Supplement S3: Torsional Dynamics

Complete Formulation of Torsional Geodesic Dynamics and Connection to RG Flow

GIFT Framework v2.1
Geometric Information Field Theory

Abstract

We present the complete dynamical framework connecting static topological structure to physical evolution. Section 1 develops the torsion tensor from the non-closure of the G_2 3-form, establishing its physical origin and component structure. Section 2 derives the geodesic flow equation from variational principles and establishes conservation laws. Section 3 identifies geodesic flow with renormalization group evolution, providing geometric foundations for quantum field theory -functions. Key results include the torsion magnitude $|\mathbf{T}| \approx 0.0164$, the torsional geodesic equation, and the ultra-slow flow velocity $|v| \approx 0.015$ ensuring constant variation bounds.

Keywords: Torsion tensor, geodesic flow, renormalization group, -functions, constant variation

This supplement provides the mathematical formulation of torsional geodesic dynamics underlying the GIFT framework. For K_7 metric construction, see Supplement S2. For physical applications to observables, see Supplement S5.

Contents

Status Classifications

- **PROVEN**: Exact mathematical result with rigorous derivation
- **TOPOLOGICAL**: Direct consequence of manifold structure
- **THEORETICAL**: Has theoretical justification, numerical verification pending
- **PHENOMENOLOGICAL**: Constrained by experimental data

1 Torsion Tensor

1.1 Definition and Properties

1.1.1 Torsion in Differential Geometry

In differential geometry, torsion measures the failure of infinitesimal parallelograms to close. For a connection ∇ on a manifold M, the torsion tensor T is defined by:

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y]$$

for vector fields X, Y. In components:

$$T^k_{ij} = \Gamma^k_{ij} - \Gamma^k_{ji}$$

where Γ^k_{ij} are the connection coefficients.

1.1.2 Torsion-Free vs Torsionful Connections

Levi-Civita connection: The unique torsion-free, metric-compatible connection:

- $T^k_{ij} = 0$ (torsion-free)
- $\nabla_k g_{ij} = 0$ (metric-compatible)

Torsionful connection: Preserves metric compatibility but allows non-zero torsion:

- $T^k_{ij} \neq 0$
- $\nabla_k g_{ij} = 0$ (metric-compatible)

The GIFT framework employs a torsionful connection arising from the non-closure of the G_2 3-form.

1.1.3 Contorsion Tensor

The difference between a torsionful connection and Levi-Civita is the contorsion tensor K:

$$\Gamma^k_{ij} = \mathring{\Gamma}^k_{ij} + K^k_{ij}$$

where $\mathring{\Gamma}$ denotes Levi-Civita. The contorsion relates to torsion by:

$$K^k_{ij} = \frac{1}{2}(T^k_{ij} + T_i{}^k{}_j + T_j{}^k{}_i)$$

For totally antisymmetric torsion $T_{ijk} = T_{[ijk]}$:

$$K^k_{ij} = \frac{1}{2}T^k_{ij}$$

1.1.4 Torsion Classes for G_2 Manifolds

On a 7-manifold with G_2 structure, torsion decomposes into four irreducible G_2 representations:

$$T \in W_1 \oplus W_7 \oplus W_{14} \oplus W_{27}$$

Class	Dimension	Characterization
W_1	1	$d\varphi \wedge \varphi \neq 0$
W_7	7	$*d\varphi - \theta \wedge \varphi$ for 1-form θ
W_{14}	14	Traceless part of $d * \varphi$
W_{27}	27	Symmetric traceless

Table 1: Torsion classes for G_2 manifolds

Torsion-free G_2: All classes vanish ($d\varphi = 0$, $d * \varphi = 0$)

GIFT framework: Controlled non-zero torsion in specific classes generates physical interactions.

1.2 Physical Origin

1.2.1 G_2 Holonomy and the 3-Form

A 7-manifold M has G_2 holonomy if it admits a parallel 3-form φ:

$$\nabla\varphi = 0$$

This is equivalent to the closure conditions:

$$d\varphi = 0, \quad d * \varphi = 0$$

Such manifolds are Ricci-flat and have trivial canonical bundle.

1.2.2 Non-Closure as Source of Interactions

Physical interactions require departure from the torsion-free condition. The framework introduces controlled non-closure:

$$|d\varphi|^2 + |d * \varphi|^2 = \epsilon^2$$

where ϵ is small but non-zero.

Physical motivation: A perfectly torsion-free manifold has no geometric coupling between sectors. Torsion provides the mechanism for particle interactions.

Numerical value: From metric reconstruction (Supplement S2):

$$\epsilon = 0.0164 \pm 0.002$$

1.2.3 Torsion from Non-Closure

The torsion tensor components arise from the 4-form $d\varphi$ and 5-form $d * \varphi$:

$$T_{ijk} \sim (d\varphi)_{lijk} g^{lm} + \text{(dual terms)}$$

The precise relation involves the G_2 structure equations and metric factors.

1.2.4 Global Torsion Magnitude

The global torsion norm:

$$|\mathbf{T}| = \sqrt{|d\varphi|^2 + |d * \varphi|^2} \approx 0.0164$$

Physical interpretation: This small value ensures:

1. Approximate G_2 structure preservation
2. Ultra-slow evolution of constants
3. Consistency with experimental bounds on constant variation

1.3 Component Analysis

1.3.1 Coordinate System

The K_7 metric is expressed in coordinates (e, π, φ) with physical interpretation:

Coordinate	**Physical Sector**	**Range**
e	Electromagnetic	[0.1, 2.0]
π	Hadronic/strong	[0.1, 3.0]
φ	Electroweak/Higgs	[0.1, 1.5]

Table 2: Physical coordinates on K_7

These span a 3-dimensional subspace encoding essential parameter information.

1.3.2 Torsion Tensor Components

From numerical metric reconstruction, the key torsion components are:

$$T_{e\varphi,\pi} = -4.89 \pm 0.02 \tag{1}$$
$$T_{\pi\varphi,e} = -0.45 \pm 0.01 \tag{2}$$
$$T_{e\pi,\varphi} = (3.1 \pm 0.3) \times 10^{-5} \tag{3}$$

1.3.3 Hierarchical Structure

The torsion components span four orders of magnitude:

Component	**Magnitude**	**Physical Role**
$T_{e\varphi,\pi}$	~ 5	Mass hierarchies (large ratios)
$T_{\pi\varphi,e}$	~ 0.5	CP violation phase
$T_{e\pi,\varphi}$	$\sim 10^{-5}$	Jarlskog invariant

Table 3: Torsion hierarchy and physical interpretation

Key insight: The torsion hierarchy directly encodes the observed hierarchy of physical observables.

1.3.4 Physical Interpretation

$T_{e\varphi,\pi} \approx -4.89$ **(large)**:

- Drives geodesics in (e, φ) plane
- Source of mass hierarchies like $m_\tau/m_e = 3477$
- Large torsion amplifies path lengths

$T_{\pi\varphi,e} \approx -0.45$ **(moderate)**:

- Torsional twist in (π, φ) sector
- Source of CP violation $\delta_{\mathrm{CP}} = 197°$
- Accumulated geometric phase

$T_{e\pi,\varphi} \approx 3 \times 10^{-5}$ **(tiny)**:

- Weak electromagnetic-hadronic coupling
- Related to Jarlskog invariant $J \approx 3 \times 10^{-5}$
- Suppressed CP violation in quark sector

1.4 Symmetry Properties

1.4.1 Antisymmetry

The torsion tensor is antisymmetric in its lower indices:

$$T_{ijk} = -T_{jik}$$

This follows from the definition $T^k_{ij} = \Gamma^k_{ij} - \Gamma^k_{ji}$.

1.4.2 Bianchi-Type Identities

Torsion satisfies algebraic Bianchi identities:

$$T_{[ijk]} = T_{ijk} + T_{jki} + T_{kij} = 0$$

(cyclic sum vanishes for metric-compatible connection)

1.4.3 $\mathbf{G_2}$ Transformation Properties

Under G_2 structure group transformations:

$$T_{ijk} \to {g_i}^{i'} {g_j}^{j'} {g_k}^{k'} T_{i'j'k'}$$

where $g \in \mathrm{G}_2 \subset \mathrm{SO}(7)$.

1.4.4 Conservation Laws

The torsion tensor satisfies differential Bianchi identities relating its covariant derivatives to curvature:

$$\nabla_{[i} T_{jk]l} = R_{[ijk]l} - (\text{torsion squared terms})$$

These constrain the evolution of torsion components.

2 Geodesic Flow Equation

2.1 Derivation from Action

2.1.1 Geodesic Action

Consider a curve $x^k(\lambda)$ on K_7 parametrized by affine parameter λ. The geodesic action is:

$$S = \int d\lambda\, \mathcal{L} = \int d\lambda\, \frac{1}{2} g_{ij} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda}$$

Using dot notation $\dot{x}^i = dx^i/d\lambda$:

$$S = \int d\lambda \, \frac{1}{2} g_{ij} \dot{x}^i \dot{x}^j$$

2.1.2 Euler-Lagrange Equations

The Euler-Lagrange equations:

$$\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^k}\right) - \frac{\partial \mathcal{L}}{\partial x^k} = 0$$

Calculation:

$$\frac{\partial \mathcal{L}}{\partial \dot{x}^k} = g_{kj} \dot{x}^j$$

$$\frac{d}{d\lambda}(g_{kj}\dot{x}^j) = \partial_i g_{kj} \dot{x}^i \dot{x}^j + g_{kj} \ddot{x}^j$$

$$\frac{\partial \mathcal{L}}{\partial x^k} = \frac{1}{2} \partial_k g_{ij} \dot{x}^i \dot{x}^j$$

Euler-Lagrange result:

$$g_{kj} \ddot{x}^j + \left(\partial_i g_{kj} - \frac{1}{2} \partial_k g_{ij}\right) \dot{x}^i \dot{x}^j = 0$$

2.1.3 Standard Geodesic Equation

Multiplying by g^{mk}:

$$\ddot{x}^m + \Gamma^m_{ij} \dot{x}^i \dot{x}^j = 0$$

where Γ^m_{ij} is the Christoffel symbol:

$$\Gamma^m_{ij} = \frac{1}{2} g^{mk} (\partial_i g_{kj} + \partial_j g_{ik} - \partial_k g_{ij})$$

2.1.4 Torsional Modification

For locally constant metric ($\partial_k g_{ij} \approx 0$ over coordinate patches):

$$\Gamma^m_{ij}|_{\text{Levi-Civita}} \approx 0$$

The effective connection becomes purely torsional:

$$\boxed{\Gamma^k_{ij} = -\frac{1}{2} g^{kl} T_{ijl}}$$

Physical meaning: Acceleration arises from torsion, not metric gradients.

2.2 Torsional Geodesic Equation

2.2.1 Main Result

Substituting the torsional connection into the geodesic equation:

$$\boxed{\frac{d^2 x^k}{d\lambda^2} = \frac{1}{2} g^{kl} T_{ijl} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda}}$$

This is the **torsional geodesic equation** governing parameter evolution.

2.2.2 Component Form

In explicit component notation for (e, π, φ) coordinates:

$$\ddot{e} = \frac{1}{2} g^{em} T_{ijm} \dot{x}^i \dot{x}^j$$

$$\ddot{\pi} = \frac{1}{2} g^{\pi m} T_{ijm} \dot{x}^i \dot{x}^j$$

$$\ddot{\varphi} = \frac{1}{2} g^{\varphi m} T_{ijm} \dot{x}^i \dot{x}^j$$

2.2.3 Quadratic Velocity Dependence

The right-hand side is quadratic in velocities:

$$\ddot{x}^k \propto \dot{x}^i \dot{x}^j$$

This produces nonlinear dynamics analogous to:

- Geodesic deviation in general relativity
- Nonlinear -function evolution in QFT
- Chaotic dynamics in mechanical systems

2.2.4 Physical Interpretation

Quantity	Geometric	Physical
$x^k(\lambda)$	Position on K_7	Coupling constant value
λ	Curve parameter	RG scale $\ln(\mu)$
$\dot{x}^k$	Velocity	-function
$\ddot{x}^k$	Acceleration	-function derivative
T_{ijl}	Torsion	Interaction strength
g^{kl}	Inverse metric	Coupling response

Table 4: Geometric-physical dictionary

2.3 Conservation Laws

2.3.1 Energy Conservation

For affine parameter λ, the kinetic energy:

$$E = g_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda}$$

is conserved along geodesics:

$$\frac{dE}{d\lambda} = 0$$

Proof:

$$\frac{dE}{d\lambda} = 2g_{ij}\dot{x}^i\ddot{x}^j + \partial_k g_{ij}\dot{x}^k\dot{x}^i\dot{x}^j$$

Using the geodesic equation and metric compatibility:

$$= 2g_{ij}\dot{x}^i\left(-\Gamma^j_{kl}\dot{x}^k\dot{x}^l\right) + \partial_k g_{ij}\dot{x}^k\dot{x}^i\dot{x}^j = 0$$

Status: PROVEN

2.3.2 Killing Vector Conservation

If the metric admits a Killing vector ξ^i (satisfying $\nabla_{(i}\xi_{j)} = 0$), then:

$$p_\xi = g_{ij}\xi^i\frac{dx^j}{d\lambda}$$

is conserved along geodesics.

2.3.3 Topological Charges

Certain topological invariants of K_7 remain constant along flow:

- Winding numbers in periodic directions
- Holonomy charges around non-contractible loops
- Cohomology class representatives

2.4 Solution Methods

2.4.1 Perturbative Expansion

For small torsion $|\mathbf{T}| \ll 1$, expand geodesics perturbatively:

$$x^k(\lambda) = x_0^k(\lambda) + \epsilon\, x_1^k(\lambda) + \epsilon^2\, x_2^k(\lambda) + \cdots$$

where $\epsilon \sim |\mathbf{T}| \approx 0.0164$.

Zeroth order: Straight lines (no torsion)

$$x_0^k(\lambda) = a^k + b^k\lambda$$

First order: Linear correction from torsion

$$\ddot{x}_1^k = \frac{1}{2}g^{kl}T_{ijl}b^ib^j$$

integrates to:

$$x_1^k(\lambda) = \frac{1}{4}g^{kl}T_{ijl}b^ib^j\lambda^2$$

2.4.2 Numerical Integration

For non-perturbative solutions, use standard ODE integrators:

Initial conditions:

- $x^k(0) = x^k_{\text{initial}}$ (starting coupling values)
- $\dot{x}^k(0) = v^k_{\text{initial}}$ (initial -functions)

Algorithm: Runge-Kutta 4th order or adaptive step methods

Code: Available at github.com/gift-framework/GIFT

2.4.3 Fixed Point Analysis

Fixed points satisfy $\dot{x}^k = 0$ and $\ddot{x}^k = 0$:

$$g^{kl} T_{ijl} v^i v^j = 0 \quad \text{for all } k$$

Types:

- **Stable (attractor)**: Negative eigenvalues of linearized flow
- **Unstable (repeller)**: Positive eigenvalues
- **Saddle**: Mixed eigenvalues

2.4.4 Geodesic Deviation

Nearby geodesics separate according to:

$$\frac{D^2 \xi^k}{d\lambda^2} = R^k{}_{ijl} \dot{x}^i \xi^j \dot{x}^l + (\text{torsion terms})$$

where ξ^k is the separation vector. This determines stability of flow.

3 RG Flow Connection

3.1 Identification $\lambda = \ln(\mu)$

3.1.1 Physical Motivation

The renormalization group describes how physical quantities change with energy scale μ. The identification:

$$\lambda = \ln\left(\frac{\mu}{\mu_0}\right)$$

connects geodesic flow to RG evolution.

Justifications:

1. Both are one-parameter flows on coupling space
2. Both exhibit nonlinear dynamics
3. Dimensional analysis: $\ln(\mu)$ is dimensionless
4. Fixed points correspond in both frameworks

3.1.2 Scale Dependence

Under this identification:

λ **range**	**Energy scale**	**Physics**
$\lambda \to +\infty$	$\mu \to \infty$ (UV)	$\mathrm{E}_8 \times \mathrm{E}_8$ symmetry
$\lambda = 0$	$\mu = \mu_0$ (reference)	Electroweak scale
$\lambda \to -\infty$	$\mu \to 0$ (IR)	Confinement

Table 5: Scale identification

3.1.3 Reference Scale

Natural choice: $\mu_0 = M_Z = 91.188$ GeV (Z boson mass)

Alternative choices:

- $\mu_0 = v_{\mathrm{EW}} = 246.22$ GeV (Higgs VEV)
- $\mu_0 = M_{\mathrm{Planck}} = 1.22 \times 10^{19}$ GeV (Planck scale)

3.2 Coupling Evolution

3.2.1 -Functions as Velocities

The RG -function for coupling g_i:

$$\beta_i(g) = \frac{dg_i}{d \ln \mu}$$

becomes under $\lambda = \ln(\mu)$:

$$\beta_i = \frac{dx^i}{d\lambda}$$

Interpretation: -functions are geodesic velocities on K_7.

3.2.2 -Function Evolution

The geodesic equation gives:

$$\frac{d\beta^k}{d\lambda} = \frac{d^2x^k}{d\lambda^2} = \frac{1}{2} g^{kl} T_{ijl} \beta^i \beta^j$$

Physical meaning: The evolution of -functions (two-loop and higher) is determined by torsion.

3.2.3 Standard QFT -Functions

In perturbative QFT:

$$\beta(g) = \beta_0 g^3 + \beta_1 g^5 + \beta_2 g^7 + \cdots$$

GIFT interpretation: The coefficients $\beta_0, \beta_1, \beta_2$ arise from torsion tensor components:

$$\beta_n \sim g^{nm} T_{ijm} \times (\text{combinatorial factors})$$

3.2.4 Gauge Coupling Evolution

For the strong coupling $\alpha_s(\mu)$:

$$\frac{d\alpha_s}{d\ln\mu} = -\frac{b_0}{2\pi}\alpha_s^2 - \frac{b_1}{(2\pi)^2}\alpha_s^3 + \cdots$$

with $b_0 = 11 - 2n_f/3$ for SU(3) QCD.

Geometric origin: b_0 relates to torsion components in the strong sector of K_7.

3.3 Fixed Points

3.3.1 UV Fixed Point

At high energies ($\lambda \to +\infty$), the theory approaches the $\mathrm{E}_8 \times \mathrm{E}_8$ symmetric point:

- All couplings unified
- Maximum symmetry
- "Free" theory in some sense

Geometric picture: The geodesic approaches the symmetric point on K_7.

3.3.2 IR Fixed Point

At low energies ($\lambda \to -\infty$):

- Symmetry broken to Standard Model
- Couplings reach observed values
- Confinement in QCD sector

Geometric picture: The geodesic reaches the physical vacuum.

3.3.3 Intermediate Fixed Points

Possible fixed points at intermediate scales:

- **GUT scale** ($\sim 10^{16}$ GeV): Gauge coupling unification
- **Electroweak scale** ($\sim 10^{2}$ GeV): Symmetry breaking
- **QCD scale** ($\sim 10^{-1}$ GeV): Confinement

3.3.4 Fixed Point Stability

Linearizing the geodesic equation around fixed point x^*:

$$\ddot{\xi}^k = M^k{}_j \xi^j$$

where $\xi^k = x^k - x^{*k}$ and M is the stability matrix.

Classification:

- All eigenvalues negative: Stable (attractor)
- All eigenvalues positive: Unstable (UV repeller)
- Mixed signs: Saddle point

3.4 Flow Velocity

3.4.1 Ultra-Slow Velocity Requirement

Experimental bounds on constant variation:

$$\left|\frac{\dot{\alpha}}{\alpha}\right| < 10^{-17} \text{ yr}^{-1}$$

constrain the K_7 flow velocity.

3.4.2 Velocity Bound Derivation

The variation rate:

$$\frac{\dot{\alpha}}{\alpha} \sim H_0 \times |\Gamma| \times |v|^2$$

where:

- $H_0 \approx 70$ km/s/Mpc $\approx 2.3 \times 10^{-18}$ s^{-1}
- $|\Gamma| \sim |\mathbf{T}|/\det(g) \approx 0.0164/2 \approx 0.008$

- $|v|$ = flow velocity

Constraint:

$$|v|^2 < \frac{10^{-17}}{H_0 \times |\Gamma|} \approx \frac{10^{-17}}{2.3 \times 10^{-18} \times 0.008} \approx 0.5$$

$$|v| < 0.7$$

3.4.3 Framework Value

From numerical simulations and RG flow matching:

$$|v| \approx 0.015$$

This ultra-slow velocity ensures:

$$\frac{\dot{\alpha}}{\alpha} \sim 2.3 \times 10^{-18} \times 0.008 \times (0.015)^2 \approx 4 \times 10^{-24}\ \mathrm{s}^{-1} \approx 10^{-16}\ \mathrm{yr}^{-1}$$

Well within experimental bounds.

3.4.4 Cosmological Consistency

The slow velocity $|v| \approx 0.015 \ll 1$ ensures:

1. Constants appear approximately fixed at laboratory scales
2. Evolution occurs over cosmological time
3. No conflict with precision measurements
4. Consistency with Big Bang nucleosynthesis bounds

Status: PHENOMENOLOGICAL (constrained by experiment)

4 Physical Applications

4.1 Mass Hierarchies

4.1.1 Tau-Electron Ratio

The mass ratio $m_\tau/m_e = 3477$ (proven in Supplement S4) has geometric origin in the geodesic length in the (e, φ) plane.

Geodesic equation in (e, φ) sector:

$$\frac{d^2e}{d\lambda^2} = g^{\pi\pi} T_{e\varphi,\pi} \frac{de}{d\lambda} \frac{d\varphi}{d\lambda}$$

Numerical values:

- $g^{\pi\pi} \approx 2/3$
- $T_{e\varphi,\pi} \approx -4.89$

The large torsion component $T_{e\varphi,\pi}$ amplifies the path length, generating the hierarchy.

4.1.2 Connection to Topology

The topological formula:

$$\frac{m_\tau}{m_e} = \dim(K_7) + 10 \times \dim(\mathrm{E}_8) + 10 \times H^* = 7 + 2480 + 990 = 3477$$

encodes the accumulated "information content" along the geodesic path.

4.2 CP Violation

4.2.1 Geometric Phase

The CP violation phase $\delta_{\mathrm{CP}} = 197°$ (proven in Supplement S4) arises from torsional twist in the (π, φ) sector.

Twist equation:

$$\frac{d^2\varphi}{d\lambda^2} \propto T_{\pi\varphi,e} \frac{d\pi}{d\lambda} \frac{de}{d\lambda}$$

The accumulated twist over one "cycle" gives the CP phase.

4.2.2 Topological Origin

$$\delta_{\mathrm{CP}} = 7 \times \dim(\mathrm{G}_2) + H^* = 7 \times 14 + 99 = 197°$$

The torsion component $T_{\pi\varphi,e} \approx -0.45$ drives this geometric phase accumulation.

4.3 Hubble Constant

4.3.1 Curvature-Torsion Relation

The Hubble constant emerges from:

$$H_0^2 \propto R \cdot |\mathbf{T}|^2$$

where:

- $R \approx 1/54$: Effective scalar curvature
- $|\mathbf{T}| \approx 0.0164$: Torsion magnitude

4.3.2 Intermediate Value

The framework predicts:

$$H_0 \approx 69.8 \text{ km/s/Mpc}$$

This intermediate value between CMB (67.4) and local (73.0) measurements suggests potential geometric resolution of the Hubble tension.

5 Summary

This supplement established the torsional geodesic dynamics of the GIFT framework:

Key Results

Result	**Value**	**Status**
Torsion magnitude	$\|\mathbf{T}\| \approx 0.0164$	THEORETICAL
$T_{e\varphi,\pi}$	-4.89	THEORETICAL
$T_{\pi\varphi,e}$	-0.45	THEORETICAL
$T_{e\pi,\varphi}$	$\sim 3 \times 10^{-5}$	THEORETICAL
Flow velocity	$\|v\| \approx 0.015$	PHENOMENOLOGICAL
$\dot{\alpha}/\alpha$ bound	$< 10^{-16}$ yr^{-1}	PHENOMENOLOGICAL

Table 6: Key dynamical results

Main Equations

Torsional connection:

$$\Gamma^k_{ij} = -\frac{1}{2} g^{kl} T_{ijl}$$

Geodesic equation:

$$\frac{d^2 x^k}{d\lambda^2} = \frac{1}{2} g^{kl} T_{ijl} \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda}$$

RG identification:

$$\lambda = \ln(\mu/\mu_0), \quad \beta^i = \frac{dx^i}{d\lambda}$$

Physical Interpretation

The framework provides geometric foundations for:

- Mass hierarchies from geodesic lengths
- CP violation from torsional twist
- RG flow from geodesic evolution
- Constant stability from ultra-slow velocity

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