A Factor Approach to Strategic Asset Allocation for Portfolios with Alternatives

FRE-GY 6921

NYU Tandon Program in Financial Engineering

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Fall 2024

Week 4: Outline

- Quiz 1
- Private Equity (con't)
- Real Estate, Private and REITs
- Structured Finance
- Private Debt
- Hedge Funds
- Guest, Valentine de Weck: Liability-driven requirements for SAA at Insurance Companies
 - Global Insurance Solutions, JP Morgan Asset Management
 - MIT Research Affiliate
 - Head of Portfolio Strategies, AIG CIO Office
 - MIT Laboratory for Financial Engineering, B.Sc., M.Sc. Mathematik, ETH Zurick

Intrinsic versus Accounting Risk

- We will not take a view on whether PE marks are "too" smooth, however it is a fact that PE valuations that are smoothed relative to like companies in public markets
- We will recognize that two companies, identical in all respects except that one is listed and the other is private, pose identical *intrinsic* risk, however public company stock will carry higher accounting risk (creating more quarter-to-quarter volatility) for their investors
- How do we handle this from a risk input perspective in an SAA optimization?
 - We will explore two methodologies to "de-smooth" PE returns/risk estimates for the purpose of running SAA under an intrinsic risk perspective
 - In parallel, we will run an SAA leaving private assets with lower risk estimates that reflect the accounting convenience of smoother marks

Intrinsic Risk Adjustments for PE: Price/Vol Estimate Approach

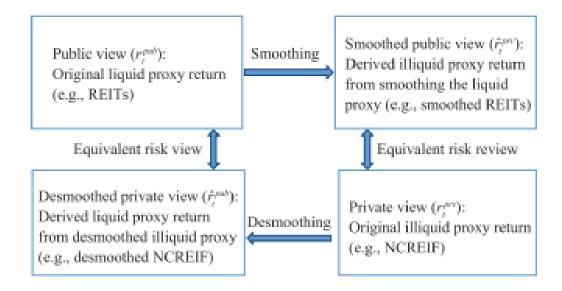
Chen and Greenberg (2017): use an MA to smooth public returns or de-smooth private returns to render return-based risk estimates consistent

Claims: public view consistent with de-smoothed private view; private view consistent with smoothed public view (similar vols, corrs); without de-smoothing, privates appear more diversifying than the actually are.

Pictorial Representation of Article

De-smoothing private returns sensible enough. Why smooth public returns? Benchmark source.

Key to their methodology: ensuring the MA is invertible



Week 4: Adding Private Assets to Public Assets Chen and Greenberg (2017)

EXHIBIT 2
Summary Statistics of Private View and Public View

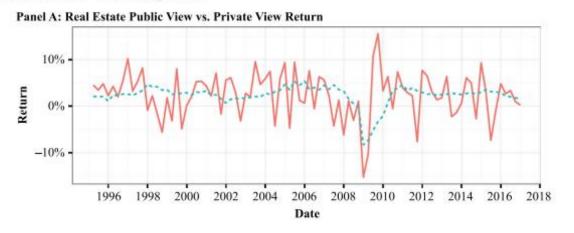
	Priva	Private View Returns			Public View Returns			
	μ	σ	ρ(1)	щ	σ	ρ(1)	$\rho(r_i^{per}, r_i^{peb})$	
Real Estate	9.5%	4.5%	85%	9.6%	9.3%	12%	7.1%	
Private Equity	13.5%	9%	38%	10%	28%	19%	71%	
Infrastructure	8%	3.5%	47%	6.7%	32%	24%	51%	

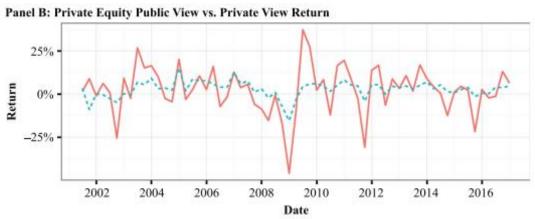
Notes: We report the mean (µ), volatility (G), and first-order autocorrelation ($\rho(1)$) of the private view and public view returns for three illiquid asset classes. The last column also reports the contemporaneous correlation between the public view and the private view performance. Private views of real estate, private equity, and infrastructure equity are from the NCREIF NPI, Cambridge Associate Global Private Equity Index, and Preqin Global Infrastructure Index, respectively. Public views are BlackRock proxy indexes using public tradable REITs or public equities to represent each asset class. Returns are quarterly and span 1995:Q1 to 2016:Q4 for real estate, 2001:Q1 to 2016:Q4 for private equity, and 2008:Q2 to 2015:Q2 for infrastructure.

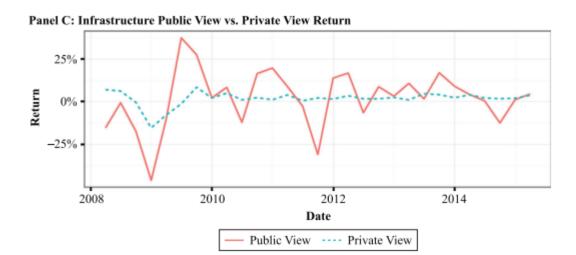
Key details: de-levered public proxies for private portfolios are non-trivial to construct (see paper).

Chen and Greenberg (2017): Basic Data

Private View and Public View Comparison







Week 4: Adding Private Assets to Public Assets Chen and Greenberg (2017): The MA Model

$$r_t^{truePvt} - rfr_t = \beta (r_t^{pub} - rfr_t) + \varepsilon_t$$

$$r_t^{obsPvt} = \sum_{l=0}^{L} \phi_l r_{t-l}^{truePvt}$$

- 1. $0 < \phi_1 < 1$
- 2. $\sum_{l=0}^{L} \phi_l = 1$
- 3. Roots of $\sum_{l=0}^{L} \phi_l z^l = 0$ outside of unit circle, i.e., |z| > 1

$$r_t^{obsPvt} = \sum_{l=0}^{L} \phi_l \left[\beta r_{t-l}^{pub} + (1-\beta)rfr_{t-l} \right] + \varepsilon_t$$

Week 4: Adding Private Assets to Public Assets Chen and Greenberg (2017): The MA Model

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- 3. Roots of $\sum_{l=0}^{L} \phi_l z^l = 0$ outside of unit circle, i.e., |z| > 1

$$r_t^{obsPvt} = \sum_{l=0}^{L} \phi_l \left[\beta r_{t-l}^{pub} + (1-\beta)rfr_{t-l} \right] + \varepsilon_t \quad or$$

$$\hat{r}_t^{pvt} = \hat{\beta} \hat{\phi}(L) r_{t-l}^{pub}$$

Chen and Greenberg (2017): The MA Model

Estimated Parameters:

Because we constrain ϕ_l to be invertible:

$$\hat{r}_t^{pub} = \hat{\beta}^{-1} \hat{\phi}(L)^{-1} r_{t-l}^{prv}$$

	Real Estate	Private Equity	Infrastructure
β	0.83***	0.46***	0.36***
	(0.20)	(0.10)	(0.13)
ϕ_0	25%***	60%***	48%***
. 0	(4%)	(3%)	(4%)
ϕ_1	20%***	17%***	30%***
	(4%)	(3%)	(4%)
ϕ_2	18%***	23%***	22%***
_	(4%)	(3%)	(4%)
ϕ_3	15%***		
	(5%)		
ϕ_4	22%***		
	(4%)		
R^2	62%	65%	57%

Notes: This table reports the estimation results of the econometrics model describing the smoothing and exposure structure of the private view return to public view return:

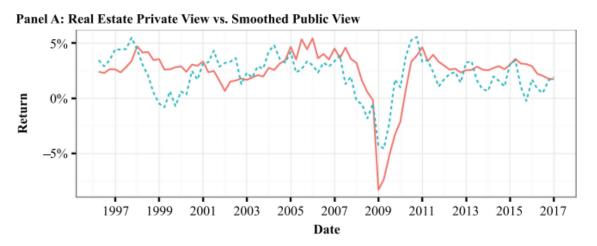
$$r_t^{pov} = \sum_{l=0}^{L} \phi_l [\beta r_{t-l}^{pub} + (1-\beta) r_{t-l}^f] + \epsilon_t$$

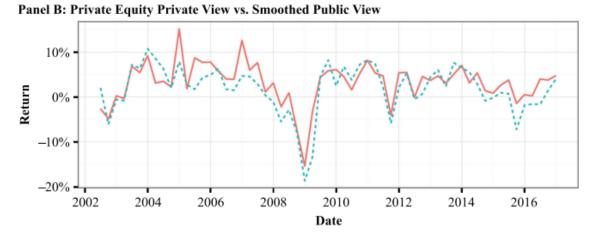
The standard errors of the estimation are noted in parentheses.

$$*P < 0.1, **P < 0.05, ***P < 0.01.$$

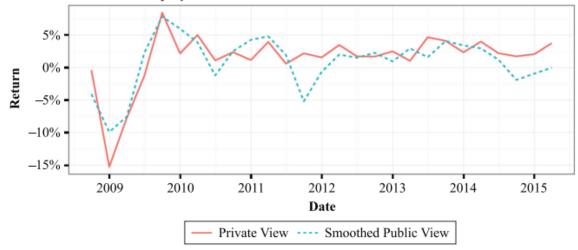
Chen and Greenberg (2017): The MA Model

Estimates:





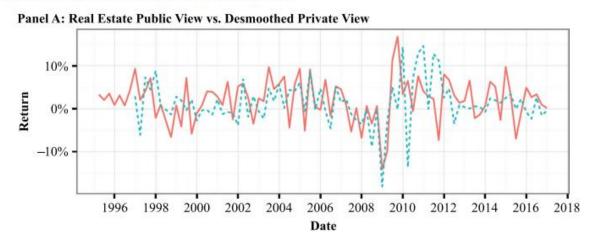
Panel C: Infrastructure Equity Private View vs. Smoothed Public View

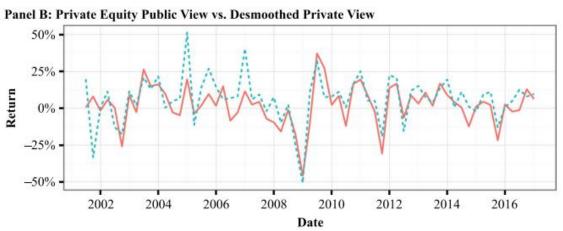


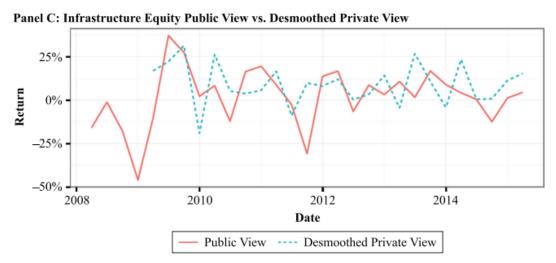
	$\sigma(r_t^{prv})$	$\sigma(\hat{r_t^{prv}})$	$Corr\left(r_{t}^{prr},\hat{r_{t}^{prr}}\right)$	$\sigma(r_t^{pub})$	$\sigma(\hat{r_t^{pub}})$	$Corr\left(r_t^{pub}, \hat{r_t^{pub}}\right)$
Real Estate	4.5%	4.3%	70%	9.3%	10.2%	54%
Private Equity	9%	10%	81%	28%	32%	77%
Infrastructure	3.5%	3.5%	78%	32%	28%	62%

Chen and Greenberg (2017): The MA Model

Public View vs. Desmoothed Private View Performance







Chen and Greenberg (2017): The MA Model

Inverting the lag function. Start with

$$\phi(L) = \phi_0 + \phi_1 L + \phi_2 L^2 + \phi_L L^K$$

We're solving for $\psi(L)$ where $\psi(L) \equiv \phi(L)^{-1}$ or $\psi(L)\phi(L) \equiv 1$ and

$$\psi(L) = \psi_0 + \psi_1 L + \psi_2 L^2 + \psi_L L^K$$

Imposing $\psi(L)\phi(L)\equiv 1$ we get:

constant:
$$\psi_0 \phi_0 = 1$$

$$B: \qquad \phi_0 \psi_1 + \phi_1 \psi_0 = 0$$

$$B^2: \qquad \phi_0 \psi_2 + \phi_1 \psi_1 + \phi_2 \psi_0 = 0$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$B^P: \qquad \sum_{k=0}^p \phi_k \psi_{p-k} = 0$$

$$\mbox{So:} \quad \psi_{_0} = \varphi_{_0}^{_{-1}}, \;\; \psi_{_1} = -\varphi_{_0}^{_{-1}}\varphi_{_1}\psi_{_0}, \;\; \psi_{_2} = -\varphi_{_0}^{_{-1}}(\varphi_{_1}\psi_{_1} + \varphi_{_2}\psi_{_0}), \;\; \mbox{etc.}$$

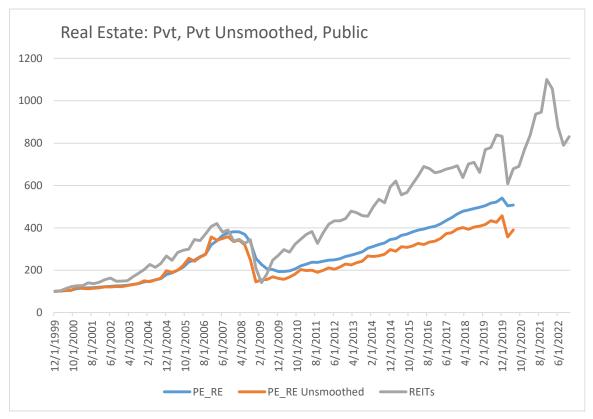
Week 4: Adding Private Assets to Public Assets Chen and Greenberg (2017): The Inverted MA Model

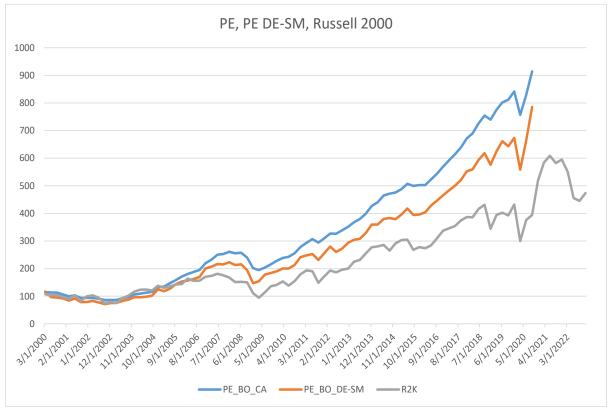
PE Filter	
Ψ12	-0.01
Ψ11	0.00
Ψ10	0.01
Ψ9	-0.02
Ψ8	-0.02
Ψ7	0.06
Ψ ₆	0.00
Ψ5	-0.15
Ψ4	0.10
Ψ3	0.32
Ψ2	-0.51
Ψ1	-0.47
Ψο	1.67

GRE Filter	
Ψ20	-0.01
Ψ19	0.01
Ψ18	-0.01
Ψ ₁₇	-0.01
Ψ ₁₆	0.03
Ψ15	-0.03
Ψ ₁₄	0.02
Ψ13	0.03
Ψ ₁₂	-0.09
Ψ11	0.09
Ψ10	0.00
Ψ9	-0.08
Ψ8	0.20
Ψ7	-0.31
Ψ ₆	0.02
Ψ5	0.47
Ψ4	-0.46
Ψ3	0.53
Ψ2	-0.53
Ψ1	-1.39
Ψο	2.53

Week 4: PE Return Adjustment

Intrinsic Risk Adjustments for PE using de-smoothing





	PE_RE	PE_RE Unsmootl	REITs
avgRtn	8.57%	8.92%	11.90%
Vol	9.69%	17.34%	21.60%
IR	0.88	0.51	0.55

	PE returns	PE_DE-SM rt	R2K returns
avgRtn	11.3%	11.3%	9.1%
vol	10.1%	16.1%	21.6%
IR	1.116	0.706	0.420

Week 4: Fundamental Approach to PE Return Adjustment

 This approach attempts to apply a consistent fundamental valuation approach to Public and Private Equities alike

• Recall from previous class on ERPB that:

$$P_{t} = \sum_{k=1}^{\infty} \left(\frac{1}{(1+r+\xi)}\right)^{k} E[PO_{t+k+1}]$$

• Substituting $E[PO_{t+k}] = (1+g)^k PO_t$, we got:

$$P_t = \sum_{k=1}^{\infty} \left(\frac{(1+g)}{(1+r+\xi)} \right)^k E[PO_{t+1}]$$

• Using infinite series we got:

$$P_t = \frac{E[PO_{t+1}]}{(r+\xi-g)}$$

This implies

$$r + \xi = \frac{E[PO_{t+1}]}{P_t} - g$$

AQR's methodology uses this as a staring point:

$$E[rtn_{equities}] = \xi + r = Payout\ Yield + g$$

Intrinsic Risk Adjustments for PE: Fundamental Approach

- This approach attempts to apply a consistent fundamental valuation approach to Public and Private Equities alike; will follow AQR's (simplified) methodology
- Public Equity LR E[return] methodology: 0.5*earningsMethod+0.5payoutMethod

$$E[r_{pubEq}] = 0.5(CAPE/2 + g_{eps}) + 0.5(POyld + g_{POyld})$$

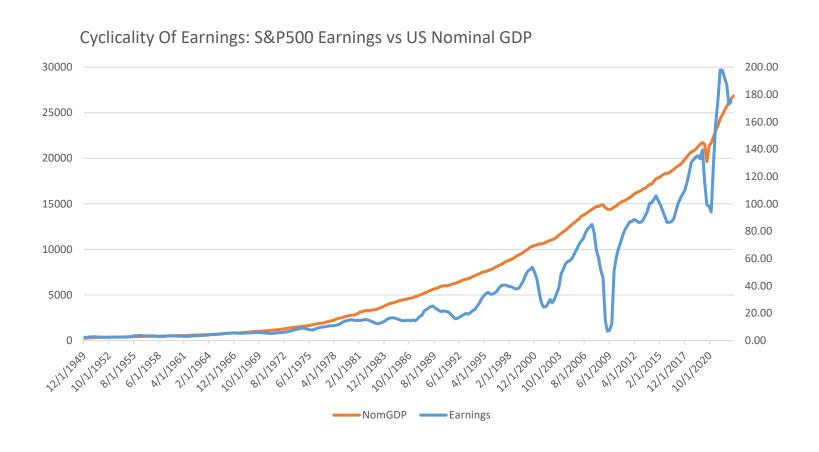
• Private Equity (Buyout) LR E[return] methodology: adjust public equity methodology for leverage, multiple expansion, fees:

$$E[r_{real_unlev}] = E[y_{unlev} + g_{unlev}]$$

$$E[r_{PE}] = E[r_{real_unlev}] + (D/E) \cdot (E[r_{real_unlev}] - \kappa_{real}) + \mu - fees$$

Week 4: Fundamental Approach to PE Return Adjustment

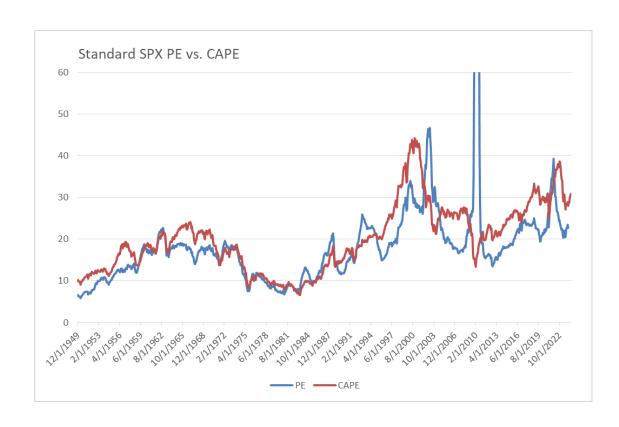
How do AQR estimate a de-cycled earnings estimate? Use Shiller's CAPE

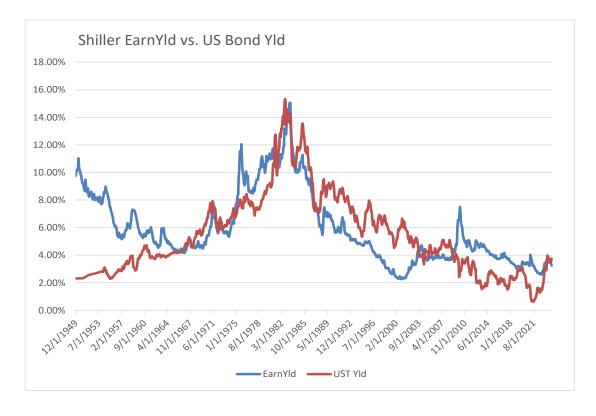


Week 4: Fundamental Approach to PE Return Adjustment

Intrinsic Risk Adjustments for PE: Fundamental Approach

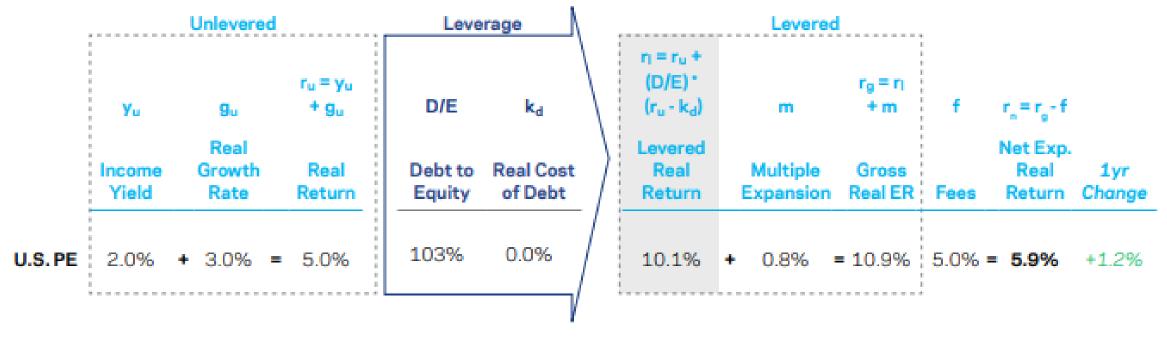
 $CAPE_yld = ma(realEarnings, 10)/realSPXprice$





Intrinsic Risk Adjustments for PE: Fundamental Approach (AQR, Q1-22)

Exhibit 6: Expected Real Returns for U.S. Private Equity



Source: AQR, Pitchbook, Bloomberg, CEM Benchmarking. Estimates as of September 30, 2021. Strictly speaking, our inputs are log returns and should be converted to simple returns before leverage is applied, then converted back to log returns. This 'round-trip' has only a small impact, so we omit it here. For real cost of debt we apply a floor at 0%. Estimates are for illustrative purposes only, are not a guarantee of performance and are subject to change.

Intrinsic Risk Adjustments for PE: Fundamental Approach (Conklin, Q2-2023)

PE Net Return	=	Gross Return	+	Alpha	-	Costs & Fees			
12.22%	=	14.66%	+	2.50%	-	4.93%			
PE Gross	_	ERP	+	Illiquidity		Return to		Term	
Return	=	EKP	+	Premium	+	Leverage	+	RFR	
14.66%	=	4.75%	+	3.50%	+	2.91%	+	3.50%	
					_	-			
Return to Leverage		Augmented Debt/Equity Ratio for Privates	х	(ERP	+	Illiquidity Premium	-	Cost of Financing Spread)
2.91%	=	70%	x (4.75%	+	3.50%	-	4.10%)
ERP	=	Payout Yield	+	LR Growth in Payouts	-	Term RFR			
4.75%	=	3.75%	+	4.50%	-	3.50%			

Week 4: Summary of Comparison Methods for Private & Public Assets

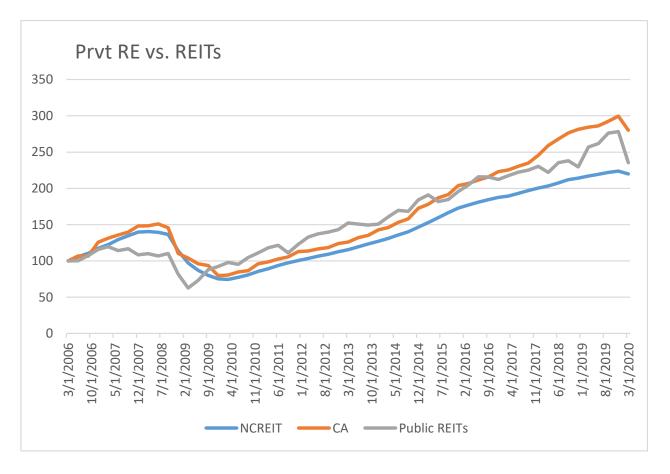
- Industry practice with Public Securities is to use time aggregation (\sqrt{T}) to scale up daily or monthly return volatilities to estimate likelihood and magnitude of loss
- A given Private Security, in all respects identical to a Public Security chosen for close comparison, will have far smaller approximated risk using this method due to valuation practices resulting in smoother marks private (non-listed) companies
- Approach 1: econometric (Chen and Greenberg, 2017). Smooth public securities returns
 (or de-smooth private security returns) to render private and public security risk estimates
 comparable
 - This approach is useful in that has implications for both volatilities and correlations
 - This approach is lacking since beyond econometric transformation of time series, it does not consider specific attributes of companies or industries that underlies securities' risks
- Approach 2: fundamental. Models used to estimate ex-ante risk premia in public securities (e.g., ERPB), are extended to private securities (as do AQR in their capital assumptions)
 - Disadvantage: this method only speaks to expected returns -- silent on loss magnitudes and likelihoods

Three Perspectives:

- 1. Historical returns
- 2. Theoretical required returns
- 3. Yield-based analysis incorporating current market conditions

Our approach combines all three

Historical Returns



Source:	Avg Rtn	StdDev	SR
NCREIT	5.59%	9.20%	0.56
Cambridge Associates	9.58%	11.03%	0.79
3 US REIT Indices (DJ, NAREIT, NCREIT)	7.82%	14.77%	0.42

	Industry	Historical	Historical	Historical
Sector	Weight	Return (CA)	StdDev (CA)	SR (CA)
Apartment / Student	28.2%	6.2%	9.1%	0.50
Office	24.9%	7.7%	18.1%	0.33
Retail	13.8%	3.0%	14.7%	0.09
Industrial	32.8%	13.8%	10.6%	1.15
Hotel	0.3%	11.3%	15.1%	0.64
Aggregate	100.0%	9.6%	11.0%	0.72

Sources of returns

- Rent operating costs ("cap rates"; like earnings yield for stocks)
- Asset appreciation
- Value added: building construction/development, conversion, refurbishment → higher rents, asset appreciation

Pricing these return components

- Real estate is pro-cyclical (correlated to stocks)...
- Proposal:

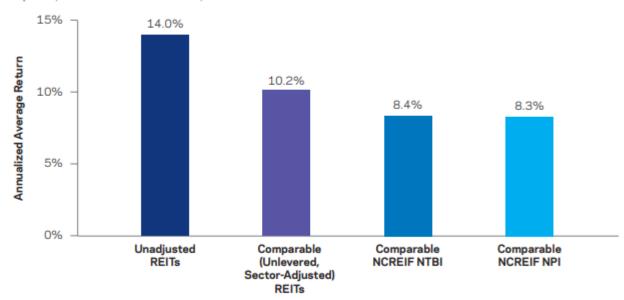
$$r_{pvtRE} = r_{REIT} + r_{leverage} + r_{illiquidity}$$

Sources of returns

$$r_{pvtRE} = r_{REIT} + r_{leverage} + r_{illiquidity}$$

Exhibit 1
Private vs. Public Real Estate: Returns Comparison Before and After Adjustments for Leverage and Sector Composition

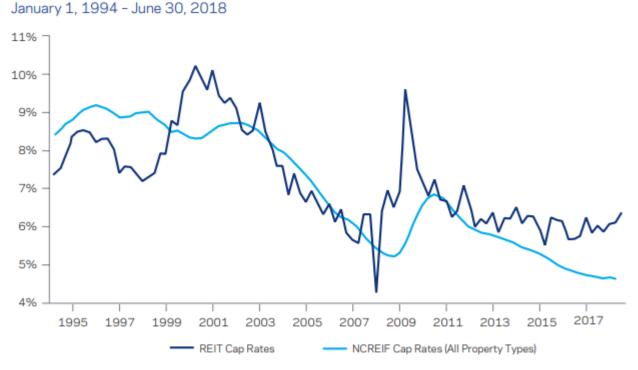
April 1, 1980 - December 31, 2012



Source: AQR, Ang et al. (2013). Available history starts in Q2 1984 for NTBI and Q2 1980 for other series. All series end in Q4 2012. The REITs series is the CRSP/Ziman Real Estate data series. Returns are annualized from the quarterly returns reported in Ang et al. (2013). Returns are gross of fees, cash, and t-costs. For illustrative purposes only and not representative of any portfolio or strategy that AQR currently manages. Please refer to footnote 5 for more detail.

Cap rates in Pvt RE more responsive to interest rates than in REITs

Exhibit 2
The Valuation Gap between Private and Public Real Estate



Source: NCREIF, CoStar Portfolio Strategy. Data as of June 30, 2018. The implied cap rate for REITs is the implied unlevered cap rate of the FTSE NAREIT index. For illustrative purposes only and not representative of any portfolio or strategy that AQR currently manages.

Final Model

• This will look very much like our model for PE:

$$r_{REIT} = r_{bond} + \zeta = capRate + g$$

$$r_{pvtRE} = r_{REIT} + r_{leverage} + r_{premium\ compression} + r_{illiquidity}$$

$$r_{pvtRE} = r_{REIT} + \frac{D}{E} \cdot (r_{REIT} - \kappa) - duration \cdot \Delta \zeta + r_{illiquidity} + (\alpha - fees)$$

		REIT	Term		Lagged Pvt	Illiquidity	Return to		Fees /
Sector	E[return]	= Risk Prer +	RFR	+	Market Correction +	Premium	+ Leverage	+ alpha -	Costs
Apartment / Student	8.47%	3.43%	3.75%		-0.51%	0.00%	1.30%	1.50%	-1.00%
Office	3.41%	1.05%	3.75%		-3.15%	0.00%	1.26%	1.50%	-1.00%
Retail	7.04%	0.99%	3.75%		1.03%	0.00%	0.77%	1.50%	-1.00%
Industrial	12.74%	2.02%	3.75%		0.82%	1.30%	4.35%	1.50%	-1.00%
Hotel	10.08%	1.62%	3.75%		0.00%	1.50%	2.71%	1.50%	-1.00%
Aggregate	10.73%	2.44%	3.75%		0.22%	0.79%	3.03%	1.50%	-1.00%

Week 4: Structured Finance

Broad Sectors

MBS: Residential Mortgages

CMBS: Commercial Mortgages

CDOs: Collateralized Debt Obligations

CLOs: Collateralized Loan Obligations

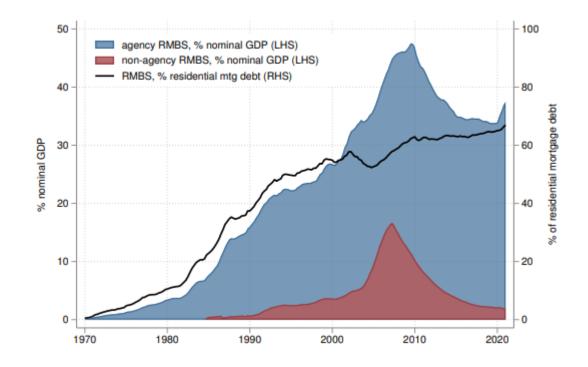
ABS: Asset Backed Securities

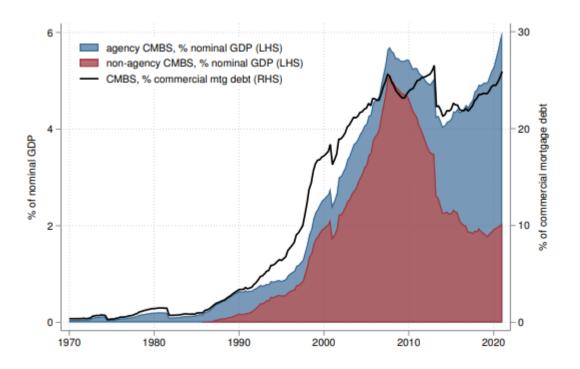
It is the same concept, different variations for each sector. Two key attributes:

- The asset generating the cash flow is isolated from the legal entity (e.g., a corporation) that originated the asset. This is the definition of a mortgage, the meaning of collateral.
- Many such assets are bundled into a pool; and groups (tranches) of securities are linked to the cash flows
 corresponding to these pools, with different seniority associated with each pool

Overview of the market

- Agency vs. Non-agency (GNMA, FNMA, FRMAC, FHA, VA): Agencies define requirements ("conforming"), create the pools, and guarantee the securities; non-agency are entirely private ("non-conforming"). E.g., JUMBOs
- Agency mortgages are not common internationally; neither are fixed rate mortgages
- Historical note: "SnL Crisis" in the US in the early 1980s; RTC and CMBS
- Historical note: "alt-A" mortgages in the 2000s





1. Role of the RTC in Liquidating Mortgage Assets

- After taking control of failed S&Ls, the RTC acquired large volumes of mortgages and real estate assets.
- One of the RTC's key tasks was to liquidate these assets to recover as much value as possible for taxpayers.
- The sheer volume of mortgage-related assets held by the RTC was unprecedented, which required creative financial solutions to sell them efficiently.

2. Securitization of Mortgage Assets

- To facilitate asset sales, the RTC bundled mortgages into securities, which could then be sold to institutional investors. These bundles of
 mortgages became part of the MBS market.
- By securitizing the mortgages and selling MBS products, the RTC was able to increase liquidity, reduce holding times, and attract a wider base of investors.
- The success of these securitizations demonstrated the viability of MBS as a financial product, helping to institutionalize and normalize the MBS market.

3. Catalyst for Broader Market Acceptance and Growth

- The RTC's large-scale use of securitization provided investors with extensive exposure to mortgage-backed securities, giving them confidence in these assets.
- With the RTC's liquidation activities and securitization strategies, the MBS market began to grow significantly. Financial institutions adopted MBS products as a new way to package and sell mortgage debt, leading to the creation of other types of MBS, including those sponsored by private issuers and eventually structured products like collateralized mortgage obligations (CMOs).

4. Influence on Financial Market Innovation

- The RTC's actions highlighted the benefits of using securitization as a risk-transfer mechanism, which became an appealing model for banks and other financial institutions looking to offload their mortgage risk.
- This innovative approach, developed out of necessity by the RTC, was instrumental in encouraging the widespread use of MBS by both private and public entities beyond just the RTC and the Federal Housing Administration (FHA).

Summary

While MBS existed in a limited form before the RTC, the RTC's activities following the S&L crisis helped to scale and normalize the MBS market, contributing directly to its institutionalization and rapid growth in the U.S. The RTC's mass sale of securitized mortgages and demonstrated success with MBS significantly increased market liquidity and set the stage for mortgage-backed securities to become a mainstream asset class.

Overview of the market: convexity

- Mortgages have "negative convexity"
- Convexity: non-linearity in returns; bonds are convex as are options
- In bonds, this is caused by duration changing as a function of the level of rates (bonds are positively convex)
- Homeowners have the option to pre-pay their mortgages, either after a set period of time or upon sale of the underlying property
- When will a homeowner pre-pay? More often than not, after rates have fallen (hence negative convexity)
- Pre-payment modelling a key part of MBS valuation / modelling

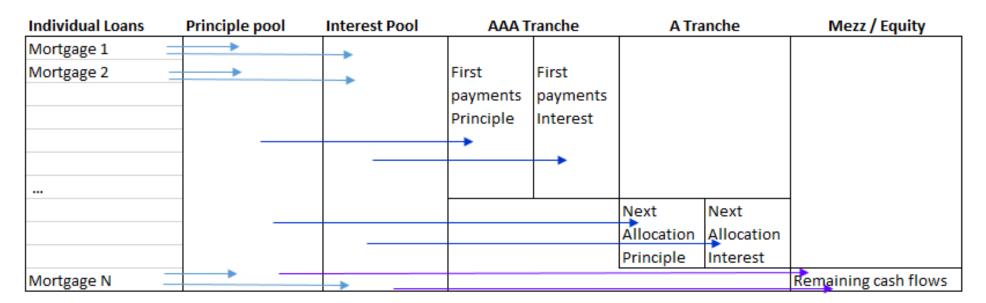
Table A.1: Investors in agency and GSE-backed securities

	\$bn	% of total
Depository institutions	3357	32%
Federal Reserve	2414	23%
Rest of the world	1145	11%
Mutual funds	713	7%
Money market funds	499	5%
State and local governments	428	4%
Life insurance companies	348	3%
Credit unions	297	3%
Pension funds	260	2%
Households and nonprofit organizations	247	2%
Government sponsored enterprises	219	2%
State and local government defined benefit pension funds	201	2%
Mortgage real estate investment trusts	188	2%
Property-casualty insurance companies	137	1%
Foreign banking offices	59	1%
Other	78	1%

Includes issues of federal budget agencies; issues of government sponsored enterprises such as Fannie Mae and FHLB; and agency- and GSE-backed mortgage pool securities issued by Ginnie Mae, Fannie Mae, Freddie Mac, and the Farmers Home Administration. Source: Financial Accounts of the United States, Table L.211, 2021:Q2.

Pass-through structures vs. more complex credit hierarchies

- Pass-through MBS (standard Agency model for residential mortgages) treats all security holders on a pro-rata basis (default costs are born by all)
- CMOs create tranches with different credit, pre-payment and duration profiles. A credit hierarchy or "senior-subordinated" structure is depicted below
- "Stripped" CMOs have PO and IO tranches, etc.
- "B-piece" buyers in CMBS: take most junior tranches and have rights to negotiate work-outs on delinquencies (are often the original issuers)



Asset performance through time: risk behind the "complexity" premium

- Duration risk: identical to Treasury securities in the case of Agency MBS, CMBS
- Pre-payment/negative convexity risk: voluntary or due to defaults (often result in no losses for MBS holder, depending on LTVs of pool)
- Credit risk: a bigger consideration for junior tranches of non-agency RMBS, CMBS, though it exists
- Liquidity risk: TBAs vs. issued MBS (see https://www.newyorkfed.org/research/epr/2013/12 <u>12vick.html</u>)

Table 2: MBS trading volume

Avg. daily
trading volume
(\$bn)

A. Residential: Agency MBS	
TBA	260.95
Specified Pool	25.34
CMO	1.37
Total	287.67
B. Residential: Non-agency MBS	
CMO (IO/PO)	0.05
CMO (P&I)	0.43
Total	0.48
C. Commercial MBS	
Agency CMBS	1.22
Non-Agency CMBS (IO/PO)	0.28
Non-Agency CMBS (P&I)	0.74
Total	2.71
Memo: other USD fixed income securities	
US Treasury	603.2
Corporate debt	38.9
Municipal bonds	12.0
Federal agency securities	5.3
Asset backed securities	1.9

Week 4: ABS

Security features

- ABS are based on assets that are not loans or securities, but other things that generate reliable cash flows:
 - Credit card receivables
 - Auto loans
 - Student loans
 - Aircraft leases
 - Shipping containers
 - Business (e.g., trade) receivables
 - RMBS, CMBS, CLOs, CDOs can be considered a subset of ABSs
- Contractual recourse to payment on the underlying asset can be more secure than a typical bank loan, though generally less standard
 - For an aircraft lease, you can seize the asset (most of the value is in the aircraft engines)
 - For a corporate receivable (e.g., trade finance), you sue the company on the basis of the contract written on the underlying cash flow or in some cases, seize the goods. The court of jurisdiction will force payment; generally, ABS receivables have prior claim to senior unsecured lenders (though not secured lenders)
- Each securitization involves a Special Purpose Vehicle (SPV) independent of the sponsor sponsor creditors do not have recourse to assets of the issue-specific SPVs

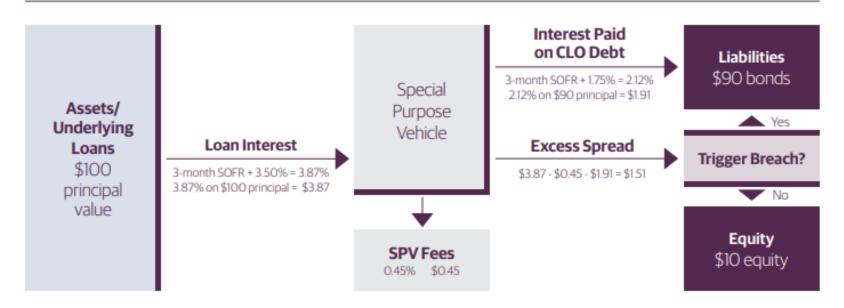
Week 4: ABSs

Security features

- Additional protections / protocols
 - The equity tranche only receives CFs in excess of a given (high) spread levels
 - If asset performance breach minimum performance triggers, CFs are withheld from equity holders and retained for ABS holders

Anatomy of a Securitization

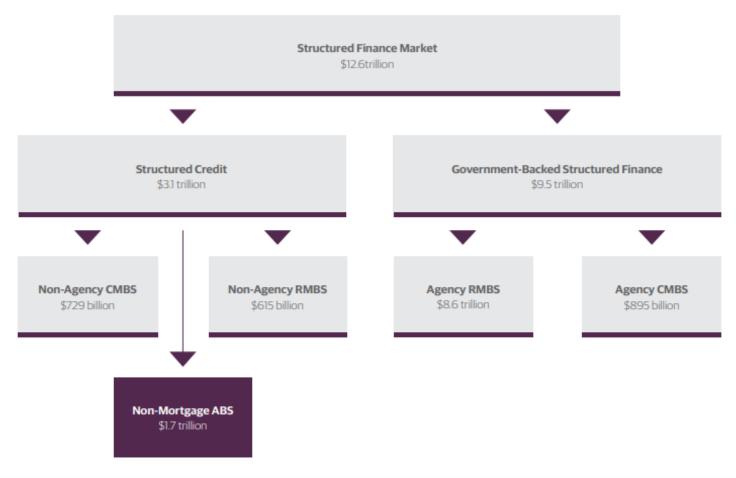
ABS may seem complex, but many embedded features are intended to act as investor protections. In this example, \$3.87 in interest income that is generated by the underlying asset pool pays \$0.45 in SPV fees and \$1.91 in interest expense to CLO debt holders. The use of excess spread is determined by trigger breaches.



Hypothetical example for illustrative purposes only. Secured Overnight Financing Rate (SOFR) is a broad measure of the cost of borrowing cash overnight collateralized by Treasury securities.

Week 4: ABSs

The ABS market in relation to the rest of structured finance

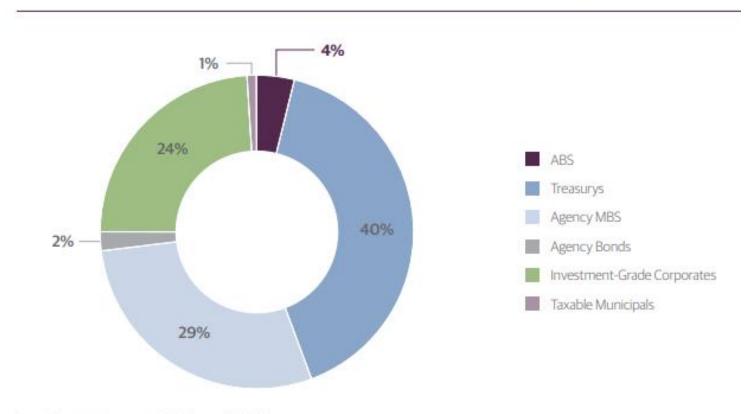


Source: Guggenheim Investments, JP Morgan, eMBS. Data as of 2.28.2023.

Week 4: ABSs

The ABS market in relation to the rest of structured finance

The U.S. fixed-income universe is dominated by Treasurys, Agency MBS and bonds, and investment-grade bonds.



Source: Guggenheim Investments; SIFMA. Data as of 9.30.2022.

Week 4: CLOs, CBOs and CDOs

Basic differences

- CLOs (Collateralized Loan Obligations) now more predominant. The main security that is "pooled" is, in fact, a non-security asset: bank loans
 - Generally below IG (question: if it is a loan, how do we know?)
 - First lien
 - Senior secured
 - Syndicated
 - Smaller fractions of "middle market" loans, second lien loans
- CDOs and CBOs: risk-transfer (pooling + tranche-ing) on top of existing securities
 - CBOs: pools of principally HY and EM bonds, w/ some lower IG bonds at times
 - CDOs: pools of ABS, CMBS, RMBS
- CLOs performed better during the financial crisis; not an accident, as the cash-flow waterfall in these vehicles had been transformed once, not potentially multiple times on lower tranches of other structured securities

Week 4: CLOs

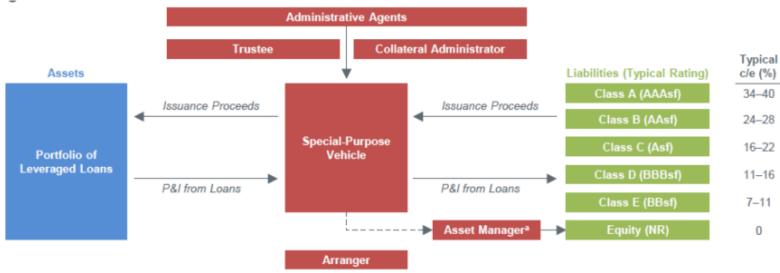
Main features

- CLOs are liabilities of a legal entity that "runs" the structure -- an SPV, as covered in the ABS section
- CLOs main collateral pool is bank loans (the SPVs's core asset pool)
- SPVs liabilities have different tranches with differing priority claims on cash from the asset pool and corresponding credit quality and credit ratings
- The CLO pool of assets is also governed by specific parameters to aid in the determination of credit quality of cashflow waterfall
 - Overcollateralization (face collateral > face CLOs)
 - Interest coverage (income on loans > coupons of CLOs)
 - Loan covenants restricting future decisions of companies issuing loans (seeks to prevent future subordination of collateral to lower rating)
- Why would an investor buy a CLO rather than just a large portfolio of bank loans?
 - Ratings "arb" w/ regulators
 - Risk transfer to more junior tranche holders

Week 4: CLOs

Main features

- Imagine a non-bank that wants to get into the loan business... i.e., without depositors, how does it raise funds? It issues a portfolio of loans with initial capital, creates an SPV, sells off the Class A – E debt securities (note: they pay S&P, Moody's, Fitch to rate their issuance), and retains the equity. Now the sponsor can turn around and issue more loans...
- The non-bank manager ("sponsor") may play any number of roles shown in red below; legal documents govern how each function is executed; the Trustee is often not the sponsor, however; Administrators are often 3rd parties



^aAsset manager typically contributes a portion of equity. P&I – Principal and interest. C/e – Credit enhancement (based on subordination). NR – Not rated. Source: Fitch Ratings.

Week 4: Hedge Fund Allocation

- 1 How should an end-investor select a Hedge Fund portfolio?
- 2 How much should an end-investor allocate to that portfolio?
- At first glance it is a bewildering problem:
 - There are probably over one hundred styles of hedge fund sub-strategies
 - There are thousands of managers...
 - ... but the indisputably good, well-known managers are closed to new investments
 - How do we even find out who the universe of fund managers is?
 - Once we do come up with a list, we find that HF managers have wildly varying track records
- Seemingly great managers in terms of "institutionality" (business platform, staff pedigree, time in business, etc.) can have unimpressive track records
- Unknown managers with unorthodox backgrounds advertise great track records

How does one proceed? Let's start by taking a look and some HF indices and returns

Week 4: Hedge Fund Allocation

Hedge Fund Indices Proliferate... there are many choices, including

- HFR
- CreditSuisse (Tremont)
- EurekaHedge
- Morningstar
- Bloomberg
- Pivotal Path
- BarclayHedge
- SocGen CTA
- There are more...

Week 4: Hedge Fund Allocation, Indices

 Eureka Hedge have the largest number of HF constituents

Eureka Hedge	Eur	eka	He	dge
--------------	-----	-----	----	-----

Bloomberg Ticker	Description	cum xs rtn	vol	SR	beta_eq	beta_UST	alpha
EHFI286 Index	Eurekahedge CTA / Managed Fut ures	6.4%	6.1%	1.05	0.00	0.25	6.0%
EHFI253 Index	EurekaHedge Macro Hedge Fund I	5.3%	3.9%	1.33	0.13	0.08	4.6%
EHFI811 Index	Eurekahedge Commodity Hedge Fund	6.9%	8.9%	0.80	0.18	-0.01	6.1%
EHFI809 Index	Eurekahedge FX Hedge Fund Ind ex	4.5%	4.3%	1.05	-0.03	0.00	4.6%
EHFI255 Index	Eurekahedge Relative Value Hedge Funds	5.8%	4.3%	1.32	0.22	-0.19	5.0%
EHFI285 Index	Eurekahedge Arbitrage Hedge Funds	4.6%	3.2%	1.41	0.13	-0.07	4.1%
EHFI252 Index	Eurekahedge Long / Short Equity	5.9%	7.6%	0.80	0.40	-0.29	4.5%
EHFI751 Index	Eurekahedge Equity Market Neutral	2.7%	2.5%	1.08	0.04	-0.08	2.6%
EHFI288 Index	Eurekahedge Event Driven Hedge Fund	6.3%	7.0%	0.90	0.35	-0.39	5.2%
EHFI287 Index	Eurekahedge Distressed Debt Hedge Fund	7.5%	6.8%	1.11	0.27	-0.37	6.8%
EHFI99 Index	Eurekahedge Global Distressed	1.2%	5.2%	0.26	0.19	-0.31	0.8%
EHFI254 Index	Eurekahedge Multi-Strategy Hedge Fund	6.0%	4.7%	1.26	0.22	-0.11	5.1%
EHFI251 Index	Eurekahedge Hedge Fund Index	5.9%	5.2%	1.13	0.26	-0.12	4.9%
BHEDGE Index	Bloomberg Hedge Fund Index Spliced	5.2%	5.6%	0.93	0.27	-0.13	4.1%
EHFI810 Index	Eurekahedge Hedge Fund Index (USD)	2.5%	12.6%	0.27	0.40	-0.08	0.7%
RU30INTR Index	Russell 3000 TR	4.6%	15.9%	0.37	1.00	-0.65	
LUATTRUU Index	Bbg-Barc UST	1.4%	4.6%	0.32	-0.06	1.00	
Averages:		5.1%	5.9%	0.98	0.20	-0.12	4.3%

Sample: 2000 - 9/2023

Week 4: Hedge Fund Allocation, Indices

• Bloomberg are a more recent entrant

Bloomberg HF Indices

Bloomberg Ticker	Description	cum xs rtn	vol	SR	beta_eq	beta_UST	alpha
BHCTA Index	Bloomberg Commodity Trading Advisors He	1.0%	7.2%	0.17	0.06	-0.05	0.8%
BHMACR Index	Bloomberg Macro Hedge Fund Index	0.7%	4.7%	0.18	0.14	0.03	0.0%
BHDMAC Index	Bloomberg Discretionary Macro Hedge Fur	1.5%	5.5%	0.29	0.29	-0.06	0.2%
BHCOM Index	Bloomberg Commodity Hedge Fund Index	3.3%	5.0%	0.68	0.06	-0.05	3.0%
BHEQLS Index	Bloomberg Equity Long/Short Hedge Fund	2.0%	7.9%	0.29	0.46	-0.09	0.0%
BHEVT Index	Bloomberg Event Driven Hedge Fund Index	2.9%	7.2%	0.43	0.38	-0.28	1.5%
BHQEMN Index	Bloomberg Quantitative Equity Market New	2.5%	4.0%	0.63	0.19	0.00	1.6%
BHSTA Index	Bloomberg Statistical Arbitrage Hedge Fund	1.2%	7.1%	0.20	0.33	0.16	-0.6%
BHFEMN Index	Bloomberg Fundamental Equity Market Ne	2.3%	3.3%	0.73	0.15	-0.16	1.8%
BHACT Index	Bloomberg Activist Hedge Fund Index	7.6%	15.6%	0.55	0.85	-0.49	4.3%
BHMARB Index	Bloomberg Merger Arbitrage Hedge Fund I	3.3%	4.7%	0.72	0.18	-0.15	2.7%
BHRV Index	Bloomberg Relative Value Hedge Fund Ind	2.2%	3.4%	0.67	0.18	-0.09	1.5%
BHFIA Index	Bloomberg Fixed Income Arbitrage Hedge	2.1%	3.7%	0.58	0.16	-0.04	1.4%
BHCBA Index	Bloomberg Convertible Bond Arbitrage He	3.3%	4.9%	0.69	0.22	-0.13	2.5%
BHVOLA Index	Bloomberg Volatility Arbitrage Hedge Fund	6.2%	7.3%	0.87	0.40	-0.22	4.7%
BHCRLS Index	Bloomberg Credit Long/Short Hedge Fund I	2.2%	6.3%	0.38	0.24	-0.14	1.3%
BHDIST Index	Bloomberg Distressed Debt Hedge Fund In	3.5%	9.0%	0.43	0.45	-0.33	1.8%
BHEQMS Index	Bloomberg Equity Multi-Strategy Hedge Fu	0.8%	7.9%	0.14	0.44	0.04	-1.3%
BHRVMS Index	Bloomberg Relative Value Multi Strategy H	3.3%	6.8%	0.50	0.32	-0.03	1.8%
BHCRMS Index	Bloomberg Credit Multi-Strategy Hedge Fu	1.6%	4.5%	0.37	0.21	0.00	0.6%
BHMMS Index	Bloomberg Macro Multi-Strategy Hedge Fu	0.1%	4.3%	0.05	0.18	0.05	-0.8%
BHEDGE Index	Bloomberg All Hedge Fund Index	1.9%	6.0%	0.35	0.34	-0.04	0.4%
EHFI251 Index	Eurekahedge Hedge Fund Index	3.7%	5.2%	0.72	0.30	0.00	2.3%
RU30INTR Index	Russell 3000 TR	8.9%	15.5%	0.63	1.00	0.01	
LUATTRUU Index	Bbg-Barc UST	-0.8%	4.5%	-0.15	0.10	1.00	
Averages:		2.7%	6.5%	0.44	0.28	-0.09	1.4%
Sample: 2013 - 9/	າດາວ						

Sample: 2013 - 9/2023

Week 4: Hedge Fund Allocation, Indices

- There are a number of problems / issues with hedge fund indices (various biases)
- Pivotal Paths is an emerging HF index provider who have made efforts to address methodological issues other providers suffer. To date, PPs have had good GP buy-in and they offer a sophisticated UI (user interface) for GP-specific and fund information
- For a similar sample period, Pivotal Paths shows comparable characteristics to Bloomberg indices

Risk Free Return	1.82%		Correlation to:				Beta to:				
Index (2011 - 2023)	Avg Return	StdDev	SR	Max DD	S&P500	BarcGlobAgg	Alpha	S&P500	BarcGlbAg	XS Rtn	
PvtPath Composite	5.91%	4.62%	0.88		0.77	0.12	1.50%	0.25	0.14	4.08%	
S&P 500	12.43%	14.40%	0.74		1.00	0.25	0.24%	1.00	0.87	10.61%	
Barclays US Agg	1.54%	4.12%	-0.07		0.25	1.00	-0.75%	0.07	1.00	-0.28%	
PvtPath Credit	5.86%	4.80%	0.84		0.58	0.05	2.01%	0.19	0.05	4.04%	
PvtPath Equity Diversified	6.34%	6.63%	0.68		0.88	0.20	0.29%	0.41	0.33	4.51%	
PvtPath Event Driven	5.60%	6.80%	0.56		0.76	0.07	0.01%	0.36	0.11	3.78%	
PvtPath Global Macro	4.91%	4.28%	0.72		0.22	-0.07	2.36%	0.07	-0.07	3.09%	
PvtPath Managed Futures	4.38%	7.98%	0.32		-0.04	-0.02	2.77%	-0.02	-0.03	2.56%	
PvtPath Multi-strat	6.09%	3.73%	1.15		0.52	0.01	2.83%	0.14	0.01	4.27%	
PvtPath Equity Quant	6.96%	4.14%	1.24		0.43	0.11	3.86%	0.12	0.11	5.14%	
Average:	5.76%	5.37%	0.80		0.52	0.06	1.95%	0.19	0.08	3.93%	

• In all of these tables we showed an "alpha" calculation:

$$(r_{HFindx} - rfr) = \alpha_{HFindx} + \beta_{S\&P} \cdot (r_{S\&P} - rfr) + \beta_{BarcAgg} \cdot (r_{BarcAgg} - rfr) + \varepsilon_{HFindx}$$

- 4.4% of your return in the Composite PvtPths index is generated by the risk free rate and beta exposures... your alpha is just 1.5%, but you pay 2 & 20 on the whole return... (Gross return was \sim 7.5%, so fees were 2% + 20%*5.5% = 3.1%)
- That is, fees were more than double alpha! What is a fair split? I would say 60-40 (not 33-67)

Risk Free Return	1.82%				Correlation to:			Beta to:				
Index (2011 - 2023)	Avg Return	StdDev	SR	Max DD	S&P500	BarcGlobAgg	Alpha	S&P500	BarcGlbAg	XS Rtn		
PvtPath Composite	5.91%	4.62%	0.88		0.77	0.12	1.50%	0.25	0.14	4.08%		
S&P 500	12.43%	14.40%	0.74		1.00	0.25	0.24%	1.00	0.87	10.61%		
Barclays US Agg	1.54%	4.12%	-0.07		0.25	1.00	-0.75%	0.07	1.00	-0.28%		
PvtPath Credit	5.86%	4.80%	0.84		0.58	0.05	2.01%	0.19	0.05	4.04%		
PvtPath Equity Diversified	6.34%	6.63%	0.68		0.88	0.20	0.29%	0.41	0.33	4.51%		
PvtPath Event Driven	5.60%	6.80%	0.56		0.76	0.07	0.01%	0.36	0.11	3.78%		
PvtPath Global Macro	4.91%	4.28%	0.72		0.22	-0.07	2.36%	0.07	-0.07	3.09%		
PvtPath Managed Futures	4.38%	7.98%	0.32		-0.04	-0.02	2.77%	-0.02	-0.03	2.56%		
PvtPath Multi-strat	6.09%	3.73%	1.15		0.52	0.01	2.83%	0.14	0.01	4.27%		
PvtPath Equity Quant	6.96%	4.14%	1.24		0.43	0.11	3.86%	0.12	0.11	5.14%		
Average:	5.76%	5.37%	0.80		0.52	0.06	1.95%	0.19	0.08	3.93%		

- At large end-investors Strategic Asset Allocation (SAA) governs allocation targets to asset classes
- BlackRock, JPMorgan AM, PIMCO, etc. all product "LTCMA"s for their clients. JPM's are used below
- What is a high alpha-to-beta ratio mean for allocation to HFs? (the optimization below uses a standard mean-variance utility function with an additional spec accounting for risk charges faced by Life Insurance companies)
- Note JPM's 2023 assumptions for diversified HFs: E[rtn] = 4.8%, E[vol] = 6.9%. This produces a very low demand for HFs (1% of portfolio), and is in line with returns in HFR, EurekaHedge indices
- On the other hand, an AbsRtn (0 beta) HF portfolio with an expected return of 7.5% and an expected vol of 6% receives an allocation of 10%. If the LT risk free rate assumption is 4%, the Sharpe of this portfolio would be 0.6 (7.5% 4%)/6%

Asset	E[rtns]	E[vol]	Opt Wts	Corre	lation	S								
'TIPS'	4.1%	5.6%	0%	1	0.58	0.69	0.46	0.63	0.26	0.1	0.03	0.31	0.23	0.0
'USTs'	4.0%	12.3%	0%	0.58	1	0.54	0.62	0.7	-0.1	-0.5	-0.3	0.09	-0.2	0.0
'IG'	5.3%	5.7%	37%	0.69	0.54	1	0.66	0.66	0.46	0.29	0	0.46	0.38	0.0
'HY'	6.6%	8.5%	8%	0.46	0.62	0.66	1	0.34	0.74	0.7	0.42	0.64	0.71	0.0
'StrcCr'	4.8%	3.1%	24%	0.63	0.7	0.66	0.34	1	0.16	-0.2	-0.2	0.25	0.03	0.0
'US_LargeCapPubEqty'	7.7%	16.1%	5%	0.26	-0.1	0.46	0.74	0.16	1	0.77	0.76	0.41	0.78	0.0
'PE'	9.7%	20.0%	9%	0.1	-0.5	0.29	0.7	-0.2	0.77	1	0.42	0.51	0.78	0.0
'US_RealEstate'	7.5%	17.6%	4%	0.03	-0.3	0	0.42	-0.2	0.76	0.42	1	0.53	0.37	0.0
'US_REITs'	6.6%	15.8%	3%	0.31	0.09	0.46	0.64	0.25	0.41	0.51	0.53	1	0.45	0.0
'HFs_highBeta'	4.8%	6.9%	1%	0.23	-0.2	0.38	0.71	0.03	0.78	0.78	0.37	0.45	1	0.0
'AbsRtn Obeta'	7.5%	6.0%	10%	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1

Portfolio Stats							
E[rtn]	7.9%						
E[vol]	8.3%						
E[Sharpe]	0.65						

LTCMA source: JPMorgan, 2023

- Optimal Strategic Asset Allocation is pretty clear about what characteristics of our Hedge Fund portfolio that would create demand for a large end-investor: good capital efficiency, and high alpha-to-beta proportions
- If we choose the following target characteristics for our hedge fund portfolio, it would be attractive in an SAA sense for a large allocator:
 - Low-beta (<0.1 on equity, rates)
 - Relatively high vol (>6%)
 - O Sharpe ratio >= 0.6
- A traditional portfolio of hedge funds with a relatively high beta (0.4) to stocks would not be very attractive in an SAA sense, at least in the evaluation of JPM

Individual manager issues

- Survivorship bias in existing funds
- Conflicts of interest between LPs and GP: vol, capacity
- Style bias creep
- Factor cheating

HF asset class issues

- Adverse selection in HF index construction
- Are HF returns persistent (can we predict HFs' future returns based on their track records)?
- Skewness vs Sharpe ratio
- The problem of too little volatility
- Do any characteristics of HFs predict alpha?
- Do HFs even deliver alpha?

The parable of 100 horses:

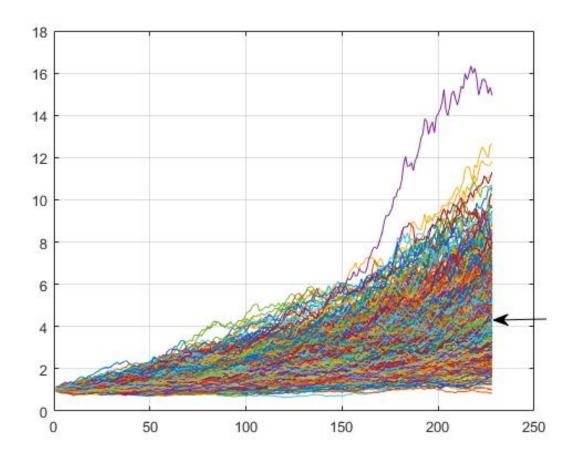
- A "con man" gets a hold of the names and cell phone numbers of 100 known horse race bettors
 who frequent Saratoga Springs
- Suppose that at Saratoga all the races have 10 horses and that there is always a race at 3p on Saturdays
- The con man calls all 100 betters. He tells the first 10 the name of the first horse as next Saturday's winner; he call another 10 bettors, telling them the name of the second horse, and so on. After the race is run, 10 bettors had been told the name of the winning horse.
- The next week the con man calls the 10 bettors who were told the winning horse the prior week.
 He tells the first, horse number 1, will win; he tells the second bettor horse number 2 will win, and
 so on. After the race is run on the second Saturday, one bettor has been told the name of the
 winning horse two weeks in a row.
- The third week, the con man calls that one better and says, just like prior weeks, "I have the name of the horse that will win on Saturday." The bettor responds, "great, which horse is it?!" The con man replies, "sure, but this time I'll need some money first."

Survivorship bias does not require agents behaving malevolently. An example:

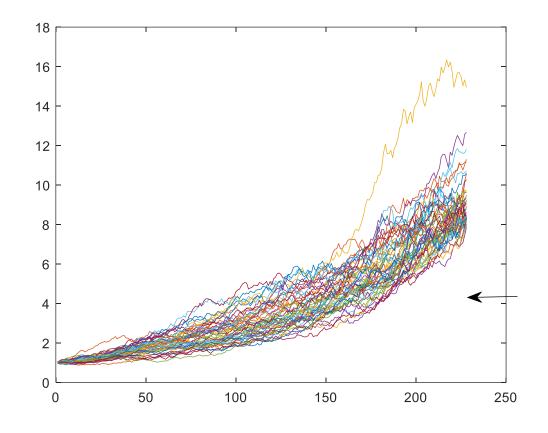
Specify a monthly return process:

$$r_t = \frac{rfr_t}{12} + \frac{0.075}{12} + \epsilon_t, \epsilon_t \sim N(0.0.1/\sqrt{12})$$

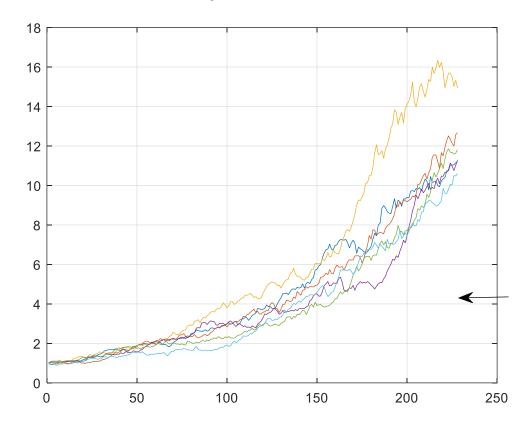
- We simulate the excess returns of a hedge fund with a vol of 10% and a Sharpe of 0.75 over 20 years, 1000 times
- (1.075)^20 = 4.25, so that is the mean of the cumulative excess return of these 1000 simulations
- Remember: the dispersion you see is just due to statistical draws... all these time series reflect an underlying process w/ a Sharpe of 0.75



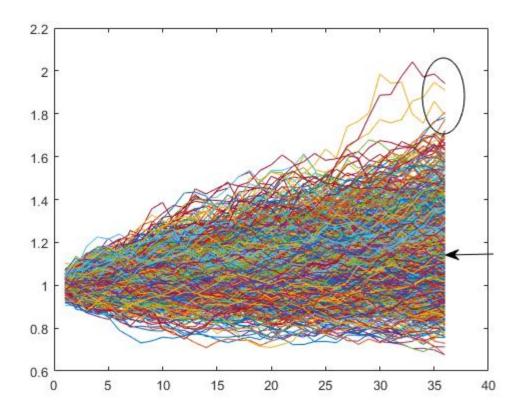
- To the right you see the best 5% draws... i.e., 1 in 20
- The average Sharpe ratio in this group is
 1.21, the average excess return is 12%
- That is, after 20 years, 50 HFs with true Sharpe's of 0.75 have "lucked into" Sharpe's greater than 1.2...



- Here are the best 6 draws
- Their average Sharpe ratio is 1.38, the average excess return is 13.8%
- Again, this is over 20 years. You can't assume that just because a fund has been around for even 10 years that its talent / process is "real"



- This simulation was done over 20 years... if you do something similar over 3 years the problem becomes SO much worse...
- Suppose we simulate 1000 Hedge Funds with a true Sharpe of 0.5 over 3 years.
- Nearly 5% of them will have Sharpes over 1.5 after that time...
- It is very easy to think you are investing in a Sharpe > 1.5 manager but they really have a Sharpe of 0.5
- In the plot, the arrow shows the mean cum return, the ellipse identifies 6 strategies w/ Sharpe ratios > 2



How should HF investors deal with the problem?

- When they see a Sharpe > 1 assume a lot of it is luck
- Try to ignore the track record and study the investment process: its originality and its rigor, the strength of the investment theses within the process, look for a repeatable, systematic logical decision making process, caliber of people executing the functions, etc.
- Many, many things...
- The truth: a new fund with a Sharpe of 1.5 for 3 years can generally find a marketing person to help them tell a compelling story... they will raise money.
- Most new hedge funds will cease having a high Sharpe within a year or two of raising a lot of money
- The vast majority of hedge funds cease to exist after 5 years

Week 4: Hedge Fund Allocation, LP-GP conflicts of interest: the problem of capacity / vol rationing

Problems arise when a new hedge fund makes good returns and raises significant capital:

- People notice and try to copy (example: AHL's "Evolution" system. Spin-offs: Gresham, Florin Court, Systematica, AQR, Campbell, Advent, Winton, ISAM, ...). At least \$40bn is being invested in markets that were considered too illiquid to trade even a single fund of \$1bn AUM 10 years ago
- The manager "pushes" market prices against himself as he trades, raising TCs. TCs can be 20, 30, even 50% of total PnL, depending on the strategy
- Plus there is liquidity: if there is a market reversal, how quickly can you get out of your positions? Many hedge funds hold positions that it could take one or two weeks to get out of... double your AUM, and now these positions take a month to get out of, and your TCs rise...
- In all cases, your Sharpe will fall unless you improve your core strategy or diversify into others
- Looking at the Eureka Hedge database, less than half of the managers there lasted more than 2 years. Less than 10% lasted more than 5 years.

Week 4: Hedge Fund Allocation, Due Diligence Primer

- 1. Bernie Madoff: a wake-up call
- 2. Manager investment skill: what is the thesis? Do we believe they are capable of executing that thesis?
- 3. Trade execution: personnel, processes (automation), pricing resources, counterparties
- 4. Trade clearing: ops
- 5. Risk management
- 6. Ownership structure: incentives of controlling interests of GP
- 7. Legal / compliance (regulatory, etc.)
- 8. Other businesses / conflicts of interest / management company longevity
- 9. IT robustness: DevOps, network security, user support, disaster recovery
- 10. 3rd Parties: Fund Admin, Audit, PB, ISDA counterparties, trade clearing

Week 4: Resources and References

AQR White Paper. (2019, 2020, etc.) "Alternative Thinking Capital Market Assumptions."

Chen and Greenburg (2017). "Consistent Risk Modeling of Liquid and Illiquid Asset Returns," Journal of Portfolio Management, Special Real Estate Issue: 73-89.

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