$ Supplement Note 3: A proof of Pythagorean Theorem

Let a, b, c be the lengths of the sides of a rectangular triangle, with a representing the length of the *hypotenuse*, i.e. the longest side opposite to the right angle. The Pythagorean theorem states that

$$c^2 = a^2 + c^2 \tag{1}$$

In Figure 1, the sum of the shaded area is equal to the square with the length of each side equal to c. Thus we have

$$(a + b)^2 + (a - b)^2 = 2\,c^2 \tag{2}$$

Equation (1) then follows from (2) by using the identities

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

This proof was presented by an ancient Chinese. The plot is obtained by using the pythonprogram *PlotPythagorean.py*.

```
# PlotPythagorean.py :plot graph for Pythagorean Theorem
import numpy as np
import matplotlib.pyplot as plt
plt.hlines(-4,-4,4)
plt.hlines(4,-4,4)
plt.vlines(4,-4,4)
plt.vlines(-4,-4,4)
plt.hlines(-1,-4,1)
plt.hlines(1,-1,4)
plt.vlines(-1,-1,4)
plt.vlines(1,-4,1)
```

```
plt.arrow(-3.5,4.2,-0.5,0, length_includes_head = True, head_width= 0.1)
plt.arrow(-1.5,4.2,0.5,0, length_includes_head = True, head_width= 0.1)
plt.arrow(-0.5,4.2,-0.5,0, length_includes_head = True, head_width= 0.1)
plt.arrow(3.5,4.2,0.5,0, length_includes_head = True, head_width= 0.1)
plt.hlines(4.2,-4,4)
plt.vlines(-4,4.1,4.3)
plt.text(4.1,2.5,'a')
plt.text(4.1,-1.5,'b')
plt.vlines(4,4.1,4.3)
plt.vlines(-4,4.1,4.3)
#plt.hlines(4.5,-1,4)
plt.vlines(4.1,-4,4)
plt.hlines(4,4,4.2)
plt.hlines(1,4,4.2)
plt.hlines(-4,4,4.2)
plt.arrow(-0.5,1,-0.5,0, length_includes_head = True, head_width= 0.2)
plt.arrow(0.5,1,0.5,0, length_includes_head = True, head_width= 0.2)
plt.arrow(1.5,1,-0.5,0, length_includes_head = True, head_width= 0.2)
plt.arrow(3.5,1,0.5,0, length_includes_head = True, head_width= 0.2)
plt.text(-0.2,1.1,'a - b')
plt.text(2.3,1.1,'a')
x1,y1 = -4, -1
x2,y2 = 1, -4
x3,y3 = 4, 1
x4,y4 = -1, 4
plt.plot([x1,x2], [y1,y2])
plt.plot([x2,x3], [y2,y3])
plt.plot([x3,x4], [y3,y4])
plt.plot([x1,x4], [y1,y4])
plt.fill_between([x1,x4],[y1,y4],4)
plt.fill_between([x1,x2], [y1,y2], -4)
plt.fill_between([x2,x3], [y2,y3], -4)
plt.fill_between([x3,x4], [y3,y4], 4)
```

```
plt.fill_between([x4,x2], y1,y3)
plt.text(1,2.3,'c')
plt.text(1.5,4.1,'b')
plt.text(-3,4.1,'a')
plt.axis('off')
plt.legend()
fig = plt.gcf()
fig.savefig('Pythagorean.eps', format='eps')
plt.show()
```
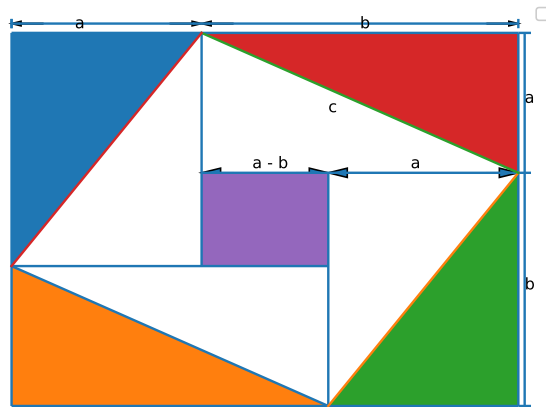
Figure 1: Pythagorean Theorem