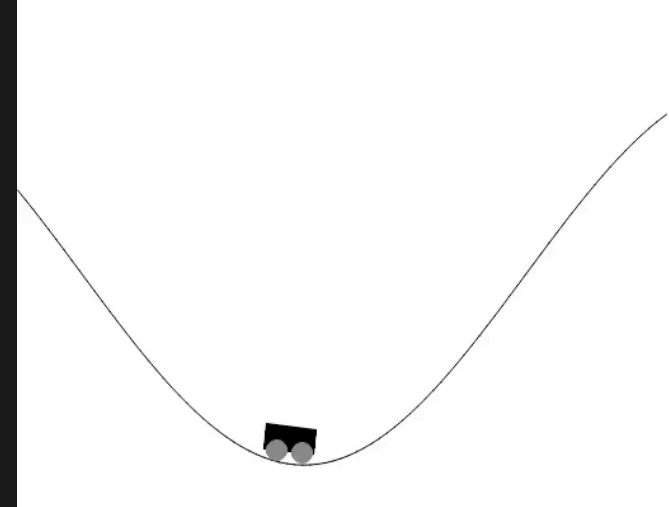
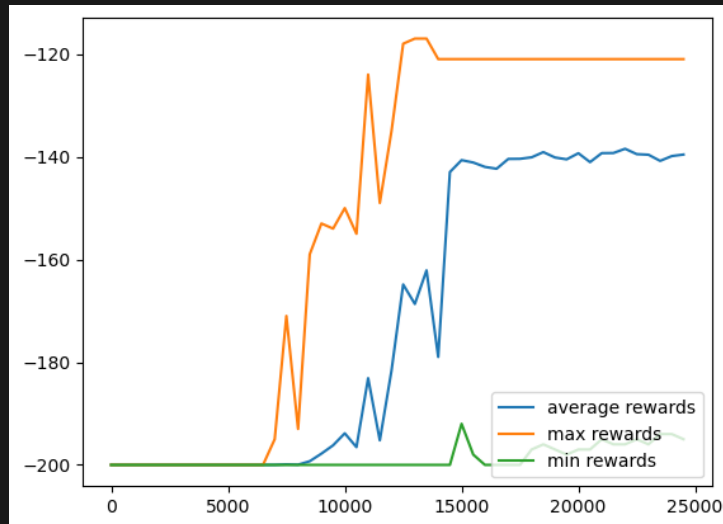
$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + \alpha \left[ r + \gamma \max_{a'} Q(s', a') \right]$$

```
1
2 for episode in range(EPIISODES):
3     state, _ = env.reset()
4     discrete_state = get_discrete_state(state)
5     done = False
6     episode_reward = 0
7
8     if episode % SHOW_EVERY == 0:
9         render = True
10        print(episode)
11    else:
12        render = False
13
14    while not done:
15        if np.random.random() > epsilon:
```

# RESULTS Q LEARNING

Parameters used - Average reward per episode - Rendering

```
# parameter setting  
LEARNING_RATE = 0.2  
DISCOUNT = 0.95  
EPISODES = 25000  
SHOW_EVERY = 500  
FRAMES_EVERY = 5000  
epsilon = 1
```



# THE EXPECTED SARSA ALGORITHM

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + \alpha \left[ r + \gamma \sum_a \pi(a|s')Q(s', a) \right]$$

```
1
2 for episode in range(EPIISODES):
3     # initialize
4     state, _ = env.reset()
5     discrete_state = get_discrete_state(state)
6     done = False
7     episode_reward = 0
8
9     # episode loop
10    while not done:
11
12        # epsilon greedy
13        action = epsilon_greedy(q_table, discrete_state, epsilon)
14        # new_state and reward
15        new_state, reward, done, _ = env.step(action)
```

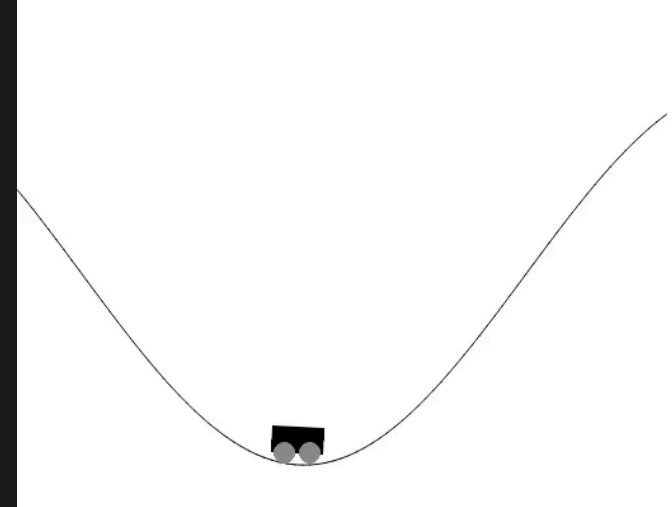
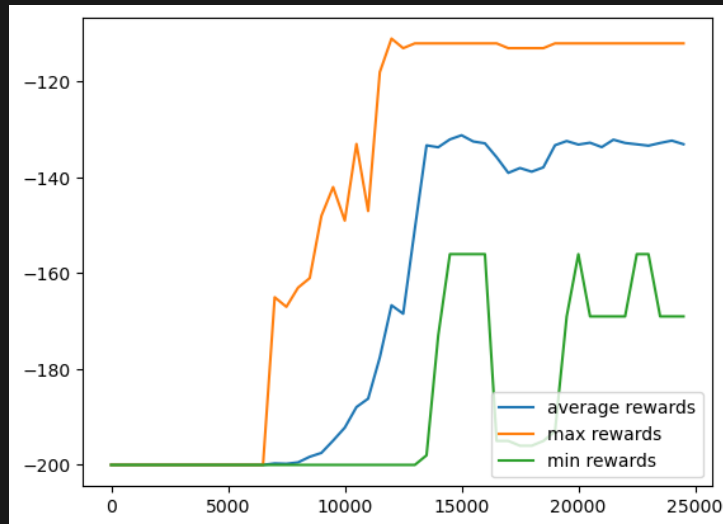




# RESULTS EXPECTED SARSA

Parameters used - Average reward per episode - Rendering

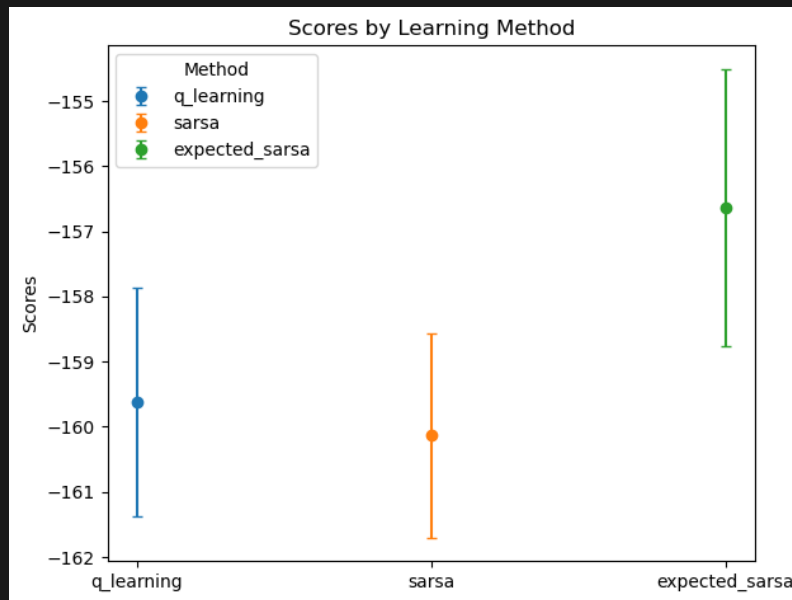
```
# parameter setting  
LEARNING_RATE = 0.2  
DISCOUNT = 0.95  
EPISODES = 25000  
SHOW_EVERY = 500  
FRAMES_EVERY = 5000  
epsilon = 1
```



# COMPARISON BETWEEN THE ALGORITHMS



# BOXPLOT



# IMPROVEMENTS

- Improve reward function
- Maximization bias
- After states methods
- Different trajectory, improve transition dynamics
- Mountain car continuous problem

# REFERENCES

- [1] [OpenAI Gym](#)
- [2] [OpenAI Gym MountainCar-v0](#)
- [3] [Richard S. Sutton \(2018\). "Reinforcement Learning". MIT Press: 119-140](#)

# GITHUB

## The mountain car discrete problem solved with TD learning

```
import gym
import numpy as np

class RL_Trainer:
    def __init__(self, env_name, learning_rate=0.1, discount=0.95,
                 show_every=50, generate_frames=False, frames_every=500,
                 epsilon=1, start_epsilon_decaying=1, end_epsilon_decaying=500, discrete_os_size=[20, 20], q_table=):
        self.env = gym.make(env_name, render_mode='rgb_array')
        self.learning_rate = learning_rate
        self.discount = discount
        self.show_every = show_every
        self.frames_every = frames_every
        self.epsilon = epsilon
        self.start_epsilon_decaying = start_epsilon_decaying
```

**THE END**