

Geometric Information Field Theory

Topological Derivation of Standard Model Parameters from G_2 Holonomy Manifolds

Brieuc de La Fournière

Independent researcher, Paris

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Abstract

The Standard Model requires 19 experimentally determined parameters lacking theoretical explanation. We explore a geometric framework in which dimensionless ratios emerge as topological invariants of a seven-dimensional G_2 holonomy manifold K_7 coupled to $E_8 \times E_8$ gauge structure, containing zero continuous adjustable parameters.

Assuming existence of a compact G_2 manifold with Betti numbers $b_2 = 21$ and $b_3 = 77$, we derive 18 dimensionless predictions with mean deviation 0.087% from experiment. The Koide parameter follows as $Q = \dim(G_2)/b_2 = 14/21 = 2/3$. The neutrino CP-violation phase $\delta_{CP} = 197^\circ$ is consistent with the recent T2K+NOvA joint analysis (Nature, 2025). Exhaustive search over 19,100 configurations confirms $(b_2, b_3) = (21, 77)$ as uniquely optimal ($> 4\sigma$ after look-elsewhere correction).

All arithmetic relations are formally verified in Lean 4 (180+ theorems). The Deep Underground Neutrino Experiment (DUNE, 2028–2040) will test δ_{CP} with resolution of a few degrees to $\sim 15^\circ$; measurement outside 182° – 212° would refute the framework. We present this as an exploratory investigation emphasizing falsifiability, not a claim of correctness.

Keywords: G_2 holonomy, exceptional Lie algebras, Standard Model parameters, topological field theory, falsifiability, formal verification

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1 Introduction

1.1 The Parameter Problem

The Standard Model describes fundamental interactions with remarkable precision, yet requires 19 free parameters determined solely through experiment [1]. These parameters—gauge couplings, Yukawa couplings spanning five orders of magnitude, mixing matrices, and Higgs sector values—lack theoretical explanation. Several tensions motivate the search for deeper structure:

- **Hierarchy problem:** The Higgs mass requires fine-tuning absent new physics [2].
- **Hubble tension:** CMB and local H_0 measurements differ by $> 4\sigma$ [3, 4].
- **Flavor puzzle:** No mechanism explains three generations or mass hierarchies [5].
- **Koide mystery:** The charged lepton relation $Q = 2/3$ holds for 43 years without explanation [6].

These challenges suggest examining whether parameters might emerge from geometric or topological structures.

1.2 Contemporary Context

The present framework connects to three active research programs:

Division algebra program (Furey, Hughes, Dixon [7, 8]): Derives Standard Model symmetries from $\mathbb{C} \otimes \mathbb{O}$ structure. GIFT adds compactification geometry and numerical predictions.

$E_8 \times E_8$ unification: Wilson (2024) shows $E_8(-248)$ encodes three fermion generations with Standard Model gauge structure [9]. Singh, Kaushik et al. (2024) develop similar $E_8 \otimes E_8$ unification [10]. GIFT extracts numerical values from this structure.

G_2 holonomy physics (Acharya, Haskins, Foscolo-Nordström [11, 12, 13]): M-theory on G_2 manifolds. Recent work (2022–2025) extends twisted connected sum constructions [14, 15]. GIFT derives dimensionless constants from topological invariants.

1.3 Framework Overview

The Geometric Information Field Theory (GIFT) proposes that dimensionless parameters represent topological invariants:

$E_8 \times E_8$ (496D) \rightarrow AdS4 x K7 (11D) \rightarrow Standard Model (4D)

The key elements:

1. $E_8 \times E_8$ **gauge structure** (dimension 496)
2. **Compact 7-manifold** K_7 with G_2 holonomy ($b_2 = 21, b_3 = 77$)

3. **Topological constraints** on the G_2 metric ($\det(g) = 65/32$)
4. **Cohomological mapping**: Betti numbers constrain field content

We emphasize this represents mathematical exploration, not a claim that nature realizes this structure. The framework's merit lies in falsifiable predictions from topological inputs.

1.4 Paper Organization

- Section 2: Mathematical framework ($E_8 \times E_8$, K_7 , G_2 structure)
- Section 3: Derivation of 18 dimensionless predictions
- Section 4: Formal verification and statistical analysis
- Section 5: Falsification criteria
- Section 6: Discussion and limitations
- Section 7: Conclusions

2 Mathematical Framework

2.1 The Octonionic Foundation

GIFT emerges from the algebraic fact that **the octonions \mathbb{O} are the largest normed division algebra**.

Algebra	Dim	Physics Role	Extends?
\mathbb{R}	1	Classical mechanics	Yes
\mathbb{C}	2	Quantum mechanics	Yes
\mathbb{H}	4	Spin, Lorentz group	Yes
\mathbb{O}	8	Exceptional structures	No

The octonions terminate this sequence. Their automorphism group $G_2 = \text{Aut}(\mathbb{O})$ has dimension 14 and acts naturally on $\text{Im}(\mathbb{O}) \cong \mathbb{R}^7$.

2.2 $E_8 \times E_8$ Structure

E_8 is the largest exceptional simple Lie group with dimension 248 and rank 8 [16]. The exceptional algebras connect to octonions through the chain established by Dray and Manogue [17]:

Algebra	Dimension	Connection to \mathbb{O}
G_2	14	$\text{Aut}(\mathbb{O})$
F_4	52	$\text{Aut}(J_3(\mathbb{O}))$
E_6	78	Collineations of $\mathbb{O}P^2$
E_8	248	Contains all lower exceptionals

Wilson (2024) demonstrates that $E_8(-248)$ encodes three fermion generations (128 degrees of freedom) with GUT structure [9]. The product $E_8 \times E_8$ arises in heterotic string theory [18], with dimension 496.

2.3 The K_7 Manifold Hypothesis

2.3.1 Statement of Hypothesis

Hypothesis: There exists a compact 7-manifold K_7 with G_2 holonomy satisfying:

- Second Betti number: $b_2(K_7) = 21$
- Third Betti number: $b_3(K_7) = 77$
- Simple connectivity: $\pi_1(K_7) = 0$

We do not claim to have constructed such a manifold explicitly. Rather, we assume its existence and derive consequences from these topological data.

2.3.2 Plausibility from TCS Constructions

The twisted connected sum (TCS) method of Joyce [19] and Kovalev [20], extended by Corti-Haskins-Nordström-Pacini [21] and recent work on extra-twisted connected sums [14, 15], produces compact G_2 manifolds with controlled Betti numbers.

TCS constructions glue asymptotically cylindrical building blocks:

$$K_7 = M_1^T \cup_{\varphi} M_2^T$$

For appropriate Calabi-Yau building blocks, Mayer-Vietoris sequences yield Betti numbers in ranges including $(b_2, b_3) = (21, 77)$. While we do not cite a specific construction achieving exactly these values, such manifolds are plausible within the TCS/ETCS landscape.

The effective cohomological dimension:

$$H^* = b_2 + b_3 + 1 = 21 + 77 + 1 = 99$$

2.4 G_2 Structure and Metric Constraints

2.4.1 Local Model: The Standard G_2 Form

On the tangent space $T_p K_7 \cong \mathbb{R}^7$, the G_2 structure is locally modeled by the standard associative 3-form φ_0 of Harvey-Lawson [22]:

$$\varphi_0 = e^{012} + e^{034} + e^{056} + e^{135} - e^{146} - e^{236} - e^{245}$$

This form has 7 non-zero components among $\binom{7}{3} = 35$ basis elements and defines a metric $g_0 = I_7$ with induced volume form.

2.4.2 Topological Constraint on the Metric

The framework imposes a **topological constraint** on the G_2 metric determinant:

$$\det(g) = \frac{65}{32}$$

This value derives from topological integers:

$$\det(g) = \frac{\text{Weyl} \times (\text{rank}(E_8) + \text{Weyl})}{2^{\text{Weyl}}} = \frac{5 \times 13}{32} = \frac{65}{32}$$

Clarification: This is a constraint that the global G_2 metric on K_7 must satisfy, not an explicit construction of that metric. The TCS method of Joyce-Kovalev provides constructions of compact G_2 manifolds; under appropriate analytic conditions, perturbation techniques yield torsion-free metrics [19, 20]. We hypothesize that a K_7 satisfying $\det(g) = 65/32$ exists within this landscape.

2.4.3 Torsion Capacity

We define the **torsion capacity** as a topological parameter:

$$\kappa_T = \frac{1}{b_3 - \dim(G_2) - p_2} = \frac{1}{77 - 14 - 2} = \frac{1}{61}$$

where $p_2 = \dim(G_2)/\dim(K_7) = 2$. This characterizes the manifold's topological structure; the actual torsion of the G_2 metric (which vanishes for holonomy exactly G_2) is a separate geometric property guaranteed by Joyce's theorem for appropriate constructions.

2.5 Topological Constraints on Field Content

2.5.1 Betti Numbers as Capacity Bounds

The Betti numbers provide **upper bounds** on field multiplicities:

- $b_2(K_7) = 21$: Bounds the number of gauge field degrees of freedom
- $b_3(K_7) = 77$: Bounds the number of matter field degrees of freedom

Important caveat: On a smooth G_2 manifold, dimensional reduction yields b_2 abelian $U(1)$ vector multiplets [11]. Non-abelian gauge groups (such as the Standard Model $SU(3) \times SU(2) \times U(1)$) require **singularities** in the G_2 manifold—specifically, codimension-4 singularities with ADE-type structure [23, 24]. We assume K_7 admits such singularities; a complete treatment would require specifying the singular locus.

2.5.2 Generation Number

The number of chiral fermion generations follows from a topological constraint:

$$(\text{rank}(\text{E}_8) + N_{\text{gen}}) \cdot b_2 = N_{\text{gen}} \cdot b_3$$

Solving: $(8 + N_{\text{gen}}) \times 21 = N_{\text{gen}} \times 77$ yields $\mathbf{N}_{\text{gen}} = \mathbf{3}$.

This derivation is formal; physically, it reflects index-theoretic constraints on chiral zero modes, which in M-theory on G_2 require singular geometries for chirality [24].

3 Derivation of the 18 Dimensionless Predictions

3.1 Methodology

Inputs (hypotheses):

- Existence of K_7 with G_2 holonomy and $(b_2, b_3) = (21, 77)$
- $\text{E}_8 \times \text{E}_8$ gauge structure with standard algebraic data
- Topological constraint $\det(g) = 65/32$

Outputs (derived quantities):

- 18 dimensionless ratios expressed in terms of topological integers

We claim that given the inputs, the outputs follow algebraically. We do **not** claim the formulas are uniquely determined by geometry; the specific combinations (e.g., $b_2/(b_3 + \dim G_2)$ for $\sin^2 \theta_W$) are empirically motivated.

3.2 Gauge Sector

3.2.1 Weinberg Angle

$$\sin^2 \theta_W = \frac{b_2}{b_3 + \dim(G_2)} = \frac{21}{91} = \frac{3}{13} = 0.230769$$

	Experimental [1]	GIFT
$\sin^2 \theta_W$	0.23122 ± 0.00004	0.230769
Deviation		0.195%

3.2.2 Strong Coupling

$$\alpha_s(M_Z) = \frac{\sqrt{2}}{\dim(G_2) - p_2} = \frac{\sqrt{2}}{12} = 0.11785$$

Experimental: 0.1179 ± 0.0009 . Deviation: **0.04%**.

3.3 Lepton Sector

3.3.1 Koide Parameter

$$Q_{\text{Koide}} = \frac{\dim(\mathbf{G}_2)}{b_2} = \frac{14}{21} = \frac{2}{3}$$

Experimental: 0.666661 ± 0.000007 . Deviation: **0.0009%**.

3.3.2 Tau-Electron Mass Ratio

$$\frac{m_\tau}{m_e} = \dim(K_7) + 10 \cdot \dim(\mathbf{E}_8) + 10 \cdot H^* = 7 + 2480 + 990 = 3477$$

Experimental: 3477.15 ± 0.05 . Deviation: **0.0043%**.

3.3.3 Muon-Electron Mass Ratio

$$\frac{m_\mu}{m_e} = \dim(J_3(\mathbb{O}))^\phi = 27^\phi = 207.01$$

where $\phi = (1 + \sqrt{5})/2$. Experimental: 206.768. Deviation: **0.118%**.

3.4 Quark Sector

$$\frac{m_s}{m_d} = p_2^2 \times \text{Weyl} = 4 \times 5 = 20$$

Experimental (PDG 2024): 20.0 ± 1.0 . Deviation: **0.00%**.

3.5 Neutrino Sector

3.5.1 CP-Violation Phase

$$\delta_{CP} = \dim(K_7) \times \dim(\mathbf{G}_2) + H^* = 7 \times 14 + 99 = 197^\circ$$

Experimental status: The T2K+NOvA joint analysis (Nature, 2025) [25] reports δ_{CP} consistent with values in the range $\sim 180^\circ$ – 220° depending on mass ordering assumptions, with best-fit regions compatible with 197° within uncertainties. This represents agreement, not exact confirmation; DUNE will provide definitive measurement.

3.5.2 Mixing Angles

Angle	Formula	GIFT	NuFIT 6.0 [26]	Dev.
θ_{12}	$\arctan \sqrt{\delta/\gamma_{\text{GIFT}}}$	33.40°	$33.41^\circ \pm 0.75^\circ$	0.03%
θ_{13}	π/b_2	8.57°	$8.54^\circ \pm 0.12^\circ$	0.37%
θ_{23}	$(\text{rank}(\mathbf{E}_8) + b_3)/H^* \text{ rad}$	49.19°	$49.3^\circ \pm 1.0^\circ$	0.22%

3.6 Higgs Sector

$$\lambda_H = \frac{\sqrt{\dim(G_2) + N_{\text{gen}}}}{2^{\text{Weyl}}} = \frac{\sqrt{17}}{32} = 0.1289$$

Experimental: 0.129 ± 0.003 . Deviation: **0.12%**.

3.7 Cosmological Observables

3.7.1 Dark Energy Density

$$\Omega_{DE} = \ln(2) \cdot \frac{b_2 + b_3}{H^*} = \ln(2) \cdot \frac{98}{99} = 0.6861$$

Experimental (Planck 2020): 0.6847 ± 0.0073 . Deviation: **0.21%**.

3.7.2 Scalar Spectral Index

$$n_s = \frac{\zeta(11)}{\zeta(5)} = 0.9649$$

Experimental: 0.9649 ± 0.0042 . Deviation: **0.004%**.

3.7.3 Fine Structure Constant

$$\alpha^{-1} = \frac{\dim(E_8) + \text{rank}(E_8)}{2} + \frac{H^*}{11} + \det(g) \cdot \kappa_T = 128 + 9 + \frac{65}{1952} = 137.033$$

This formula yields $\alpha^{-1} \approx 137.033$. The experimental value $\alpha^{-1} = 137.035999\dots$ [27] differs by **0.002%**. Note: this comparison involves subtleties regarding renormalization scale; the GIFT value should be understood as a topological target rather than a prediction at a specific energy.

3.8 Summary: 18 Derived Relations

#	Relation	Formula	Value	Exp.	Dev.
1	N_{gen}	Index constraint	3	3	exact
2	τ	$496 \times 21 / (27 \times 99)$	3472/891	—	—
3	κ_T	$1 / (77 - 14 - 2)$	1/61	—	—
4	$\det(g)$	$5 \times 13 / 32$	65/32	—	—
5	$\sin^2 \theta_W$	21/91	3/13	0.23122	0.195%
6	α_s	$\sqrt{2}/12$	0.11785	0.1179	0.04%
7	Q_{Koide}	14/21	2/3	0.666661	0.0009%
8	m_τ/m_e	$7 + 2480 + 990$	3477	3477.15	0.004%
9	m_μ/m_e	27^ϕ	207.01	206.768	0.12%
10	m_s/m_d	4×5	20	20.0	0.00%
11	δ_{CP}	$7 \times 14 + 99$	197°	$\sim 197^\circ$	compat.
12	θ_{13}	$\pi/21$	8.57°	8.54°	0.37%
13	θ_{23}	$85/99 \text{ rad}$	49.19°	49.3°	0.22%
14	θ_{12}	$\arctan \sqrt{\delta/\gamma}$	33.40°	33.41°	0.03%
15	λ_H	$\sqrt{17}/32$	0.1289	0.129	0.12%
16	Ω_{DE}	$\ln(2) \times 98/99$	0.6861	0.6847	0.21%
17	n_s	$\zeta(11)/\zeta(5)$	0.9649	0.9649	0.004%
18	α^{-1}	$128 + 9 + \text{corr}$	137.033	137.036	0.002%

Mean deviation: 0.087%

4 Formal Verification and Statistical Analysis

4.1 Lean 4 Verification

The arithmetic relations are formalized in Lean 4 [28] with Mathlib [29]:

Category	Count
Verified theorems	180+
Unproven (sorry)	0
Custom axioms	0

Example:

```
theorem weinberg_relation :
  b2 * 13 = 3 * (b3 + dim_G2) := by native_decide

theorem koide_relation :
  dim_G2 * 3 = b2 * 2 := by native_decide
```

4.2 Scope of Formal Verification

What is proven: Arithmetic identities relating topological integers. Given $b_2 = 21$, $b_3 = 77$, $\dim(G_2) = 14$, etc., the numerical relations ($21/91 = 3/13$, $14/21 = 2/3$, etc.) are machine-verified.

What is not proven:

- Existence of K_7 with the specified topology
- Physical interpretation of these ratios as Standard Model parameters
- Uniqueness of the formula assignments

The verification establishes **internal consistency**, not physical truth.

4.3 Statistical Uniqueness

Question: Is $(b_2, b_3) = (21, 77)$ special, or could many configurations achieve similar precision?

Method: Grid search over 19,100 configurations with $b_2 \in [1, 100]$, $b_3 \in [10, 200]$.

Metric	Value
GIFT rank	#1 of 19,100
Grid search metric*	0.23%
Second-best (21, 76)	0.50%
Improvement factor	$2.2\times$
LEE-corrected significance	$> 4\sigma$

*Note: The grid search uses a simplified metric over a subset of observables for computational efficiency. The full 18-prediction mean deviation (0.087%) reported elsewhere uses all observables with experimental uncertainties.

The configuration (21, 77) occupies a **sharp minimum**: adjacent values perform significantly worse.

5 Falsifiable Predictions**5.1 The δ_{CP} Test**

- **GIFT prediction:** $\delta_{CP} = 197^\circ$
- **Current data:** T2K+NOvA joint analysis consistent with $\sim 197^\circ$ within uncertainties [25]
- **DUNE sensitivity:** Resolution of a few degrees to $\sim 15^\circ$ depending on exposure and true δ_{CP} value [30, 31]

Falsification criterion: If DUNE measures δ_{CP} outside $[182^\circ, 212^\circ]$ at 3σ , the framework is refuted.

5.2 Fourth Generation

The derivation $N_{\text{gen}} = 3$ admits no flexibility. Discovery of a fourth-generation fermion would immediately falsify the framework.

5.3 Experimental Timeline

Experiment	Observable	Timeline	Test Level
DUNE Phase I	δ_{CP} (3σ)	2028–2030	Critical
DUNE Phase II	δ_{CP} (5σ)	2030–2040	Definitive
Lattice QCD	m_s/m_d	2028–2030	Strong
FCC-ee	$\sin^2 \theta_W$	2040s	Definitive

6 Discussion

6.1 Relation to M-Theory

The $E_8 \times E_8$ structure and G_2 holonomy connect to M-theory [32, 33]:

- Heterotic string theory requires $E_8 \times E_8$ for anomaly cancellation [18]
- M-theory on G_2 manifolds preserves $N = 1$ SUSY in 4D [34]

GIFT differs from standard M-theory phenomenology [35] by focusing on topological invariants rather than moduli stabilization.

6.2 Comparison with Other Approaches

Criterion	GIFT	String Landscape	Lisi E_8
Falsifiable	Yes	No	No
Adjustable parameters	0	$\sim 10^{500}$	0
Formal verification	Yes	No	No

Distler-Garibaldi obstruction [36]: Lisi’s E_8 theory attempted direct particle embedding, which is impossible. GIFT uses $E_8 \times E_8$ as algebraic scaffolding; particles emerge from cohomology, not representation decomposition.

6.3 Limitations and Open Questions

Issue	Status
K_7 existence proof	Hypothesized, not constructed
Singularity structure	Required but unspecified
$E_8 \times E_8$ selection principle	Input assumption
Formula selection rules	Empirically motivated
Quantum gravity completion	Not addressed

We do not claim to have solved these problems. The framework’s value lies in producing falsifiable predictions from stated assumptions.

6.4 Numerology Concerns

Integer arithmetic yielding physical constants invites skepticism. Our responses:

1. **Falsifiability:** If DUNE measures $\delta_{CP} \notin [182^\circ, 212^\circ]$, the framework fails regardless of arithmetic elegance.
2. **Statistical analysis:** The configuration (21, 77) is the unique optimum among 19,100 tested, not an arbitrary choice.
3. **Epistemic humility:** We present this as exploration, not established physics. Only experiment decides.

7 Conclusion

7.1 Summary

We have explored a framework deriving 18 dimensionless Standard Model parameters from topological invariants of a hypothesized G_2 manifold K_7 with $E_8 \times E_8$ gauge structure:

- **18 derived relations** with mean deviation 0.087%
- **Formal verification** of arithmetic consistency (180+ Lean 4 theorems)
- **Statistical uniqueness** of $(b_2, b_3) = (21, 77)$ at $> 4\sigma$
- **Falsifiable prediction** $\delta_{CP} = 197^\circ$, testable by DUNE

7.2 Epistemic Status

We do not claim this framework is correct. It may represent:

- (a) Genuine geometric insight
- (b) Effective approximation
- (c) Elaborate coincidence

Only experiment—particularly DUNE—can discriminate.

7.3 Invitation for Scrutiny

We invite critical examination. The purpose of publication is peer review and error identification, not truth claims. If falsified, we learn what nature is not. If confirmed, deeper investigation is warranted.

The ultimate arbiter is experiment.

Acknowledgments

The mathematical foundations draw on Joyce, Kovalev, Haskins, Nordström, and collaborators on G_2 geometry. Harvey and Lawson’s calibrated geometry provides the standard G_2 form. Lean 4 verification uses Mathlib. Experimental data from PDG, NuFIT, T2K, NOvA, Planck, and DUNE collaborations.

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Data Availability

- Paper and data: <https://doi.org/10.5281/zenodo.17979433>
- Code: <https://github.com/gift-framework/GIFT>
- Lean proofs: <https://github.com/gift-framework/core/tree/main/Lean>

Competing Interests

The author declares no competing interests.

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Appendix A: Topological Input Constants

Symbol	Definition	Value
$\dim(E_8)$	Lie algebra dimension	248
$\text{rank}(E_8)$	Cartan subalgebra dimension	8
$\dim(K_7)$	Manifold dimension	7
$b_2(K_7)$	Second Betti number	21
$b_3(K_7)$	Third Betti number	77
$\dim(G_2)$	Holonomy group dimension	14
$\dim(J_3(\mathbb{O}))$	Jordan algebra dimension	27

Appendix B: Derived Structural Constants

Symbol	Formula	Value
p_2	$\dim(G_2)/\dim(K_7)$	2
Weyl	From $ W(E_8) $ factorization	5
H^*	$b_2 + b_3 + 1$	99
τ	$(496 \times 21)/(27 \times 99)$	3472/891
κ_T	$1/(b_3 - \dim G_2 - p_2)$	1/61
$\det(g)$	$(5 \times 13)/32$	65/32