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6 DOF REACTION WHEEL PENDULUM

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Abstract. *In this paper, we present a different kind of reaction wheel pendulum. The main novelty is that it's articulation point consists of a ball joint that allows the pendulum to rotate around it's three axes. Furthermore, three reaction wheels are used for it's control and stabilization. First, a model of the system is obtained from Euler-Lagrange equations. After that, a control law that assure asymptotic stabilization of the system in a large domain is proposed. This system has several interesting features, both from a pedagogical standpoint as from a research standpoint. From a pedagogical standpoint, the system is non-linear and it's inputs are tightly coupled. From a research standpoint, two non-linear control techniques are considered: standard linearization and Flatness based control (exact linearization). These two different control laws are combined for obtaining a sufficiently large domain of stability. Simulation results are presented.*

Keywords: *Inverted pendulum, Reaction wheel, Flatness based control, Exact linearization and Euler-Lagrange*

1. INTRODUCTION

Bipedal robots are often modelled based on various versions of inverted pendulum. These simplified models have been very useful in the analysis and control of humanoid dynamics (Pons, 2008). However, the inverted pendulum models in the literature represent the whole body as a mass point and only concentrate into the center of pressure and the center of mass. By doing this it does not take into account the moment of inertia from the arms and trunk, a key component for the humanoid balance (Goswami, 2008). In order to study a similar situation, a different kind of inverted pendulum will be analyzed.

Inverted pendulums are commonly used as a way to explore non-linear mechatronic systems with fast dynamics. It is particularly suitable for the learning process in the control area in order to design and implement several of the pendulum's stabilization techniques. There are many possible schemes of a inverted pendulum system, each one with a different mathematical model. An inverted pendulum controlled by a reaction wheel is perhaps one of the simplest types of inverted pendulum in terms of their dynamic properties, and hence their controllability properties. At the same time, this kind of pendulum has several properties, such as sub-actuation and non-linearity, which make it an attractive and useful system for research and advanced training (Block *et al.*, 2007).

Firtly, the dynamics of the reaction wheels will be disregarded. For the rest of the system (the pendulum itself) exact linearization will be applied in a special way. The main issue is to assure that the system obtained with exact linearization will coincide with the system obtained with standard linearization. This assure that the non-linear state feedback from the exact linearization will virtually do nothing when the system is close to it's equilibrium point (in this case, the linear controller that stabilize the exact linearized system will coincide with the linear controller that stabilized the standard linearized system). After that a small feedback gain from the reaction wheels speeds will be added, assuring stabilization of the complete system.

2. MATHEMATICAL MODELING

The first step of any control system design is to deduce a mathematical model of the system to be controlled. A tridimensional diagram of the system is presented in "Fig. 1".

At the end of the pendulum (point O) there is a ball joint that provides three degrees of freedom to the system. At the opposite end (point P) there are three reaction wheels, which one providing another one d.o.f. and so totalizing six degrees of freedom in the system.

The position and orientation of the pendulum is described by three generalized coordinates ϕ, θ and ψ that correlates the mobile coordinate system xyz of the pendulum with the global coordinate system XYZ. The pendulum has mass m and moments of inertia J_x, J_y and J_z[1]. It's center of mass G is within a distance z_G from the ball joint, where, due to the

[1]The mass and moments of inertia of the pendulum take into account the reaction wheels.

gravitational acceleration g, the weight force is originated always pointing downwards (Z axis) and with module mg.

The reaction wheels are driven by electric motors whose axes are orthogonal to each other. Torques responsible for controlling the position and orientation of the pendulum arises when there are variations in the speeds of the reaction wheels. These torques are proportional to the moment of inertia J_r of the reaction wheels and the angular accelerations $\ddot{\alpha}$, $\ddot{\beta}$ and $\ddot{\gamma}$ of each wheel. The reaction wheels have a counterweight to keep the pendulum symmetrical and stable around it's equilibrium point.

Thus, the system inputs are the three torques τ_x, τ_y and τ_z, the states are all the generalized coordinates ϕ, θ, ψ, α, β and γ and their time derivatives $\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$, $\dot{\alpha}$, $\dot{\beta}$ and $\dot{\gamma}$ and the outputs are only the generalized coordinates ϕ, θ and ψ that describes the position and orientation of the pendulum.

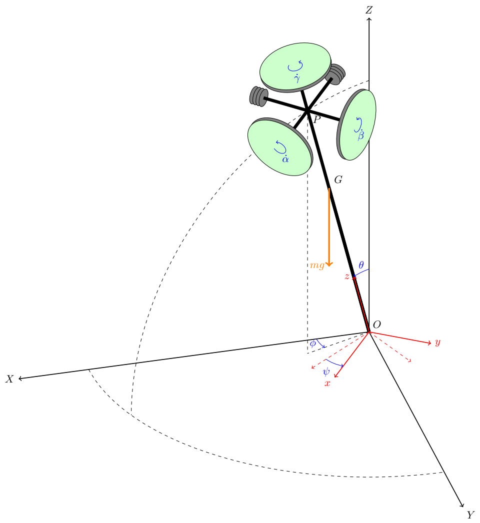
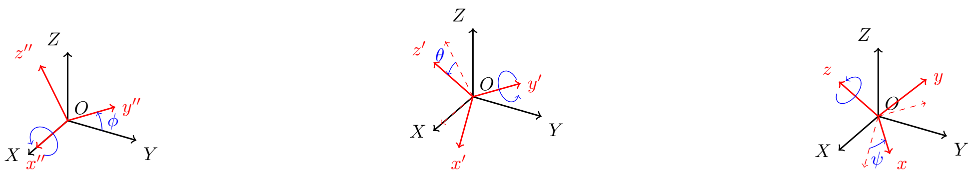


Figure 1: Tridimensional diagram of the system

The three generalized coordinates that describes the position and orientation of the pendulum are given by the Euler angles (Meirovitch, 2003). There is no agreement on the notation used by the Euler angles, the notation used in this paper is the $x-y-z$ and it can be seen in "Fig. 2". It was chosen to avoid singularity problems around the equilibrium point of the pendulum.



(a) Rotation around the X axis by ϕ (b) Rotation around the y'' axis by θ (c) Rotation around the z' axis by ψ

Figure 2: Euler angles notation

The system was then modeled by Euler-Lagrange equations and six second-order ordinary differential equations (non-linear and coupled to each other) that represents the system dynamics were obtained:

$$\begin{cases} J_x\left(\ddot{\phi}cos^2\theta - \dot{\phi}\dot{\theta}sen\left(2\theta\right)\right) + J_z\left(\ddot{\phi}sen^2\theta - \ddot{\psi}sen\theta - \dot{\theta}\dot{\psi}cos\theta + \dot{\phi}\dot{\theta}sen\left(2\theta\right)\right) - mgz_G sen\phi cos\theta = \\ \qquad\qquad -\tau_x cos\theta cos\psi - \tau_y cos\theta sen\psi + \tau_z sen\theta \\ J_x\left(\ddot{\theta} + \dot{\phi}^2 sen\theta cos\theta\right) + J_z\left(\dot{\psi} - \dot{\phi}sen\theta\right)\dot{\phi}cos\theta - mgz_G cos\phi sen\theta = \tau_x sen\psi - \tau_y cos\psi \\ J_z\left(\ddot{\psi} - \ddot{\phi}sen\theta - \dot{\phi}\dot{\theta}cos\theta\right) = -\tau_z \\ J_r\ddot{\alpha} = \tau_x \\ J_r\ddot{\beta} = \tau_y \\ J_r\ddot{\gamma} = \tau_z \end{cases} \quad (1)$$

A generic system of second-order ordinary differential equations with n generalized coordinates can be written as follows:

$$M(q)\ddot{q} + L(q,\dot{q}) + K(q) = J(q)u \quad (2)$$

Where q is the generalized coordinates vector, u is the inputs vector, M is the mass matrix $n \times n$ which is always symmetric and invertible, L is the Coriolis effects and viscous friction[2] matrix, K is the spring and gravity effects matrix and J is the inputs coupling matrix (Craig, 2005). A common way to represent a system of second-order ordinary differential equations is rewriting the equations so that the second derivatives depend only on the first derivatives and the function itself. This means rewriting "Eq. (2)" as:

$$\ddot{q} = -M^{-1}\left[L + K\right] + M^{-1}Ju \tag{3}$$

Initially the reaction wheels dynamics will be disregarded, so the generalized coordinates vector q is only given by ϕ, θ and ϕ, while the input vector u is given by the three torques τ_x, τ_y and τ_z:

$$q = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \qquad u = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{4}$$

With a little algebra the system is now described as follows:

$$\begin{cases} \ddot{\phi} = 2\frac{\dot{\phi}\dot{\theta}sen\theta}{cos\theta} + \frac{J_z}{J_x}\frac{\dot{\theta}\left(\dot{\psi}-\dot{\phi}sen\theta\right)}{\cos\theta} + \frac{mgz_G}{J_x}\frac{sen\phi}{cos\theta} - \frac{1}{J_x}\frac{\tau_x cos\psi + \tau_y sen\psi}{cos\theta} \\ \ddot{\theta} = -\frac{1}{2}\dot{\phi}^2 sen\left(2\theta\right) - \frac{J_z}{J_x}\dot{\phi}cos\theta\left(\dot{\psi} - \dot{\phi}sen\theta\right) + \frac{mgz_G}{J_x}cos\phi sen\theta + \frac{1}{J_x}\left(\tau_x sen\psi - I_y cos\psi\right) \\ \ddot{\psi} = -\frac{\dot{\phi}\dot{\theta}\left(cos^2\theta - 2\right)}{\cos\theta} + \frac{J_z}{J_x}\frac{\dot{\theta}sen\theta\left(\dot{\psi}-\dot{\phi}sen\theta\right)}{\cos\theta} + \frac{mgz_G}{J_x}\frac{sin\phi sin\theta}{cos\theta} - \frac{1}{J_x}\frac{sin\theta(\tau_x cos\psi + \tau_y sin\psi)}{cos\theta} - \frac{1}{J_z}\tau_z \end{cases} \tag{5}$$

3. LINEARIZATION

A system of n second-order ordinary differential equations can also be expressed as a system of $2n$ first-order ordinary differential equations as follows:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \tag{6}$$

Where x is the states vector, u is the inputs vector, y is the outputs vector, $f(x)$ is the system dynamics function vector, $g(x)$ is the inputs coupling function matrix and $h(x)$ is output function vector. All these parameters can be easily obtained by:

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \qquad f = \begin{bmatrix} \dot{q} \\ -M^{-1}\left[L+K\right] \end{bmatrix}, \qquad g = \begin{bmatrix} 0 \\ M^{-1}J \end{bmatrix}, \qquad h = \begin{bmatrix} q \end{bmatrix} \tag{7}$$

Thus, the non-linear system can be represented as the block diagram from "Fig. 3".

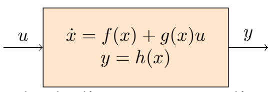


Figure 3: Block diagram - non-linear system

Much of control system design theories were developed for linear systems, but virtually all real systems are non-linear. Control design techniques for non-linear systems are complicated, complex and mainly guarantee stability. However, there are some techniques to linearize such systems. Two fairly common and that will be addressed in this paper are the standard linearization and the exact linearization (Isidori, 1995).

The standard linearization technique consist on finding a simplified and approximated version that best represent the system dynamics around a certain nominal operating point (that must be an equilibrium point). Thus, the first step for applying the standard linearization consists in determining the equilibrium point of the system. In the inverted pendulum case this task is very intuitive, as the goal is to stabilize the pendulum vertically, the nominal operating point of the system is given by:

$$\begin{cases} \phi_0 = \dot{\phi}_0 = \ddot{\phi}_0 = 0 \\ \theta_0 = \dot{\theta}_0 = \ddot{\theta}_0 = 0 \\ \psi_0 = \dot{\psi}_0 = \ddot{\psi}_0 = 0 \end{cases} \qquad \begin{cases} \tau_{x0} = 0 \\ \tau_{y0} = 0 \\ \tau_{z0} = 0 \end{cases} \tag{8}$$

[2] The viscous friction are not being considered in the system model.

The system then becomes represented by:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{9}$$

Where A is the state transmission matrix, B is the inputs matrix and C is the output matrix. All these matrices can be obtained computing the Jacobian matrices for the non-linear functions in the nominal operating point:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{mgz_G}{J_x} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{mgz_G}{J_x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{1}{J_x} & 0 & 0 \\ 0 & -\frac{1}{J_x} & 0 \\ 0 & 0 & -\frac{1}{J_z} \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Analyzing the matrices A, B and C obtained and given that $x = \begin{bmatrix} q & \dot{q} \end{bmatrix}^T$, it is possible to conclude that $\dot{q}$ do not depend of the other states, while $\ddot{q}$ depends of q and u. So another way of representing this same system with standard linearization is as the block diagram from "Fig. 4".

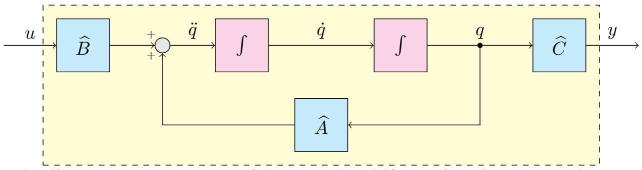


Figure 4: Block diagram - system with standard linearization (another representation)

Where:

$$\widehat{A} = \begin{bmatrix} \frac{mgz_G}{J_x} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{mgz_G}{J_x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \widehat{B} = \begin{bmatrix} -\frac{1}{J_x} & 0 & 0 \\ 0 & -\frac{1}{J_x} & 0 \\ 0 & 0 & -\frac{1}{J_z} \end{bmatrix}, \qquad \widehat{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

The exact linearization technique consists on finding a non-linear state feedback that allows the exact cancellation of non-linearities of the system. Thus, the closed loop system becomes linear at least when it is described in a suitable coordinate system. In order to successfully design a single controller to serve both the system with standard linearization as the system with exact linearization, the idea is that the system after linearized exactly matches the standard linearization. The state feedback that accomplishes this task has the form:

$$u = j(x) + k(x)v \tag{12}$$

Where:

$$j = J^{-1}\left[L + K + M\widehat{A}x\right], \qquad k = J^{-1}M\widehat{B} \tag{13}$$

Thus, the system with exact linearization can be represented as the block diagram from "Fig. 5".

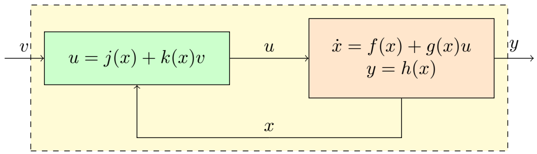


Figure 5: Block diagram - system with exact linearization

The system with the non-linear state feedback then becomes represented by:

$$\begin{cases} \dot{x} = Ax + Bv \\ y = Cx \end{cases} \tag{14}$$

It can be noticed that the system with standard linearization is almost equal to the system with exact linearization, the only difference is that while the input of the first is u the input of the second is v. It can be shown that the non-linear state feedback $u(x,v)$ defined by "Eq. (12)" and "Eq. (13)" has the following properties:

$$\left.\frac{\partial u(x,v)}{\partial x}\right|_{(0,0)} = 0 \tag{15}$$

$$\left.\frac{\partial u(x,v)}{\partial v}\right|_{(0,0)} = I \tag{16}$$

This means that the effect of u and v are very close when the system is not far from the equilibrium point. So for control design the system with standard linearization will be used, while for simulations the system with exact linearization will be used.

4. CONTROLLER DESIGN

A simple method to design a controller for a system in which all states are accessible to measurement is known as state feedback. In a controllable system where all states are accessible, it is possible to allocate the closed loop poles anywhere in the complex plane. This means it is possible, in principle, to completely specify the closed loop system dynamic behaviour (Friedland, 2005).

The first step in designing a state feedback controller is to find out if the system is controllable. In case of the inverted pendulum the system has dimension 6 so the controlable matrix is given by:

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B & A^3B & A^4B & A^5B \end{bmatrix} \tag{17}$$

For any value of m, g, z_G, J_x, J_z e J_r different from zero, the controllable matrix will always have rank 6. Since the rank of the matrix is the same as the system's dimension, the system is said to be controllable.

A state feedback controller consists of a closed loop where the input of the system u is a function of a reference r less the states x multiplied by a gain matrix K:

$$v = -Kx + r \tag{18}$$

Thus, the linearized system with state feedback can be represented as the block diagram from "Fig. 6".

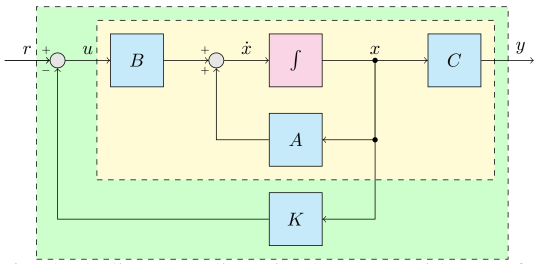


Figure 6: Block diagram - linearized system with state feedback

There are different methods to calculate the gain matrix K. Perhaps the simplest is to choose the gains that allocate the closed loop poles in the desired locations in the complex plane. However, the desired closed loop poles might not always be known. Another method is the optimum control (also know as LQR controller). In the design of an optimal controller, rather than calculating the gains which guarantees desired closed loop poles, it is obtained a gain matrix K that minimizes a performance criterion J (also know as cost function) expressed as the integral of a quadratic term of system states x plus another second quadratic term of system inputs u:

$$J = \int \left[x^T(\tau)Q(\tau)x(\tau) + u^T(\tau)R(\tau)u(\tau) \right] d\tau \tag{19}$$

The simplest case is to assume $Q = C^TC$ and $R = I$, meaning that the cost function places equal importance on the states which are outputs (in this case, the angles ϕ, θ and ψ) and the control efforts (in this case, the torques τ_x, τ_y and τ_z). By doing this the following gain matrix was obtained[3]:

[3]The LQR method is a numerical algorithm, thus numerical values were adopted for the system's parameters: $941.3g$ for m, $9.81m/s^2$ for g, $18.2cm$ for z_G, $2.28 \times 10^{-2} kg \cdot m^2$ for J_x, $2.30 \times 10^{-3} kg \cdot m^2$ for J_z and $2.54 \times 10^{-5} kg \cdot m^2$ for J_r.

$$K = \begin{bmatrix} -3,64 & 0 & 0 & -0,49 & 0 & 0 \\ 0 & -3,64 & 0 & 0 & -0,49 & 0 \\ 0 & 0 & -1,00 & 0 & 0 & -0,07 \end{bmatrix} \tag{20}$$

To validate the controller a few simulations were performed. Assuming that the system reference r is zero, meaning that ϕ_r, θ_r and ψ_r are 0, and that the system is initially at a small offset from it's equilibrium point but without speed, meaning that ϕ_0, θ_0 and ψ_0 are $5°$ while $\dot{\phi}_0$, $\dot{\theta}_0$ and $\dot{\psi}_0$ are $0 rad/s$, the result of this simulation can be observed in "Fig. 7".

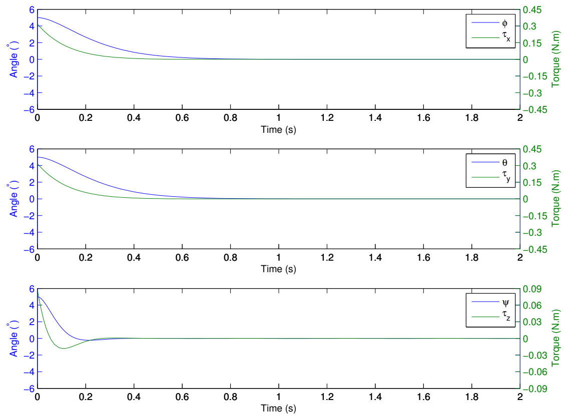


Figure 7: Simulation - closed loop system

As can be seen the system is stable, the response time is low and the control efforts are not high, meaning that the controller is doing a good job. Let's assume now that the ϕ angle's sensor has a small additive error of $1°$, either because it is not perfectly aligned with the pendulum or because the pendulum is not symmetrical. By doing the same simulation the following results arrive:

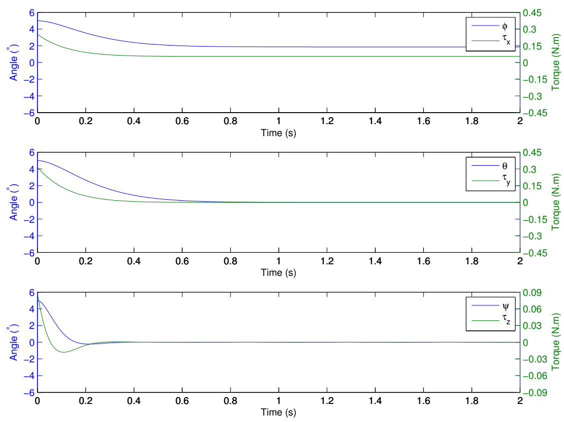


Figure 8: Simulation - closed loop system with additive measurement error

As can be seen in "Fig. 8", the system is still stable, but this time the angle ϕ stabilizes out of it's equilibrium point, and the control effort τ_x never goes to zero. This happens because the controller is trying to stabilize the pendulum in a point that is not the system's equilibrium point, so the reaction wheel corresponding to τ_x will accelerate independently. To solve this problem the reaction wheels will need to be considered again and their speeds $\dot{\alpha}$, $\dot{\beta}$ and $\dot{\gamma}$ will also be feedbacked.

The matrices A, B and C obtained by standard linearization and the gain matrix K obtained by LQR method generates a closed loop system where the channels ϕ, θ and ψ are all decoupled to each other, meaning that the system can also be represented as three individual sub-systems that are not coupled to each other, as can be seen in "Fig. 9"[4].

[4]For simplicity, only the sub-system from channel ϕ is being represented, but the same applies to the channels θ and ψ.

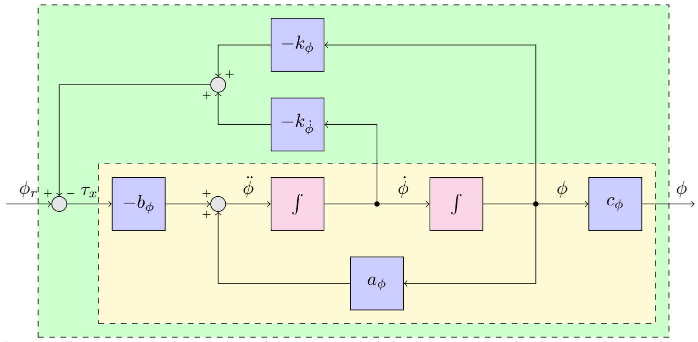


Figure 9: Block diagram - linearized system with decoupled channels and state feedback

The gains a_ϕ, a_θ, a_ψ, b_ϕ, b_θ, b_ψ, c_ϕ, c_ψ and c_ψ are given by the system's matrices $\widehat{A}$, $\widehat{B}$ and $\widehat{C}$:

$$\widehat{A} = \begin{bmatrix} a_\phi & 0 & 0 & 0 & 0 & 0 \\ 0 & a_\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & a_\psi & 0 & 0 & 0 \end{bmatrix}, \qquad \widehat{B} = \begin{bmatrix} -b_\phi & 0 & 0 \\ 0 & -b_\theta & 0 \\ 0 & 0 & -b_\psi \end{bmatrix}, \qquad \widehat{C} = \begin{bmatrix} c_\phi & 0 & 0 \\ 0 & c_\theta & 0 \\ 0 & 0 & c_\psi \end{bmatrix} \tag{21}$$

While the gains k_ϕ, k_θ, k_ψ, $k_{\dot\phi}$, $k_{\dot\theta}$ and $k_{\dot\psi}$ are given by the state feedback gain matrix K:

$$K = \begin{bmatrix} -k_\phi & 0 & 0 & -k_{\dot\phi} & 0 & 0 \\ 0 & -k_\theta & 0 & 0 & -k_{\dot\theta} & 0 \\ 0 & 0 & -k_\psi & 0 & 0 & -k_{\dot\psi} \end{bmatrix} \tag{22}$$

Since the reaction wheels are also decoupled to each other, their speeds will be feedbacked into which one of the sub-systems, as shown in “Fig. 10”.

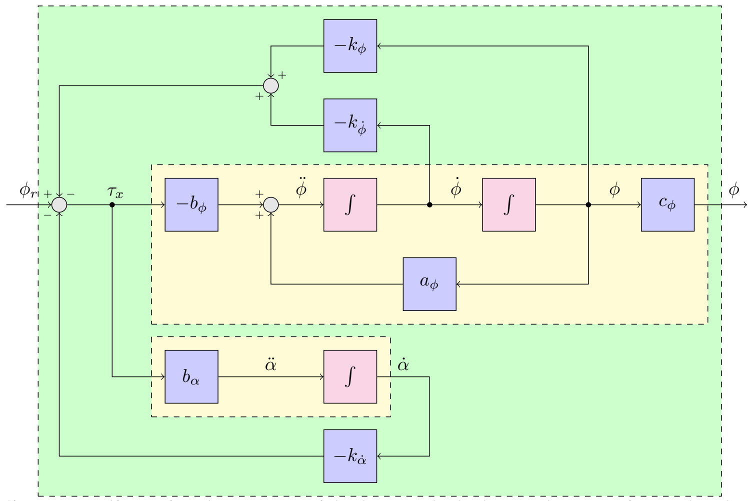


Figure 10: Block diagram - linearized system with decoupled channels, reaction wheels and state feedback

The gains b_α, b_β and b_γ are given by the system diferential equations:

$$b_\alpha = \frac{1}{J_r}, \qquad b_\beta = \frac{1}{J_r}, \qquad b_\gamma = \frac{1}{J_r} \tag{23}$$

While the gains k_α, k_β and k_γ need to be determined. One way to determine these gains would be applying again the LQR method, but now for the complete system (considering the dynamics of the reaction wheels), and thereby obtaining a new gain matrix K. However, our intention is to introduce an small additice feedback gain of the reaction wheels speed of each axis, thus the gains already determined will be kept the same while the news gains $k_{\dot\alpha}$, $k_{\dot\beta}$ and $k_{\dot\gamma}$ will be determined from Routh-Hurwitz criteria only to keep the system stable.

By applying the Routh-Hurwitz criteria into the system the following conditions need to be satisfied for the system to be kept stable:

$$k_{\dot{\alpha}} \leq \frac{k_{\dot{\phi}}\left(a_{\phi} + b_{\phi}k_{\phi}\right)}{b_{\alpha}k_{\phi}}, \qquad k_{\dot{\beta}} \leq \frac{k_{\dot{\theta}}\left(a_{\theta} + b_{\theta}k_{\theta}\right)}{b_{\beta}k_{\theta}}, \qquad k_{\dot{\gamma}} \leq \frac{k_{\dot{\psi}}\left(a_{\psi} + b_{\psi}k_{\psi}\right)}{b_{\gamma}k_{\psi}} \tag{24}$$

Adopting values for the gains $k_{\dot{\alpha}}$, $k_{\dot{\beta}}$ and $k_{\dot{\gamma}}$ that satisfies this condition with a good margin, the result from a new simulation can be seen in “Fig. 11”.

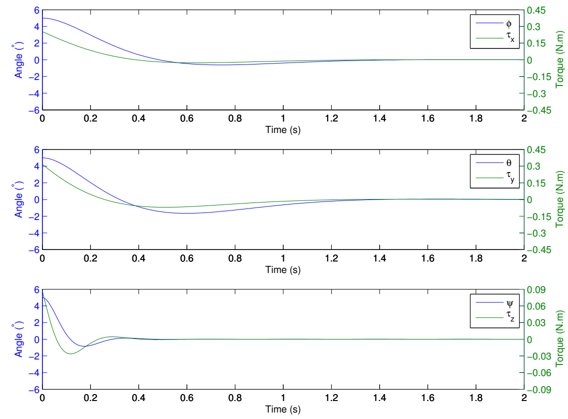


Figure 11: Simulation - closed loop system with reaction wheels and additive measurement error

As can be seen, the dynamics of the closed loop system changed a bit. The response time is a little higher and a small overshoot that was not present now appear. However, the angle ϕ now stabilizes on it’s equilibrium point and the control effort τ_x goes to zero, meaning that the problem with the additive measurement error have been solved.

5. CONCLUSIONS AND FUTURE WORKS

A different kind of reaction wheel pendulum was presented and modelled through Euler-Lagrange equations. The system was then linearized under two different linearization techniques: standard linearization and exact linearization. After that a state feedback controller was design based on the concepts of an optimum controller (LQR) and the system was tested and validated with simulations. By taking into consideration an additive measurement error, a few adjustment were needed on the controller. It was possible to conclude that when the system is close to it’s equilibrium point the non-linear state feedback from the exact linearization virtually do nothing, meaning that a controller can be developed for a standard linearized system but implemented into a exact linearized system.

As for future works, some improvements are in scope. Firstly, the dynamics of the electric motors that drive the reaction wheels must be considered in the system. Although it is expected for the dynamics of the motors to be very fast if compared to the dynamics of the pendulum, they must be considered and the controller may need to be adjusted. Secondly, a state observer will be designed and implemented into the controller. Although all states of the system can be easily measured through an IMU[5] nowadays, an state observer will be designed to operate as a Kalman filter. This way not only additive measurement errors would be compensated by the controller as well as random measurement errors.

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7. RESPONSIBILITY NOTICE

[5] IMU means Inertial Measurement Unit, an electronic device mainly compose by accelerometers, gyroscopes and magnetometers.