$ Supplement Note 13

$1: **Lorentz Contraction in Special Relativity**

Problem 1 *A* moving frame *L' is moving with a constant velocity v in the x'-direction relative to the* lab frame *L. A rod of length l_0 is at rest and lying on the x-axis of the frame L'. Calculate its length as observed in the lab frame L.*

Let x'_1, x'_2 be the x'-coordinates of the two ends of the rod as it rests in the L' frame. The length of the rod is then given by $l_0 \equiv x'_2 - x'_1$. From Lorentz Transformation (Eq.(6.24) in the Lecture Note):

$$
\begin{aligned}
t' &= \frac{t - \frac{v}{c^2}\, x}{\sqrt{1 - \frac{v^2}{c^2}}} \\
x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\
y' &= y \\
z' &= z
\end{aligned}
$$

the relevant coordinates of the rod in the lab frame L may be obtained as

$$
x'_1 = \frac{x_1 - vt_1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad x'_2 = \frac{x_2 - vt_2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}
$$

The length of the rod in the lab frame L is given by $l \equiv x_2 - x_1$ *together with the condition* $t_2 = t_1$. It follows from (1) that

$$
x'_2 - x'_1 = \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \Rightarrow \quad l \equiv x_2 - x_1 = l_0 \sqrt{1 - \frac{v^2}{c^2}}
$$

In other words, as seen from the lab frame, the length of the moving rod becomes shorter. This is referred as the *Lorentz contraction* or *Lorentz-Fitzgerald contraction*.

Note that if the rod is orthogonal to the moving direction, there is no contraction.

$2: **Time Dilation**

Problem 2 *An elementary particle is at rest in the* moving frame L' *which is moving relative to the* lab frame L *with a constant velocity v in the x-direction. The* intrinsic life time of the particle $\tau_0 \equiv t'_2 - t'_1$ *is the difference between t'_1, the time it was produced, and t'_2, the time it decays, in the frame L'. Calculate its life time τ as observed in the lab frame L.*

The inverse of Lorentz transform Eq.(6.24) is given by(which is equivalent to replacing v by $-v$!):

$$t = \frac{t' + \frac{v}{c^2}\, x}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2a}$$

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2b}$$

The corresponding times in the L frame may then be obtained from (2b)

$$t_1 = \frac{t'_1 + \frac{v}{c^2}\, x'_1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad ;\text{and} \quad t_2 = \frac{t'_2 + \frac{v}{c^2}\, x'_2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

By definition, the position of the particle in L' is the same: $x'_2 = x'_1$. This leads to the life time τ in the lab frame L:

$$\tau \equiv t_2 - t_1 = \frac{t'_2 - t'_1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\tau_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is *time dilation* from special relativity.