Stellar Migration and Chemical Enrichment in the Milky Way Disc: The Impact on Enrichment from Asymptotic Giant Branch Stars

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CCSN and AGB Star Yields of Nitrogen

Core-Collapse Supernova Yields

Empirically, nitrogen-to-oxygen ratios exhibit a plateau at log(N/O) $\approx$ -1.5 for log(O/H) $\lesssim 8$ (see Fig. 1 of Vincenzo et al., 2016 comparing Berg et al., 2012, Izotov, Thuan & Guseva, 2012, and James et al., 2015 measurements).

What is the implied relation between the IMF integrated CCSN yields of nitrogen and oxygen?

The ratio of their yields can be related to the number densities of the two nuclei in the supernova ejecta via:

$$\frac{y_{\text{N}}^{\text{CC}}}{y_{\text{O}}^{\text{CC}}} = \frac{\mu_{\text{N}} n_{\text{N}}}{\mu_{\text{O}} n_{\text{O}}} \tag{1}$$

where μ_x is the mean molecular weight of a species x and n_x is the number of nuclei. Taking the ratio $n_{\text{N}}/n_{\text{O}}$ from these observed results yields:

$$\frac{y_{\text{N}}^{\text{CC}}}{y_{\text{O}}^{\text{CC}}} = \frac{\mu_{\text{N}}}{\mu_{\text{O}}} 10^{\log(\text{N/O})} \tag{2}$$

Though supernova ejecta may produce different isotopic ratios of N than AGB stars, potentially altering the ratio $\mu_{\text{N}}/\mu_{\text{O}}$, taking $\mu_{\text{N}} = 14.007$ and $\mu_{\text{O}} = 15.999$ from a periodic table suggests that, with the previously adopted $y_{\text{O}}^{\text{CC}} = 0.015$ (e.g. Weinberg, Andrews & Freudenburg, 2017; Johnson & Weinberg, 2020; Johnson et al., 2021), this suggests

$$y_{\text{N}}^{\text{CC}} = \frac{\mu_{\text{N}}}{\mu_{\text{O}}} 10^{\log(\text{N/O})} y_{\text{O}}^{\text{CC}} = \frac{14.007}{15.999} 10^{-1.5}(0.015) \approx 4.15 \times 10^{-4} \tag{3}$$

Can this be understood from theoretically predicted yields?

The left panel of Figure 1 presents the IMF-integrated yields y_{N}^{CC} computing with VICE using the Limongi & Chieffi (2018), Sukhbold et al. (2016), Nomoto et al. (2013), and Woosley & Weaver (1995) CCSN yield tables, with Limongi & Chieffi (2018) being the only study which reports yields for rotating progenitors. Broadly, the non-rotating predictions are consistent with one another, and predict a significant metallicity dependence; the lowest metallicity progenitor from Woosley & Weaver (1995) predict somewhat higher yields overall, but this could have to do with this being the only yield set for which we can calculate only *gross* yields rather than *net* yields. The rotating progenitors from Limongi & Chieffi (2018), however, predict that the yield should be considerably enhanced by rotation. They interpret this as being due to the interplay between the core helium and hydrogen burning shells triggered by rotation-induced instabilities, which drives the synthesis of all products of CNO, not

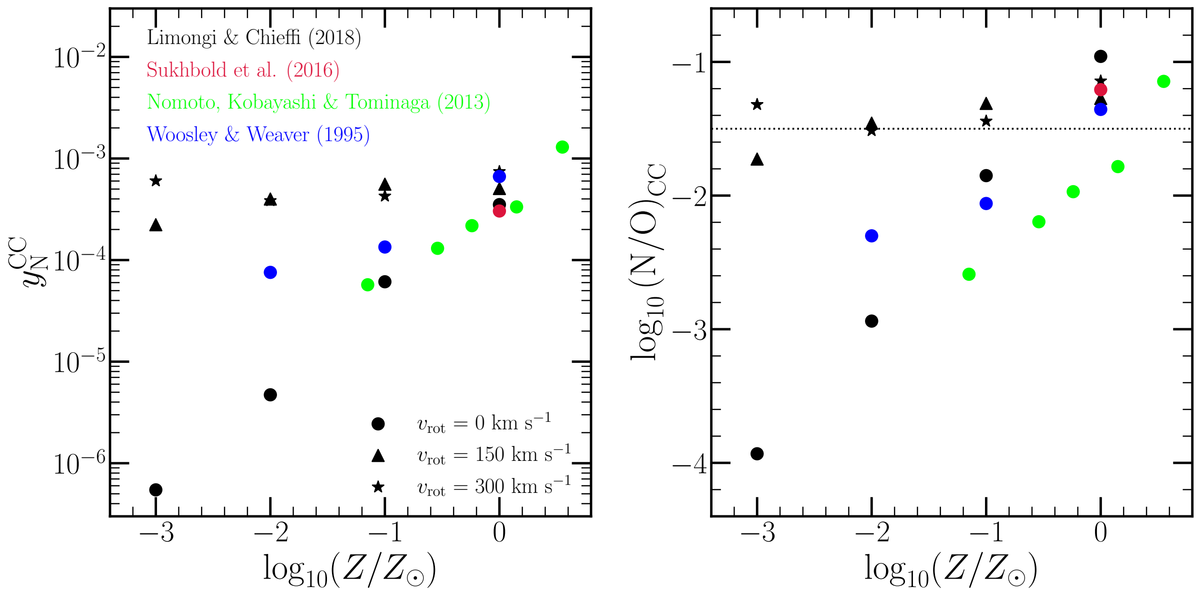

Figure 1: **Right**: IMF-integrated CCSN yields of N computed with VICE using the Limongi & Chieffi (2018) (black), Sukhbold et al. (2016) (crimson), Nomoto et al. (2013) (lime), and Woosley & Weaver (1995) (blue) yield sets. **Left**: IMF-integrated nitrogen-to-oxygen yield ratios computed with VICE for the same studies. The Limongi & Chieffi (2018) yields are calculated with progenitor rotational velocities of $v_{\rm rot} = 0$ (circles), 150 (triangles), and 300 km/s (stars). All other studies only report yields for non-rotating progenitors.

just ^{14}N (see their abstract). These yields predict a relatively metallicity-independent y_N^{CC} of $\sim 5\times10^{-4}$, in surprisingly good agreement with the empirically derived value of 4.15×10^{-4}.

The right panel of Figure 1 presents the IMF-integrated nitrogen-to-oxygen ratios predicted by the same studies for the same combinations of metallicity and rotational velocity. The flat, dotted black line denotes $\log_{10}$ (N/O)$_{CC}$ = -1.5, the value empirically derived from observations (Vincenzo et al., 2016; Berg et al., 2012; Izotov et al., 2012; James et al., 2015). The rotating progenitor models from Limongi & Chieffi (2018) do the best job of reproducing this ratio in theoretical models of core collapse supernova ejecta; the other models do not include rotation, which appears to play a key role in establishing this empirical result.

What is the implied plateau in [N/O]?

[N/O] and log(N/O) are directly related, but one is relative to the sun while the other is just a ratio of number densities. Expanding [N/O]:

$$[\mathrm{N/O}] = [\mathrm{N/H}] - [\mathrm{O/H}] \tag{4a}$$

$$= \log_{10}\left(\frac{Z_N}{Z_{N,\odot}}\right) - \log_{10}\left(\frac{Z_O}{Z_{O,\odot}}\right) \tag{4b}$$

$$= \log_{10}\left(\frac{Z_N}{Z_O}\right) - \log_{10}\left(\frac{Z_{N,\odot}}{Z_{O,\odot}}\right) \tag{4c}$$

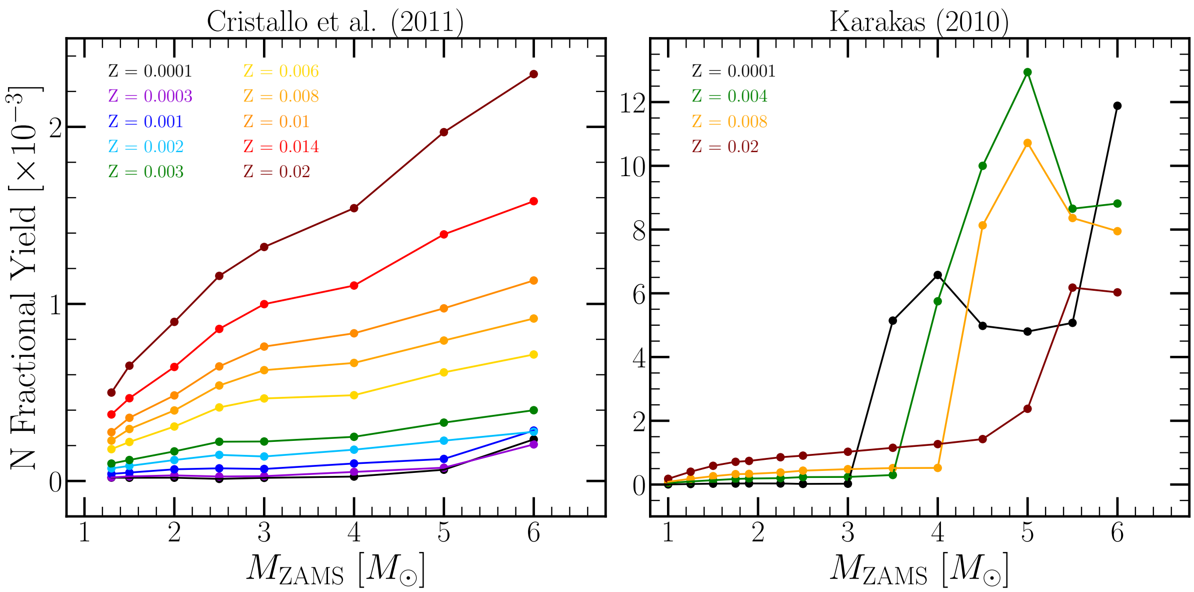

Figure 2: Fractional yields of N as a function of progenitor zero age main sequence mass at the metallicities at which Cristallo et al. (2011) (left) and Karakas (2010) report yields.

Zooming in on the $Z_{\mathrm{N}}/Z_{\mathrm{O}}$ term:

$$\log_{10}\left(\frac{Z_{\mathrm{N}}}{Z_{\mathrm{O}}}\right) = \log_{10}\left(\frac{\mu_{\mathrm{N}} n_{\mathrm{N}}}{\mu_{\mathrm{O}} n_{\mathrm{O}}}\right) \tag{5a}$$

$$= \log_{10}\left(\frac{\mu_{\mathrm{N}}}{\mu_{\mathrm{O}}}\right) + \log_{10}\left(\frac{n_{\mathrm{N}}}{n_{\mathrm{O}}}\right) \tag{5b}$$

where μ and n are again the mean molecular weight and number of some species x. The term $\log_{10}(n_{\mathrm{N}}/n_{\mathrm{O}})$ is exactly the log(N/O) value which Berg et al. (2012), Izotov, Thuan & Guseva (2012), and James et al. (2015) measured to be ∼-1.5. Plugging this in:

$$[\mathrm{N/O}] = \log_{10}\left(\frac{\mu_{\mathrm{N}}}{\mu_{\mathrm{O}}}\right) + \log_{10}\left(\frac{n_{\mathrm{N}}}{n_{\mathrm{O}}}\right) - \log_{10}\left(\frac{Z_{\mathrm{N},\odot}}{Z_{\mathrm{O},\odot}}\right) \tag{6a}$$

Taking $\mu_{\mathrm{N}} = 14.007$ and $\mu_{\mathrm{O}} = 15.999$ again, with the empirical result of $\log_{10}(n_{\mathrm{N}}/n_{\mathrm{O}}) = $ -1.5 and the Asplund et al. (2009) solar photospheric composition of $Z_{\mathrm{N},\odot} = 6.91\times10^{-4}$ and $Z_{\mathrm{O},\odot} = 0.00572$, yields the following:

$$[\mathrm{N/O}]_{\mathrm{plateau}} = -0.64 \tag{7}$$

Asymptotic Giant Branch Star Yields

Figure 2 presents the fractional yields of N as a function of progenitor zero age main sequence (ZAMS) mass and metallicity as reported in the FRUITY database (Cristallo et al., 2011) and by Karakas (2010). Both models show the metallicity-dependent nature of seconday nitrogen production, whose fractional yields increase with progenitor mass at fixed metallicity. Secondary nitrogen production refers to the production of ^{14}N at the expense of C and O in the CNO cycle; the nuclear reaction network of the CNO cycle:

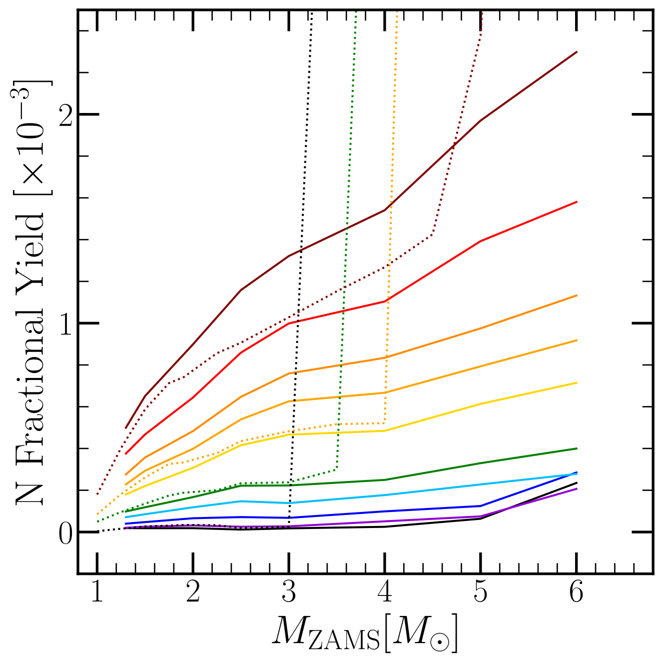

Figure 3: The same as figure 2, but with the Cristallo et al. (2011) (solid) and Karakas (2010) yields plotted on the same set of axes for comparison.

^{12}C(p,γ)^{13}N(,β^+)^{13}C(p,γ)^{14}N(p,γ) ^{15}O(,β^+)^{15}N(p,α)^{12}C

In this chain, the ^{14}N(p,γ)^{15}O reaction is particularly slow, and for this reason, the net effect of the CNO cycle is to turn all of the C and O into ^{14}N at their expense.

By omparing the left- and right-hand panels of Fig. 2, it's clear that the Karakas (2010) predicts secondary nitrogen production to be a much stronger effect than Cristallo et al. (2011); the yields from high mass stars are over an order magnitude larger in Karakas (2010) than in Cristallo et al. (2011) at all reported metallicities except solar. Karakas (2010) also predicts a much more complicated mass-dependence than does Cristallo et al. (2011). Intuitively, I would think that secondary N yields should be nothing but monotonic with metallicity. Rotation may induce some interesting effects, but unfortunately the Karakas (2010) paper makes no mention of rotation. They were primarily interested in the effect of updates to the ^{13}C(α,n)^{16}O reaction on light-element nucleosynthesis. I'm assuming this means they were using non-rotating models, but if this ends up being discussed in a paper I should email Amanda Karakas to verify.

How consistent are the two studies up to ∼3-4 $M_\odot$? Figure 3 compares the two sets of yields on the same plot, with the Cristallo et al. (2011) set shown in solid lines and the Karakas (2010) set in dotted lines. The two are broadly consistent with one another up this threshold mass, at which point the secondary nitrogen yields as reported by Karakas (2010) become considerably large in comparison.

Single Stellar Population Production of Nitrogen

Production Timescales Relative to Fe

Using $y_\mathrm{N}^\mathrm{CC} = 5 \times 10^{-4}$, the AGB star yields of N from the FRUITY database (Cristallo et al., 2011), and supernova yields of Fe as in Johnson & Weinberg (2020) and Weinberg et al. (2017) (i.e. $y_\mathrm{Fe}^\mathrm{CC} = 0.0012$ and $y_\mathrm{Fe}^\mathrm{Ia} = 0.0017$), **what is the net production of N and Fe as a function of stellar population age and metallicity?**

Figure 4 shows the net production of N and Fe as a function of stellar population age and metallicity. Since Fe has metallicity-independent yields under these assumptions, it's plotted with only one curve, whereas N has different production timescales at different metallicities. In general, the CCSN yields of N under these assumptions make up a substantially larger fraction of the N production than the CCSN yields of Fe does for its production. This means that the characteristic timescales for N production are significantly shorter than for Fe.

The AGB yields of N are also significantly weighted toward high masses such that even at solar metallicity, $\gtrsim$90% of the N production is complete by the time the population is $\tau =$ 1 Gyr old. Although the fractional yields are higher for more massive AGB stars, this does not mean that the total N produced in low-mass AGB stars is lower than that produced by high mass AGB stars due to the steep nature of the initial mass function. In a window of progenitor mass $[m, m + dm]$ at a metallicity Z, the total mass of N produced is given by:

$$dm_\mathrm{N} = y(m|Z)m\frac{dN}{dm} = y(m|Z)\xi m^{1-\alpha} \tag{8}$$

where α is the power-law index of the IMF. If the production at two masses m_1 and m_2 are comparable, then the scaling of the yield y with progenitor mass can be derived:

$$dm_\mathrm{N}|_{m=m_1} = dm_\mathrm{N}|_{m=m_2} \tag{9a}$$

$$\implies y(m_1|Z)\xi m_1^{1-\alpha} = y(m_2|Z)\xi m_2^{1-\alpha} \tag{9b}$$

$$\implies \frac{y(m_1|Z)}{y(m_2|Z)} = \left(\frac{m_1}{m_2}\right)^{\alpha-1} \tag{9c}$$

If the yield y scales with $m^{-\gamma}$ and the IMF-integrated mass production is to be mass-independent, then $\gamma = 1-\alpha =$ -1.3. Only when $y \propto m^{1.3}$ will the IMF-integrated contribution of high mass stars be comparable to that of low-mass stars. The weight of low-mass stars increases with increasing γ, and conversely for high-mass stars with decreasing γ. Based on these investigations of the Cristallo et al. (2011) yields (see `latex/yields/yields.pdf`), it appears that $\gamma \approx$ -1 for nitrogen (if anything else, the $y - m$ relation appears to me that it might be slightly sub-linear), indicating that the IMF-integrated production is marginally dominated by low-mass stars.

How can the production be dominated by high-mass stars if the IMF-integrated yields are dominated by low-mass stars? To understand this, it is enlightening to consider the scenario in which the production of some element x in AGB stars is time-independent. That is,

$$\dot{M}_x^\mathrm{AGB} = \mathrm{constant} \tag{10}$$

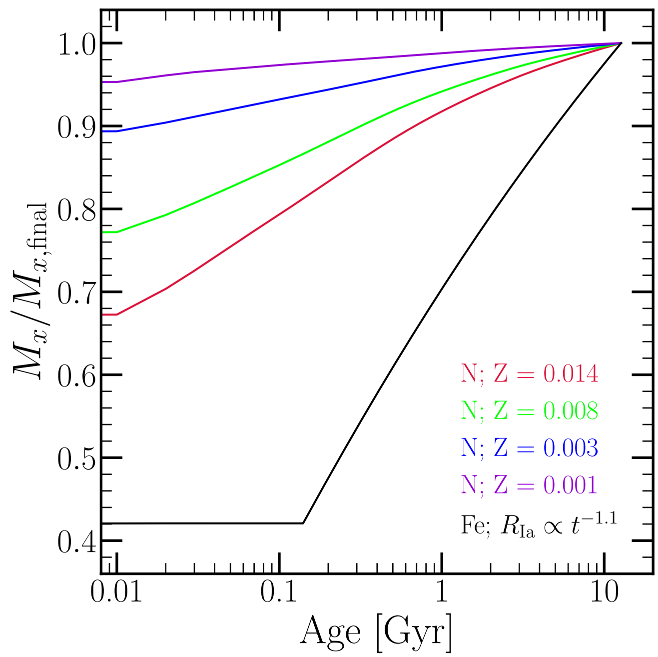

Figure 4: Net mass of N (colored lines) and Fe (black) as a function of stellar population age and metallicity (color-coded according to legend for N), in units of the mass produced at $\tau = 12.2$ Gyr.

Under this assumption, in any time interval from τ to $\tau + d\tau$, the amount of production of some element x is always the same. This corresponds to an idealized scenario in which stars of all masses contribute equally to enrichment when stellar lifetimes are taken into account. The question then becomes, **what is the required dependence of the AGB yield on mass at a given metallicity for the production rate to be constant?** For this calculation, we'll assume the IMF, lifetime, and yield scale with progenitor stellar mass m in the following way:

$$\frac{dN}{dm} \propto m^{-\alpha} \tag{11a}$$

$$\tau \propto m^{-\beta} \tag{11b}$$

$$y \propto m^{-\gamma} \tag{11c}$$

where to answer this question we will need to calculate the appropriate value of γ.

The rate of production of some element x can be expressed according to:

$$\dot{M}_x^{\mathrm{AGB}} = y(m_{\mathrm{to}}|Z) M_\star \dot{h} \tag{12}$$

where $y(m_{\mathrm{to}}|Z)$ is the yield at the main sequence turnoff mass at some metallicity Z, $M_\star$ is the total initial mass of the progenitor stellar population, and $\dot{h}$ is the time-derivative of the *main sequence mass fraction* (see Johnson & Weinberg (2020), or section 3 of VICE's science documentation[1]). In detail, VICE takes into account post main-sequence lifetimes

[1] https://vice-astro.readthedocs.io/en/latest/science_documentation/SSPs/index.html

by simply inflating the lifetime τ by some amount (10% by default), but for the purposes of this calculation, I'm assuming it to be zero. It is defined by:

$$h = \frac{\int_l^{m_\text{to}} m\frac{dN}{dm}dm}{\int_l^u m\frac{dN}{dm}dm} \tag{13}$$

where l and u are the lower and upper mass limits of star formation, respectively, and dN/dm is the IMF. Taking the time-derivative of h may appear non-trivial, but it is simplified greatly by the usage of the chain rule and the fundamental theorem of calculus. Conventiently, the demoninator of h is simply the initial mass of the stellar population and is time-independent; for ease, I'll denote it here simply as $M_\star$.

$$\begin{aligned}
\dot{h} &= M_\star^{-1}\frac{d}{dm_\text{to}}\left(\int_l^{m_\text{to}} m\frac{dN}{dm}dm\right)\frac{dm_\text{to}}{d\tau} && \text{(14a)}\\
&= M_\star^{-1}m_\text{to}\frac{dN}{dm}\bigg|_{m_\text{to}}\frac{dm_\text{to}}{d\tau} && \text{(14b)}\\
&\propto M_\star^{-1}m_\text{to}^{1-\alpha}\frac{d}{d\tau}(\tau^{-1/\beta}) && \text{(14c)}\\
&\propto M_\star^{-1}m_\text{to}^{1-\alpha}\frac{-1}{\beta}\tau^{-(1+\beta)/\beta} && \text{(14d)}\\
&\propto M_\star^{-1}\tau^{(\alpha-1)/\beta}\frac{-1}{\beta}\tau^{-(1+\beta)/\beta} && \text{(14e)}\\
&\propto M_\star^{-1}\frac{-1}{\beta}\tau^{(\alpha-2-\beta)/\beta} && \text{(14f)}
\end{aligned}$$

Therefore, under this formalism, the main sequence mass fraction should decrease monotonically with time (as expected) according to $\dot{h} \sim \tau^{(\alpha-2-\beta)/\beta}$.

Next, if the yield y scales with $m_\text{to}^{-\gamma}$, then it should scale with time according to $y \sim \tau^{\gamma/\beta}$. Plugging all of this in yields the following time-dependence for the AGB enrichment rate:

$$\begin{aligned}
\dot{M}_x^\text{AGB} &\sim \tau^{\gamma/\beta}\tau^{(\alpha-2-\beta)/\beta} && \text{(15a)}\\
&= \tau^{(\alpha+\gamma-2-\beta)/\beta} && \text{(15b)}
\end{aligned}$$

and if the enrichment rate is to be a constant, the power-law index must be equal to zero:

$$\alpha + \gamma - 2 - \beta = 0 \implies \gamma = 2 + \beta - \alpha \tag{16}$$

If α = 2.3 and β = 3.5, as adopted in VICE, then γ = 3.2. If all stellar masses are to contribute equally to the AGB enrichment rate when lifetimes are taken into account such that the rate is constant, then the yields must increase sharply with decreasing stellar mass. I note that the high value of γ is truly a consequence of the high value of β ($2 - \alpha$ = -0.3). Under this parameterization, for higher values of γ, the AGB enrichment rate increases with time as the yields of low-mass stars become more and more important. For lower values, the production rate falls with time due to the lifetimes of low-mass stars preventing them from producing yields quickly enough to maintain a constant production rate. This is an indication that even though the IMF-integrated AGB stars yields of N are marginally weighted toward low-mass AGB stars, the amount which is produced on the order of a hubble time will be dominated by high mass stars unless the yields have a very strong, inverse dependence on progenitor stellar mass.

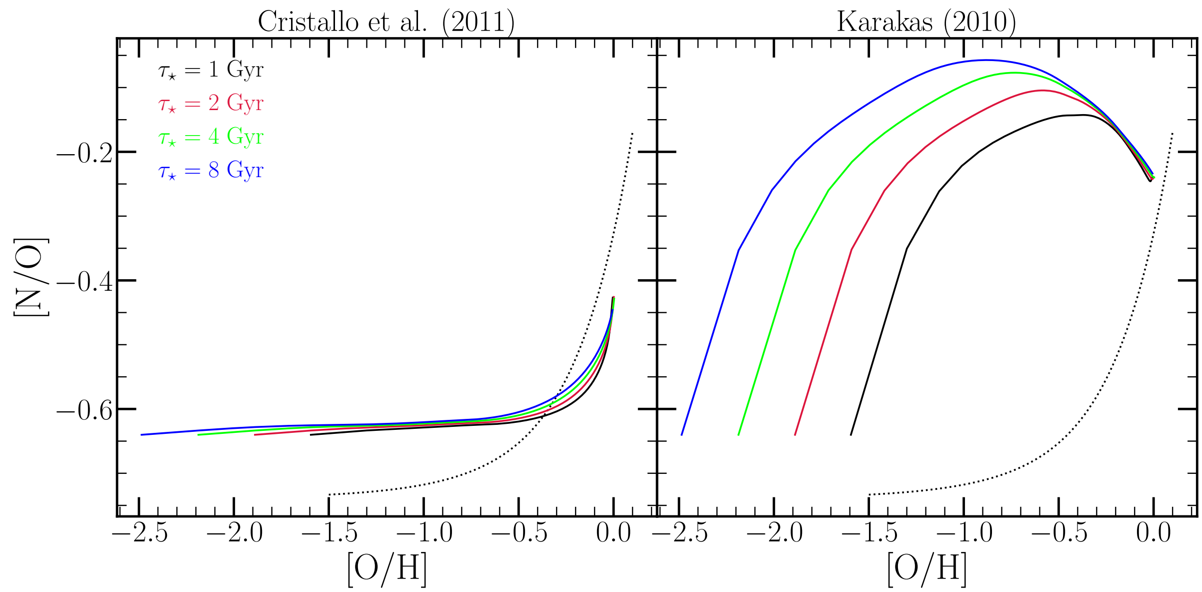

Figure 5: [N/O]-[O/H] tracks as predicted by Cristallo et al. (2011) (left) and Karakas (2010) (right). Colored lines denote different SFE timescales denoted in the legend in the upper left. The black dotted line denotes the fit to the observed [N/O]-[O/H] relation published in Henry, Edmunds & Köppen (2000).

One-Zone Models

The Impact of the AGB Star Yields

How do the abundances predicted by one-zone models differ between the Cristallo et al. (2011) and Karakas (2010) yield sets?

Taking $\dot{M}_\star =$ constant, $\eta = 2.0$, and $y_\text{N}^\text{CC} = 4.15 \times 10^{-4}$ as suggested by supernova yields and the observed [N/O] plateau at low [O/H], the [N/O]-[O/H] tracks for $\tau_\star = 1$, 2, 4, and 8 Gyr for both studies is shown in Fig. 5. The black dotted line denotes the population-averaged trend published in Henry, Edmunds & Köppen (2000). The model recovers the qualitative observational trend with the Cristallo et al. (2011) yields, though the increase in [N/O] at high [O/H] isn't as large as in the observations. The Karakas (2010) yields predict a trend in tension with the observations, but this makes sense when considering how these yields depend on mass and metallicity (see Fig. 2). Their highest yields are for higher mass stars at low metallicity; although the Cristallo et al. (2011) yields also predict the yields to increase with stellar mass, the metallicity trend appears to be what's most important here. At early times when [O/H] is low, the high mass AGB stars dump a lot of N, getting the ISM off the plateau more or less immediately. The peak and subsequent decrease in [N/O] at high [O/H] can be understood from the decrease in the N yields near solar Z as reported by Karakas (2010).

Since the Cristallo et al. (2011) yields recover the correct trend, but the Karakas (2010) yields appear to have a more realistic magnitude, better agreement with the observations can be achieved by simply amplifying the Cristallo et al. (2011) yields by some factor; this is demonstrated in Fig. 6. The model with amplified yields agrees with the observational

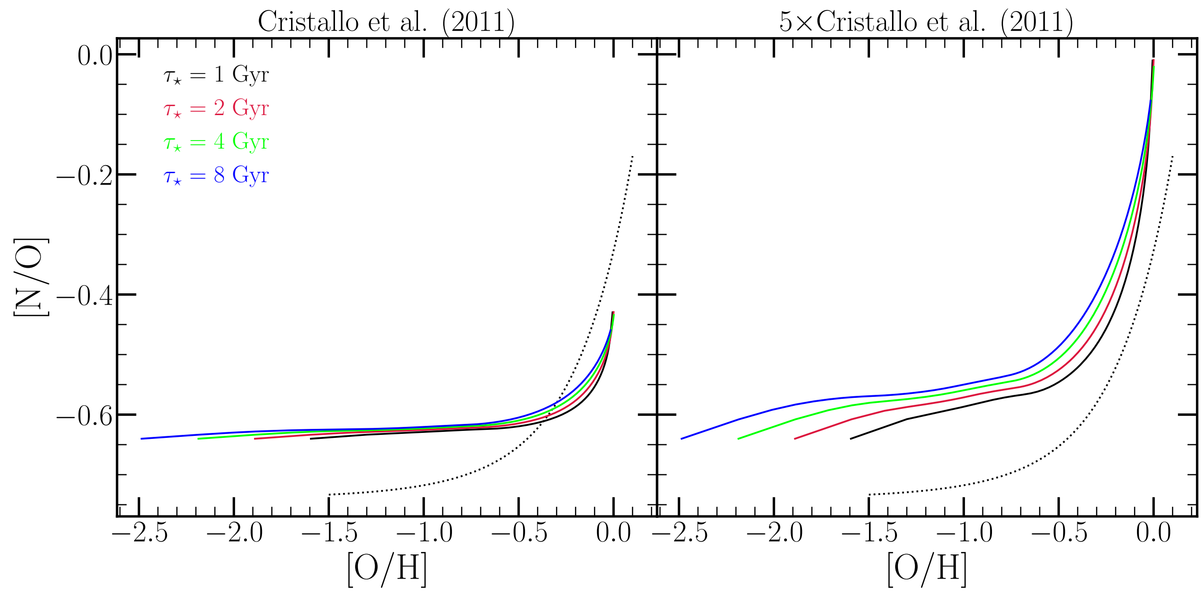

Figure 6: [N/O]-[O/H] tracks as predicted with the Cristallo et al. (2011) yields (left), and with the same yield set but amplified by a factor of 5 (right).

result better, even though it overestimates [N/O] and all [O/H]. A better match could be achieved by lowering $y_{\rm N}^{\rm CC}$, which predicts a plateau at a marginally higher [N/O] than Henry, Edmunds & Köppen (2000). Nonetheless, this is a good demonstration that even taking the more realistic of the two yield sets, the yields must be artificially amplified by a substantial prefactor in order to predict ∼solar [N/O] at ∼solar [O/H]. **This re-raises the question of the timescales of N enrichment from a single stellar population: if AGB stars make up a more substantial fraction of the nitrogen enrichment in the universe, will that increase the characteristic delay times of nitrogen production seen with the base set of yields from Cristallo et al. (2011)? This should impact the amplitude of variability in its production.**

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