sfhs

February 3, 2022

[1]:
```
import matplotlib.pyplot as plt
from plots.mpltoolkit import named_colors, mpl_loc, fancy_legend,␣
↪load_mpl_presets
import numpy as np
import math as m
load_mpl_presets()
```

1 How many SN Ia events per unit stellar mass would be expected given an exponential SFH and an exponential DTD?

In my GCE models I usually take a $t^{-1.1}$ power-law DTD, but an exponential with e-folding timescale τ_{Ia} allows analytic solutions to this question. The SN Ia rate given this SFH and DTD, for a minimum delay time t_{D} and present-day at time T:

$$\begin{aligned}
\dot{N}_{\text{Ia}} &= \int_{t_{\text{D}}}^{T} \dot{M}_\star(t) R_{\text{Ia}}(T - t) dt \\
&= \int_{t_{\text{D}}}^{T} \dot{M}_{\star,0} e^{-t/\tau_{\text{sfh}}} \xi e^{-(T-t)/\tau_{\text{Ia}}} dt \\
&= \dot{M}_{\star,0} \xi \int_{t_{\text{D}}}^{T} e^{-t/\tau_{\text{sfh}}} e^{-T/\tau_{\text{Ia}}} e^{t/\tau_{\text{Ia}}} dt \\
&= \dot{M}_{\star,0} \xi e^{-T/\tau_{\text{Ia}}} \int_{t_{\text{D}}}^{T} e^{-t(\tau_{\text{Ia}} - \tau_{\text{sfh}})/\tau_{\text{Ia}}\tau_{\text{sfh}}} dt \\
&= \dot{M}_{\star,0} \xi e^{-T/\tau_{\text{Ia}}} \frac{\tau_{\text{Ia}}\tau_{\text{sfh}}}{\tau_{\text{sfh}} - \tau_{\text{Ia}}} e^{-t(\tau_{\text{Ia}} - \tau_{\text{sfh}})/\tau_{\text{Ia}}\tau_{\text{sfh}}} \Big|_{t_{\text{D}}}^{T} \\
&= \dot{M}_{\star,0} \xi e^{-T/\tau_{\text{Ia}}} \frac{\tau_{\text{Ia}}\tau_{\text{sfh}}}{\tau_{\text{sfh}} - \tau_{\text{Ia}}} \left[e^{-T(\tau_{\text{Ia}} - \tau_{\text{sfh}})/\tau_{\text{Ia}}\tau_{\text{sfh}}} - e^{-t_{\text{D}}(\tau_{\text{Ia}} - \tau_{\text{sfh}})/\tau_{\text{Ia}}\tau_{\text{sfh}}} \right]
\end{aligned}$$

where $\dot{M}_{\star,0}$ is the initial height of the SFH at $t = 0$ and ξ is some overall normalizing factor on the SN Ia DTD. The stellar mass:

$$
\begin{aligned}
M_\star &= \int_0^T (1-r)\dot{M}_\star(t)dt \\
&= \int_0^T (1-r)\dot{M}_{\star,0}e^{-t/\tau_\text{sfh}}dt \\
&= -(1-r)\dot{M}_{\star,0}\tau_\text{sfh}e^{-t/\tau_\text{sfh}}\Big|_0^T \\
&= -(1-r)\dot{M}_{\star,0}\tau_\text{sfh}\left[e^{-T/\tau_\text{sfh}} - 1\right] \\
&= (1-r)\dot{M}_{\star,0}\tau_\text{sfh}\left[1 - e^{-T/\tau_\text{sfh}}\right]
\end{aligned}
$$

where r is the recycling fraction ($r \approx 0.4$ for a Kroupa IMF). This assumes instantaneous recycling, which is a relatively good approximation - massive stars return their envelopes quickly, after which the full time-dependent recycling rate slows down considerably due to the lifetimes of lower mass stars. Taking into account time-dependent recycling would require a numerical calculation, but I would expect it to affect the computed specific Ia rates only at the \$ \$10% level.

Taking the ratio of the two, the normalization of the SFH and the e-folding timescale τ_sfh cancel:

$$
\begin{aligned}
\frac{\dot{N}_\text{Ia}}{M_\star} &= (1-r)\xi e^{-T/\tau_\text{Ia}}\frac{\tau_\text{Ia}}{\tau_\text{sfh} - \tau_\text{Ia}}\left[e^{-T(\tau_\text{Ia} - \tau_\text{sfh})/\tau_\text{Ia}\tau_\text{sfh}} - e^{-t_\text{D}(\tau_\text{Ia} - \tau_\text{sfh})/\tau_\text{Ia}\tau_\text{sfh}}\right]\left[1 - e^{-T/\tau_\text{sfh}}\right]^{-1} \\
&\sim e^{-T/\tau_\text{Ia}}\frac{\tau_\text{Ia}}{\tau_\text{sfh} - \tau_\text{Ia}}\left[e^{-T(\tau_\text{Ia} - \tau_\text{sfh})/\tau_\text{Ia}\tau_\text{sfh}} - e^{-t_\text{D}(\tau_\text{Ia} - \tau_\text{sfh})/\tau_\text{Ia}\tau_\text{sfh}}\right]\left[1 - e^{-T/\tau_\text{sfh}}\right]^{-1}
\end{aligned}
$$

For the last line I simply ignore the normalization as only the trend of $\dot{N}_\text{Ia}/M_\star$ with τ_sfh is relevant. The value of ξ could be calculated by taking in a time-integrated value from a previous study on the SN Ia DTD (e.g. ~ 2.2 SN per 1000 $M_\odot$ of star formation over the duty cycle of the DTD - Maoz & Mannucci 2012).

Typical values for these numbers are $\tau_\text{Ia} = 1.5$ Gyr, $t_\text{D} = 150$ Myr, and $T = 13.7$ Gyr.

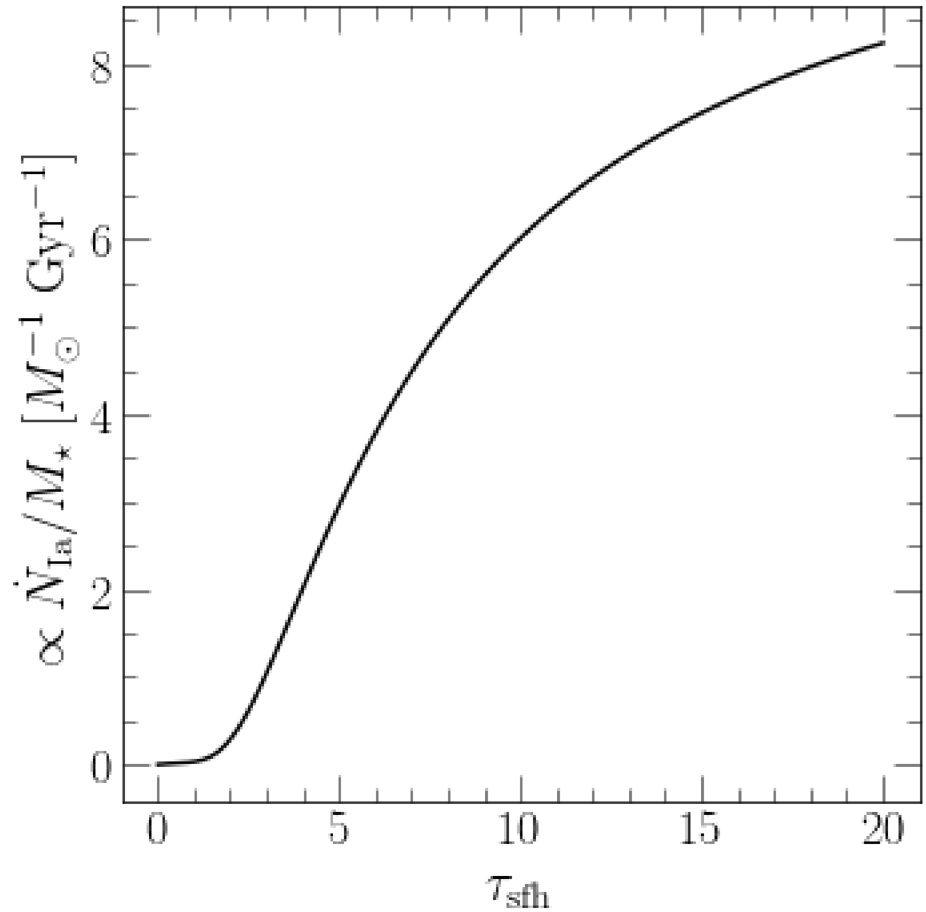
[7]:
```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.set_xlabel(r"$\tau_\text{sfh}$")
ax.set_ylabel(r"$\propto \dot{N}_\text{Ia}/M_\star$ [$M_\odot^{-1}$␣
 ↪Gyr$^{-1}$]")

def specia(tausfh, tauia = 1.5, td = 0.15, T = 13.7):
    specia_ = m.exp(-T / tauia)
    specia_ *= tauia / (tausfh - tauia)
    specia_ *= (
        m.exp(-T * (tauia - tausfh) / (tauia * tausfh)) -
        m.exp(-td * (tauia - tausfh) / (tauia * tausfh))
    )
    specia_ /= 1 - m.exp(-T / tausfh)
    return specia_

xvals = np.linspace(0.01, 20, 1000)
```

```
yvals = [100 * specia(_) for _ in xvals]
ax.plot(xvals, yvals, c = named_colors()["black"])

plt.tight_layout()
plt.show()
```



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