Single Stellar Population Production of Nitrogen

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Production Timescales Relative to Fe

Using $y_{\text{N}}^{\text{CC}} = 5 \times 10^{-4}$, the AGB star yields of N from the FRUITY database (Cristallo et al., 2011), and supernova yields of Fe as in Johnson & Weinberg (2020) and Weinberg et al. (2017) (i.e. $y_{\text{Fe}}^{\text{CC}} = 0.0012$ and $y_{\text{Fe}}^{\text{Ia}} = 0.0017$), **what is the net production of N and Fe as a function of stellar population age and metallicity?**

Figure 1 shows the net production of N and Fe as a function of stellar population age and metallicity. Since Fe has metallicity-independent yields under these assumptions, it's plotted with only one curve, whereas N has different production timescales at different metallicities. In general, the CCSN yields of N under these assumptions make up a substantially larger fraction of the N production than the CCSN yields of Fe does for its production. This means that the characteristic timescales for N production are significantly shorter than for Fe.

The AGB yields of N are also significantly weighted toward high masses such that even at solar metallicity, $\gtrsim$90% of the N production is complete by the time the population is $\tau =$ 1 Gyr old. Although the fractional yields are higher for more massive AGB stars, this does not mean that the total N produced in low-mass AGB stars is lower than that produced by high mass AGB stars due to the steep nature of the initial mass function. In a window of progenitor mass $[m, m + dm]$ at a metallicity Z, the total mass of N produced is given by:

$$dm_{\text{N}} = y(m|Z) m \frac{dN}{dm} = y(m|Z) \xi m^{1-\alpha} \tag{1}$$

where α is the power-law index of the IMF. If the production at two masses m_1 and m_2 are comparable, then the scaling of the yield y with progenitor mass can be derived:

$$dm_{\text{N}}|_{m=m_1} = dm_{\text{N}}|_{m=m_2} \tag{2a}$$

$$\implies y(m_1|Z)\xi m_1^{1-\alpha} = y(m_2|Z)\xi m_2^{1-\alpha} \tag{2b}$$

$$\implies \frac{y(m_1|Z)}{y(m_2|Z)} = \left(\frac{m_1}{m_2}\right)^{\alpha-1} \tag{2c}$$

If the yield y scales with $m^{-\gamma}$ and the IMF-integrated mass production is to be mass-independent, then $\gamma = 1 - \alpha =$ -1.3. Only when $y \propto m^{1.3}$ will the IMF-integrated contribution of high mass stars be comparable to that of low-mass stars. The weight of low-mass stars increases with increasing γ, and conversely for high-mass stars with decreasing γ. Based on these investigations of the Cristallo et al. (2011) yields (see `../yields/yields.pdf`), it appears that $\gamma \approx$ -1 for nitrogen (if anything else, the $y - m$ relation appears to me that it might be slightly sub-linear), indicating that the IMF-integrated production is marginally dominated by low-mass stars.

How can the production be dominated by high-mass stars if the IMF-integrated yields are dominated by low-mass stars?

To understand this, it is enlightening to consider the scenario in which the production of some element x in AGB stars is time-independent. That is,

$$\dot{M}_x^{\text{AGB}} = \text{constant} \tag{3}$$

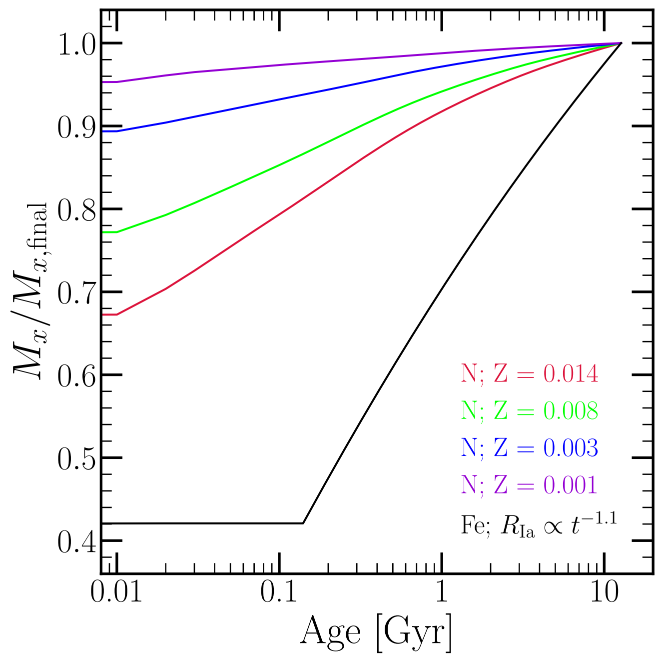

Figure 1: Net mass of N (colored lines) and Fe (black) as a function of stellar population age and metallicity (color-coded according to legend for N), in units of the mass produced at $\tau = 12.2$ Gyr.

Under this assumption, in any time interval from τ to $\tau + d\tau$, the amount of production of some element x is always the same. This corresponds to an idealized scenario in which stars of all masses contribute equally to enrichment when stellar lifetimes are taken into account. The question then becomes, **what is the required dependence of the AGB yield on mass at a given metallicity for the production rate to be constant?** For this calculation, we'll assume the IMF, lifetime, and yield scale with progenitor stellar mass m in the following way:

$$\frac{dN}{dm} \propto m^{-\alpha} \tag{4a}$$

$$\tau \propto m^{-\beta} \tag{4b}$$

$$y \propto m^{-\gamma} \tag{4c}$$

where to answer this question we will need to calculate the appropriate value of γ.

The rate of production of some element x can be expressed according to:

$$\dot{M}_x^{\mathrm{AGB}} = y(m_{\mathrm{to}}|Z) M_\star \dot{h} \tag{5}$$

where $y(m_{\mathrm{to}}|Z)$ is the yield at the main sequence turnoff mass at some metallicity Z, $M_\star$ is the total initial mass of the progenitor stellar population, and $\dot{h}$ is the time-derivative of the *main sequence mass fraction* (see Johnson & Weinberg (2020), or section 3 of VICE's science documentation[1]). In detail, VICE takes into account post main-sequence lifetimes

[1] https://vice-astro.readthedocs.io/en/latest/science_documentation/SSPs/index.html

by simply inflating the lifetime τ by some amount (10% by default), but for the purposes of this calculation, I'm assuming it to be zero. It is defined by:

$$h = \frac{\displaystyle\int_l^{m_{\text{to}}} m\frac{dN}{dm}dm}{\displaystyle\int_l^{u} m\frac{dN}{dm}dm} \tag{6}$$

where l and u are the lower and upper mass limits of star formation, respectively, and dN/dm is the IMF. Taking the time-derivative of h may appear non-trivial, but it is simplified greatly by the usage of the chain rule and the fundamental theorem of calculus. Conventiently, the demoninator of h is simply the initial mass of the stellar population and is time-independent; for ease, I'll denote it here simply as $M_\star$.

$$\dot{h} = M_\star^{-1}\frac{d}{dm_{\text{to}}}\left(\int_l^{m_{\text{to}}} m\frac{dN}{dm}dm\right)\frac{dm_{\text{to}}}{d\tau} \tag{7a}$$
$$= M_\star^{-1}m_{\text{to}}\frac{dN}{dm}\bigg|_{m_{\text{to}}}\frac{dm_{\text{to}}}{d\tau} \tag{7b}$$
$$\propto M_\star^{-1}m_{\text{to}}^{1-\alpha}\frac{d}{d\tau}(\tau^{-1/\beta}) \tag{7c}$$
$$\propto M_\star^{-1}m_{\text{to}}^{1-\alpha}\frac{-1}{\beta}\tau^{-(1+\beta)/\beta} \tag{7d}$$
$$\propto M_\star^{-1}\tau^{(\alpha-1)/\beta}\frac{-1}{\beta}\tau^{-(1+\beta)/\beta} \tag{7e}$$
$$\propto M_\star^{-1}\frac{-1}{\beta}\tau^{(\alpha-2-\beta)/\beta} \tag{7f}$$

Therefore, under this formalism, the main sequence mass fraction should decrease monotonically with time (as expected) according to $\dot{h} \sim \tau^{(\alpha-2-\beta)/\beta}$.

Next, if the yield y scales with $m_{\text{to}}^{-\gamma}$, then it should scale with time according to $y \sim \tau^{\gamma/\beta}$. Plugging all of this in yields the following time-dependence for the AGB enrichment rate:

$$\dot{M}_x^{\text{AGB}} \sim \tau^{\gamma/\beta}\tau^{(\alpha-2-\beta)/\beta} \tag{8a}$$
$$= \tau^{(\alpha+\gamma-2-\beta)/\beta} \tag{8b}$$

and if the enrichment rate is to be a constant, the power-law index must be equal to zero:

$$\alpha + \gamma - 2 - \beta = 0 \implies \gamma = 2 + \beta - \alpha \tag{9}$$

If $\alpha = 2.3$ and $\beta = 3.5$, as adopted in VICE, then $\gamma = 3.2$. If all stellar masses are to contribute equally to the AGB enrichment rate when lifetimes are taken into account such that the rate is constant, then the yields must increase sharply with decreasing stellar mass. I note that the high value of γ is truly a consequence of the high value of β ($2 - \alpha$ = -0.3). Under this parameterization, for higher values of γ, the AGB enrichment rate increases with time as the yields of low-mass stars become more and more important. For lower values, the production rate falls with time due to the lifetimes of low-mass stars preventing them from producing yields quickly enough to maintain a constant production rate. This is an indication that even though the IMF-integrated AGB stars yields of N are marginally weighted toward low-mass AGB stars, the amount which is produced on the order of a hubble time will be dominated by high mass stars unless the yields have a very strong, inverse dependence on progenitor stellar mass.

Bibliography

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