

Manipolatore di Stanford

$$\tau = K_p e + K_d \dot{e} + G(q) \quad (1)$$

$$K_p = \begin{bmatrix} 1000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \quad (2)$$

$$K_d = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \quad (3)$$

$$\tau = B(q)[\ddot{q}_d + K_p e + K_d \dot{e}] + C(q, \dot{q})\dot{q} + G(q) \quad (4)$$

$$K_p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$K_d = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$\tau = Y\hat{\pi} + K_d \dot{e} + K_p e \quad (7)$$

$$u_\pi = R^{-1}Y^T M^{-T} B^T P x \quad (8)$$

$$K_p = \begin{bmatrix} 1000 & 00 & 0 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \quad (9)$$

$$K_d = \begin{bmatrix} 100 & 00 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \quad (10)$$

$$R = I_6 \quad (11)$$

$$A = \begin{bmatrix} 0_6 & I_6 \\ -K_p & -K_d \end{bmatrix} \quad (12)$$

$$B = \begin{bmatrix} 0_6 \\ I_6 \end{bmatrix} \quad (13)$$

$$Q = I_{12} \quad (14)$$

$$A^T P + P A = -Q \quad (15)$$

Gantry crane

$$\ddot{\theta} D + [\ddot{x} l \cos \theta + g l \sin \theta] B = -b_1 \dot{\theta} \quad (16)$$

$$\ddot{x} C + [\ddot{x} l \cos \theta + g l \sin \theta] B = F - b_2 \dot{x} \quad (17)$$

$$B = m_1 + \frac{1}{3} m_2 \quad (18)$$

$$C = m_1 + m_2 + M \quad (19)$$

$$D = \frac{2}{5} m_1 r^2 \quad (20)$$

$$\mathbf{q} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} \quad (21)$$

$$\begin{cases} \dot{q}_0 = q_1 \\ \dot{q}_1 = \frac{-b_1 q_1}{D} - \frac{g l \sin(q_0) B}{D} + \frac{(b_2 q_3 + g l \sin(q_0) B)(l \cos(q_0) B)}{(C + l \cos(q_0) B) D} - \frac{F l \cos(q_0) B}{(C + l \cos(q_0) B) D} \\ \dot{q}_2 = q_3 \\ \dot{q}_3 = \frac{(-b_2 q_3 - g l \sin(q_0) B)}{C + l \cos(q_0) B} + \frac{F}{C + l \cos(q_0) B} \end{cases}$$

$$\dot{q} = f(q) + g(q)u \quad (22)$$

$$y = h(q) \quad (23)$$

$$\mathbf{f}(\mathbf{q}) = \begin{bmatrix} \frac{-b_1 q_1}{D} - \frac{g l \sin(q_0) B}{D} + \frac{q_1}{(C + l \cos(q_0) B) D} \\ \frac{q_3}{C + l \cos(q_0) B} \end{bmatrix} \quad (24)$$

$$\mathbf{g}(\mathbf{q}) = \begin{bmatrix} 0 \\ -\frac{F l \cos(q_0) B}{(C + l \cos(q_0) B) D} \\ 0 \\ \frac{F}{C + l \cos(q_0) B} \end{bmatrix} \quad (25)$$

$$\mathbf{h}(\mathbf{q}) = q_0 = \theta \quad (26)$$

$$\mathbf{h}(\mathbf{q}) = q_2 = x \quad (27)$$

$$\mathbf{h}(\mathbf{q}) = q_2 + l \sin(q_0) \quad (28)$$

$$\Delta = \begin{bmatrix} 0 & * & * & * & * \\ * & * & * & * & * \\ 0 & * & * & * & * \\ * & * & * & * & * \end{bmatrix} \quad (29)$$

$$\mathbf{dO} = \begin{bmatrix} * & 0 & 1 & 0 \\ * & * & 0 & 1 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & 0 & 0 \\ * & 0 & 0 & 0 \end{bmatrix} \quad (30)$$

$$\mathbf{dO} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ * & 0 & 0 & * \\ * & 0 & 0 & 0 \end{bmatrix} \quad (31)$$

$$\mathbf{dO} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ * & * & 0 & * \\ * & 0 & 0 & 0 \end{bmatrix} \quad (32)$$

$$\mathbf{z} = \begin{bmatrix} h \\ L_f h \\ q_0 \\ \frac{-1}{l \cos q_0 B} q_1 + q_3 \end{bmatrix} \quad (33)$$

$$\dot{\mathbf{z}} = \begin{bmatrix} L_f h \\ L_f^2 h + L_g L_f h u \\ \dot{q}_0 \\ \frac{\partial z_3}{\partial q}(f + gu) \end{bmatrix} \quad (34)$$

$$\mathbf{u} = \frac{v + L_f^2 h}{L_g L_f h} \quad (35)$$

Zero Dinamica

$$\eta_1 = \frac{l B l \eta_2 \cos \eta_1}{B l^2 \cos^2 \eta_1 - 1} \quad (36)$$

$$\begin{aligned} \dot{\eta}_2 = & \frac{F}{D(C + B l \cos \eta_1)} - \frac{B g l \sin \eta_1 + \frac{B b_2 l^2 \eta_2 \cos^2 \eta_1}{B l^2 \cos^2 \eta_1 - 1}}{C + B l \cos \eta_1} \\ & + \frac{\frac{B g l \sin \eta_1}{D} + \frac{B l F \cos \eta_1}{D(C + B l \cos \eta_1)} + \frac{B l \cos \eta_1 (B g l \sin \eta_1 + \frac{B b_2 l^2 \eta_2 \cos^2 \eta_1}{B l^2 \cos^2 \eta_1 - 1})}{D(C + B l \cos \eta_1)} + \frac{B b_1 l \eta_2 \cos \eta_1}{D(B l^2 \cos^2 \eta_1 - 1)}}{B l \cos \eta_1} \\ & - \frac{B l \eta_2^2 \sin \eta_1}{(B l^2 \cos^2 \eta_1 - 1)^2} \end{aligned}$$

$$Q = \begin{bmatrix} 0 & \frac{B l}{B l^2 - 1} \\ \frac{\frac{B g l}{D} + \frac{B^2 g l^2}{D(C + B l)}}{B l} - \frac{B g l}{C + B l} & \frac{\frac{B b_1 l}{D(B l^2 - 1)} + \frac{B^2 b_2 l^3}{D(C + B l)(B l^2 - 1)}}{B l} - \frac{B b_2 l^2}{(C + B l)(B l^2 - 1)} \end{bmatrix} \quad (37)$$