A Factor Approach to Strategic Asset Allocation for Portfolios with Alternatives

FRE-GY 6921, Week 6

NYU Tandon Program in Financial Engineering

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Spring 2025

Weeks 6&7: Outline

- Overview/review of end-investors and asset classes for SAA
- Generalized optimization problem: objective, E[rtns], E[risk], constraints
- E[rtns]
- E[risk]
- Problems with Optimizers: Robust Optimization
- Optimal SAA
  - Endowments/SWFs
  - Pensions
  - Life Insurance
- Next week:
  - Wednesday (4/30): Project first drafts due. 5/1? You will get comments back by CoB 5/3 (Sat)
  - Quiz 2
  - Final Drafts of projects due by 5:00p, and we'll do presentations during the 2nd half of class
  - Problem Set 2 due May 12th; I will post it on Thursday (not hard)

Week 6: Endowments, Pensions, L&Rs Review

- Despite having seemingly similar objectives…
  - SWFs, Endowments: invest for the welfare of citizens, beneficiaries, present and future
  - Pensions: invest for the retirement of current working age members
  - L&R Insurers: invest for retirement / end of life for policy holders
- … the end-investors we are studying hold very different asset allocations. Why?

Asset Class	Endowments	Pensions	L&Rs
Public Equity	40.5%	36.8%	8.7%
PE / VC	19.7%	6.1%	9.6%
Hedge Funds	17.1%	5.3%	0.0%
Real Assets	6.7%	6.6%	0.4%
Public and Private Credit	9.6%	22.9%	68.1%
Fixed Income & Cash	1.8%	22.2%	13.2%

- Large end-investors' portfolio allocations are large relative to available liquidity – moving an allocation even 1% might take a year—so the investment time horizon is 10 years or beyond… imagine an elephant chasing a fly
- Experience and finance theory tell us the best way to diversify is through **Factors**

Week 6: Asset Classes

- Rates – cash (T-bills, deposits, CDs, etc.), government bonds, and rate derivatives (gov't bond futures, IR swaps, etc.)
- Public Credit – Corporate bonds, sovereign bonds (IG, HY, EM)
- Structured Credit – MBS, CMBS, ABS, CLOs, etc. Securities funded on pools of underlying loans, where each security class's payouts are determined by a hierarchy of payment-priorities (e.g., "waterfall") structured to create an array of risk profiles from that single pool
- Private Credit – LPs (limited partnerships) and direct investment in whole loans from "middle market" (small cap) corporates, Private Equity investors (leveraged loans), and enterprises deemed to have steady future cash flows
- Public Equity – listed stocks
- Private Equity (PE) – LPs (limited partnerships) and direct investment in firm ownership, where the firms are not publically listed (e.g., pre-IPO). Many different contexts: LBO, growth (e.g., post-VC funding rounds, anticipating an IPO exit), etc.
- Private Equity Real Estate – Sub-set of PE, focused on particular collateral and cash-flow characteristics of commercial real estate firms
- Hedge Funds – LPs on managers that pursue active trading ("alpha" oriented) strategies on behalf of LP members

	E[rtns]	**E[vol]**
Asset Class	**Historical**	**Historical**
USTs	3.72%	5.39%
TIPS	4.94%	5.34%
US Corp Credit, IG	4.76%	4.92%
US Corp Credit, HY	6.91%	10.0%
EM IG	6.04%	7.84%
EM Broad	7.31%	8.97%
US MBS Agency	3.77%	3.87%
US CMBS	4.95%	6.72%
US ABS	3.67%	3.88%
Private Credit	7.41%	3.30%
US Public Equity	8.84%	17.1%
US Private Equity	12.0%	9.85%
US Real Estate	7.73%	9.50%
Global Hedge Funds	8.14%	6.17%
AbsRtn Funds	8.39%	5.88%
Sample: 2000-Q1-2025		

Week 6: A General Optimal SAA Set-up

The canonical problem:

$$\max \omega \cdot \mu' - \lambda_1 \cdot \omega\Sigma\omega'$$
$$s.t.$$
$$\omega \cdot \iota' = 1$$
$$\omega\Sigma\omega' \leq \bar{\sigma}^2$$
$$\omega_n \geq lb_n, \forall\, n$$
$$\omega_n \leq ub_n, \forall\, n$$

μ = $E[rtn]$ = vector of N asset-specific expected returns

$\omega \cdot \mu'$= $\sum_{n=1}^{N} \omega_n \cdot E[rtn_n]$ = expected portfolio return

Σ= the N x N variance-covariance matrix of the underlying assets

$\omega\Sigma\omega'$= the expected variance of the portfolio

Week 6: A General Optimal SAA Set-up

Examples of some parameter values and solutions (reduced universe):

Horizon: 10 years

Asset Class	**E[rtn]**	**E[vol]**	**E[SR]**	**Correlations**				
Global Public Equities	8.8%	17.4%	0.35	1	0.75	0.17	-0.25	0.49
Private Equity -- Buy-Out	11.2%	32.1%	0.27	0.75	1	0.14	0.65	0.35
Corporate Credit -- BBB	4.70%	5.90%	0.35	0.17	0.14	1	0.65	0.2
Government Securities, US	3.60%	5.00%	0.19	-0.25	0.65	0.65	1	-0.1
Hedge Funds -- Traditional	8.30%	6.00%	0.94	0.49	0.35	0.2	-0.1	1

Source: BlackRock Investment Insitutue

Output:

Output:						
Weights: \ Lambda:	0	5	7.5	10	12.5	15
Global Public Equities	40%	22%	13%	9%	6%	5%
Private Equity -- Buy-Out	6%	3%	3%	3%	3%	3%
Corporate Credit -- BBB	12%	33%	40%	39%	33%	29%
Government Securities, US	3%	3%	4%	10%	18%	23%
Hedge Funds -- Traditional	40%	40%	40%	40%	40%	40%
Portfolio E[rtn]	8.23%	7.26%	6.89%	6.64%	6.47%	6.36%
Portfolio E[vol]	10.00%	6.63%	5.64%	5.11%	4.79%	4.61%
Portfolio E[SR]	0.55	0.68	0.73	0.76	0.78	0.78

Week 6: A General Optimal SAA Set-up

Same inputs used to trace the efficient frontier:

$$\overrightarrow{\omega(i)} = \operatorname{argmin} \omega\Sigma\omega'$$

$$s.t.$$

$$\omega \cdot \iota' = 1$$

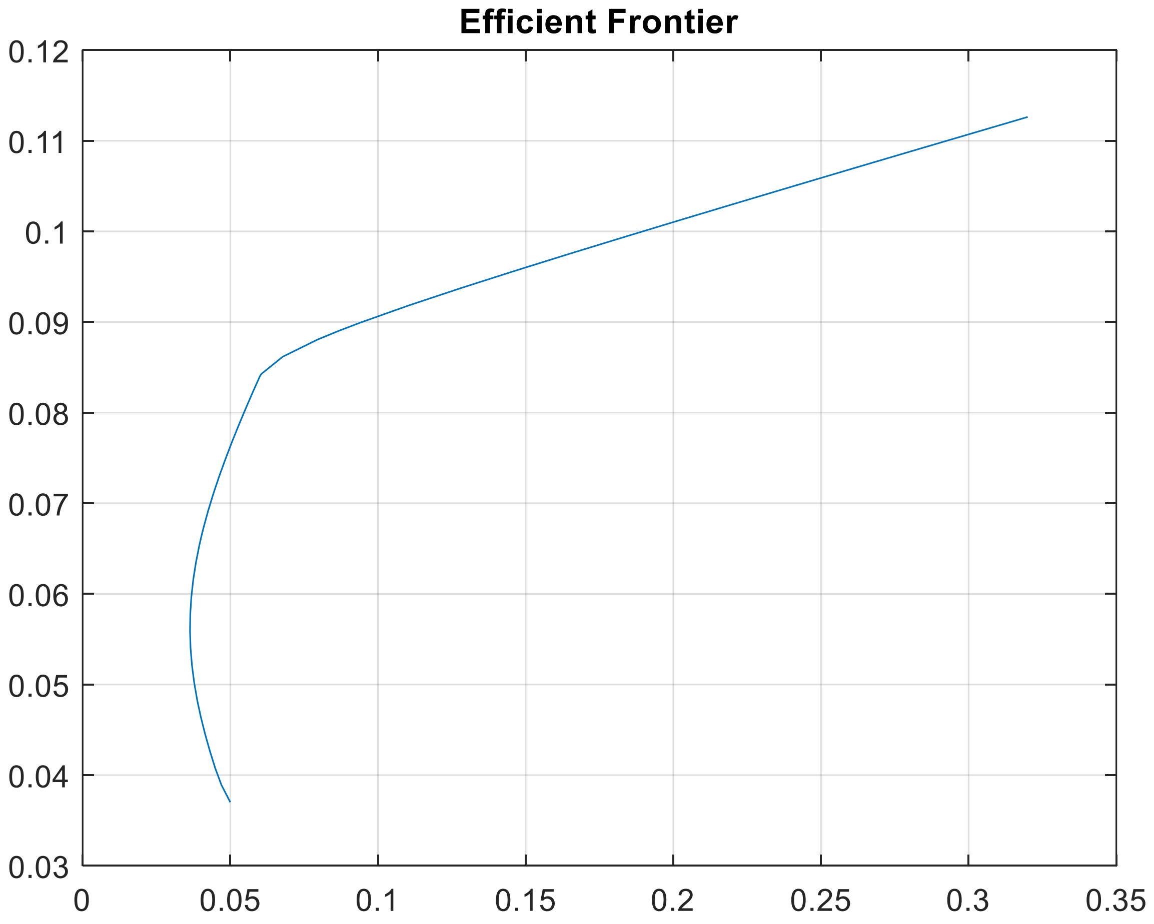
$$\omega_n \geq 0, \forall\, n$$

$$\omega_n \leq 1, \forall\, n$$

$$\omega \cdot E[rtn]' = \mu_{port,i}$$

Plot the pairs $\{\mu_{port,i}, \sigma_{port,i}\}$, where:

$$\sigma_{port,i} = \sqrt{\omega(i) \cdot \Sigma \cdot \omega(i)'}$$



Week 6: Expected Returns

2 key inputs to our optimizer: $E[rtn]$ and Σ

4 approaches for $E[rtn]$:

- Historical averages
- Industry publications (LTCMAs)
- Econometric / APT
- Fundamental factor approach

Week 6: Expected Returns

Historical Approach

- Past ≠ Future…
- …BUT all we have is data from the past

Advantages:

- Simplicity: complexity often introduces errors, biases we are unaware of
- Representative: last 25 years includes 3 full cycles, various monetary/fiscal/global integration cycles
  - Does the sample begin at non-extreme valuations? (stocks no; rates, no; credit spreads, yes)
  - Is there a reasonable variety of economic environments during the sample? (yes)
- It is a long sample: 101 quarters
- It boils down to, is our past sample representative for environments we are likely to encounter in the future?

	E[rtns]	E[vol]
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USTs	3.72%	5.39%
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Sample: 2000-Q1-2025		

Week 6: Expected Returns

Disadvantages:

- Starting points matter: ex-post, 2011-2021 investment performance was MUCH BETTER than it was in 2008 – 2018
- If 10-yr real yields are > 3%, i.e., in 2000, 2023, the prospects for fixed income out-performance much be must better than when real yields are < 1%, i.e., in 2010, 2016, or 2020. Similar observations apply to stocks.
- Isn’t there a more “structural” approach to the data allowing that we account for initial period valuation considerations?

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Week 6: Expected Returns

Industry Publications

- Almost all financial institutions have income/capital budgeting process that require Long Term Capital Market Assumptions (LTCMAs)… There is big demand
- Who fulfills demand? Asset managers (BLK, BX, JPM AM, etc.), consultants (Mercer, Willis Towers Watson, AON, Russell, NEPC, Cambridge Associates, Callon, Wilshire, etc.)
- What methodology do they use?
  - On the surface, not very systematic
  - A combination of historical averages with judgement, some market calls

29th annual edition

2025 Long-Term Capital Market Assumptions

Time-tested projections to build stronger portfolios

J.P.Morgan
ASSET MANAGEMENT

Week 6: Expected Returns

JPM LTCMAs

In brief

- We publish the 29th edition of our Long-Term Capital Market Assumptions (LTCMAs) as a new economic era comes into focus. Gone is the low investment, low growth and low interest rate world of the 2010s. In its place is a healthier economy set to deliver higher growth, strong capital investment trends and higher interest rates.
- Higher policy rates reinforce strong projected bond returns, while higher growth underpins equity returns. Our return projection of 6.4% for a USD global 60/40 stock-bond portfolio dips 60 basis points (bps) from last year – a forecast that is in line with the long-run average. Alternatives are emerging from a period of asset markdowns and offer compelling returns and diversification options.
- Governments have shifted from austerity toward fiscal activism. For this shift to boost real growth without merely fueling inflation will require investments that stimulate supply rather than simply stoke demand. Avoiding risks from higher inflation may also require labor market reform and thoughtful migration policies. In any case, elevated bond market and currency volatility seem inevitable.

- An increase in tendencies toward economic nationalism – despite stopping short of deglobalization – means our estimate of inflation volatility remains elevated and underscores the utility of assets with positive gearing to inflation, such as commodities and real assets.
- In the coming decade, the benefits of artificial intelligence (AI) and automation will accrue increasingly to the wider economy and likely support corporate earnings. We now project a 20bps annual boost to developed market growth from AI – a potentially conservative estimate. Ultimately, we expect AI to improve total factor productivity, putting downside pressure on inflation.
- Investors will need to manage a range of risks, not least from the geopolitical tensions that currently dominate headlines. But overall, our 2025 LTCMAs offer an optimistic outlook. As investment levels pick up and rates normalize, a healthy – even buoyant – economy will emerge, providing a strong foundation for asset markets.

Question: is this a long-term or medium-term forecast?

Week 6: Expected Returns

JPM LTCMAs

Exhibit 16: Key structural risks affecting our long-term forecasts and asset return assumptions

Risk	Upside or downside?	Description	Macro or asset class implications
Concern about rising debt causes a return to post GFC-style austerity	Downside	Governments reduce spending plans and cull investment spending to focus on day-to-day current spending	Growth falls and tax rates increase to meet government liabilities. Positive for bonds, negative for equity
Regional conflicts extend or spill over, sucking in NATO or China	Downside	Russia-Ukraine invasion, Middle East war or other conflicts deepen or broaden and suck in superpowers	Potential for supply chain and energy shocks, with risk of retaliatory sanctions further disrupting trade. Positive for bonds and USD, supports commodities, negative for stocks, hits Europe hardest
Trade tensions between U.S. and China reignited	Downside	Washington and Beijing find themselves in a renewed trade dispute with tit-for-tat tariffs and sanctions on a wide range of goods	Further retrenchment to regional blocs damages growth and is inflationary at the margin. Commodity prices remain elevated, industrial sectors under pressure as supply chains compromised
Rapid abandonment of USD as key reserve currency	Downside	Challenger to USD (from either crypto or an alternative fiat currency) emerges and pulls reserve assets away from USD, diminishes demand for U.S. assets and refocuses attention on U.S. deficit	Negative for growth, USD, bonds, credit and stocks; positive for real assets and commodities
Debt default by U.S.	Downside	Debt ceiling and other budget issues in U.S. reach stalemate, leading to default; debt repayments consume too much from budget to be politically palatable	Deeply negative for risk assets, and risks causing liquidity crunch as uncertainty around definitions for riskless assets are challenged. Initially positive for bonds, but may rapidly see non-U.S. bonds outperform; gold, CHF and JPY positive
European energy independence through renewables investment	Upside	European countries double down on investments after reducing reliance on Russian gas to speed up adoption of renewable energy sources and sustainable infrastructure	Uncertainty removed from European energy grid; skills deepening from investment a positive boost to productivity while infrastructure improvements add a further positive support; EUR and EU equities net winners

Week 6: Expected Returns

JPM LTCMAs

Exhibit 16: Key structural risks affecting our long-term forecasts and asset return assumptions

Risk	Upside or downside?	Description	Macro or asset class implications
Worsening climate or environmental situation	Downside	More frequent or more extreme weather events, leading to destruction of productive assets and disruptions to food and basic materials supply	Supply disruption in short run, then pressure on scarce resources during rebuild leads to higher inflation. Positive for commodities, real assets; negative for bonds, stocks, credit
Stronger than expected investment and capex cycle	Upside	Surge in fiscal spending and upswing in capex that followed pandemic lead to building of productive capacity and upskilling in labor	Positive for real GDP while limits inflation; supportive for stocks, credit and other risk assets; mitigates some right tail inflation risks from bond markets; may favor DM over EM assets
Accelerated adoption of artificial intelligence	Upside	Labor scarcity that is limiting growth in some regions mitigated; scope for productivity to rebound strongly, improving trend growth	Positive for real GDP while limits inflation; supportive for developed market stocks, credit and other risk assets; mitigates some right tail inflation risks
Successful cyberattack on financial system or national infrastructure	Downside	Economically essential infrastructure knocked out in cyberattack, leading to tangible losses and damaging confidence in the system	Negative for growth, as it potentially slows technology adoption or reduces confidence in the system. Positive for bonds at the margin, short-run negative for equity, but sectors and firms seen as having resilience or a solution could rally
Secondary pandemics or emergence of vaccine-resistant strains	Downside	Vaccine-resistant strain of current pandemic or entirely new pathogen emerges, necessitating rolling lockdowns and creating disruption to supply chains globally	Negative for growth, but likely leads to further stimulus, leading to cyclical volatility and risking further expansion of deficits; positive for bonds in short run, but risks longer period of financial repression in longer term; increases volatlity in equities
Next-generation lifestyle and chronic illness therapies become widely available	Upside	GLP-1 style drugs to combat chronic sickness in working-age populations improve efficacy and are more widely adopted, increasing working lifetime	Equivalent to increasing labor pool by extending productive working life, but without the negative impact of simply raising retirement age. Better growth supports risk assets such as equity and credit

Week 6: Expected Returns

JPM LTCMAs

		Compound Return 2025 (%)	Arithmetic Return 2025 (%)	Annualized Volatility (%)	Compound Return 2024 (%)	U.S. Inflation	U.S. Cash	U.S. Intermediate Treasuries	U.S. Long Treasuries	TIPS	U.S. Aggregate Bonds	U.S. Securitized	U.S. Short Duration Government/Credit	U.S. Long Duration Government/Credit	U.S. Inv Grade Corporate Bonds	U.S. Long Corporate Bonds	U.S. High Yield Bonds	U.S. Leveraged Loans	World Government Bonds hedged	World Government Bonds	World ex-U.S. Government Bonds hedged	World ex-U.S. Government Bonds	Emerging Markets Sovereign Debt	Emerging Markets Local Currency Debt	Emerging Markets Corporate Bonds	U.S. Muni 1-15 Yr Blend	U.S. Muni High Yield
Fixed Income	U.S. Inflation	2.40	2.41	1.60	2.50	1.00																					
	U.S. Cash	3.10	3.10	0.65	2.90	-0.03	1.00																				
	U.S. Intermediate Treasuries	3.80	3.85	3.34	3.90	-0.27	0.17	1.00																			
	U.S. Long Treasuries	4.30	5.07	12.83	5.20	-0.22	0.03	0.83	1.00																		
	TIPS	4.10	4.26	5.78	4.60	-0.02	0.02	0.62	0.60	1.00																	
	U.S. Aggregate Bonds	4.60	4.70	4.52	5.10	-0.24	0.08	0.84	0.85	0.76	1.00																
	U.S. Securitized	4.90	4.97	3.82	5.30	-0.23	0.10	0.78	0.74	0.69	0.93	1.00															
	U.S. Short Duration Government/Credit	3.90	3.91	1.55	3.90	-0.30	0.28	0.84	0.59	0.63	0.82	0.76	1.00														
	U.S. Long Duration Government/Credit	4.70	5.29	11.19	5.70	-0.22	0.02	0.74	0.90	0.69	0.94	0.81	0.66	1.00													
	U.S. Inv Grade Corporate Bonds	5.00	5.25	7.28	5.80	-0.19	0.01	0.52	0.60	0.72	0.87	0.75	0.66	0.85	1.00												
	U.S. Long Corporate Bonds	4.90	5.58	12.08	6.00	-0.19	0.00	0.51	0.67	0.67	0.87	0.74	0.60	0.91	0.97	1.00											
	U.S. High Yield Bonds	6.10	6.44	8.52	6.50	0.00	-0.05	-0.02	0.00	0.48	0.38	0.38	0.28	0.33	0.66	0.60	1.00										
	U.S. Leveraged Loans	6.60	6.88	7.80	6.50	0.18	-0.05	-0.37	-0.28	0.18	0.03	0.04	-0.05	0.03	0.36	0.31	0.77	1.00									
	World Government Bonds hedged	3.90	3.97	3.87	4.20	-0.29	0.10	0.86	0.87	0.62	0.87	0.78	0.72	0.84	0.66	0.68	0.12	-0.20	1.00								
	World Government Bonds	4.20	4.44	7.06	4.80	-0.17	0.09	0.75	0.66	0.69	0.80	0.72	0.74	0.74	0.69	0.68	0.33	-0.05	0.72	1.00							
	World ex-U.S. Government Bonds hedged	3.80	3.87	3.81	4.00	-0.28	0.10	0.72	0.74	0.58	0.79	0.70	0.63	0.77	0.64	0.66	0.19	-0.10	0.96	0.66	1.00						
	World ex-U.S. Government Bonds	4.20	4.57	8.76	4.90	-0.16	0.08	0.65	0.55	0.65	0.74	0.67	0.68	0.68	0.68	0.67	0.40	0.04	0.64	0.98	0.61	1.00					
	Emerging Markets Sovereign Debt	5.80	6.24	9.71	6.80	-0.14	0.03	0.31	0.34	0.62	0.66	0.60	0.49	0.62	0.82	0.78	0.75	0.45	0.44	0.60	0.46	0.64	1.00				
	Emerging Markets Local Currency Debt	6.10	6.78	12.15	6.00	-0.05	0.09	0.22	0.18	0.45	0.47	0.42	0.41	0.43	0.61	0.58	0.62	0.35	0.26	0.61	0.28	0.67	0.81	1.00			
	Emerging Markets Corporate Bonds	6.20	6.58	8.96	6.70	-0.07	-0.02	0.19	0.23	0.55	0.57	0.50	0.44	0.52	0.80	0.73	0.73	0.57	0.31	0.48	0.32	0.51	0.89	0.73	1.00		
	U.S. Muni 1-15 Yr Blend	3.60	3.68	4.04	4.00	-0.17	0.06	0.52	0.51	0.58	0.72	0.69	0.55	0.64	0.69	0.66	0.42	0.18	0.60	0.55	0.59	0.54	0.63	0.39	0.50	1.00	
	U.S. Muni High Yield	4.70	5.05	8.61	5.80	0.17	-0.05	0.15	0.25	0.47	0.43	0.43	0.20	0.39	0.55	0.49	0.50	0.49	0.29	0.26	0.30	0.28	0.57	0.31	0.55	0.64	1.00

Week 6: Expected Returns

JPM LTCMAs

		Compound Return 2025 (%)	Arithmetic Return 2025 (%)	Annualized Volatility (%)	Compound Return 2024 (%)																						
Equities	U.S. Large Cap	6.70	7.91	16.26	7.00	0.00	0.00	-0.06	-0.03	0.31	0.26	0.26	0.12	0.25	0.49	0.47	0.74	0.58	0.09	0.29	0.16	0.37	0.62	0.60	0.57	0.23	0.33
	U.S. Mid Cap	7.00	8.51	18.30	7.60	0.00	-0.04	-0.10	-0.05	0.31	0.25	0.25	0.10	0.24	0.50	0.47	0.78	0.62	0.05	0.26	0.13	0.34	0.62	0.60	0.58	0.25	0.33
	U.S. Small Cap	6.90	8.82	20.73	7.20	-0.03	-0.05	-0.13	-0.10	0.22	0.18	0.20	0.05	0.17	0.40	0.38	0.71	0.55	0.00	0.20	0.08	0.27	0.53	0.54	0.48	0.18	0.24
	Euro Area Large Cap	8.50	10.64	22.06	9.70	-0.02	0.05	-0.02	-0.04	0.29	0.29	0.29	0.22	0.26	0.50	0.48	0.73	0.52	0.06	0.41	0.12	0.49	0.68	0.73	0.62	0.27	0.30
	Japanese Equity	9.00	10.10	15.68	9.30	-0.07	0.00	-0.01	0.02	0.28	0.30	0.30	0.21	0.30	0.52	0.50	0.67	0.49	0.08	0.33	0.13	0.40	0.60	0.62	0.58	0.25	0.29
	Hong Kong Equity	7.40	9.36	20.96	9.90	-0.04	-0.05	-0.06	-0.03	0.23	0.26	0.24	0.18	0.24	0.48	0.44	0.59	0.49	-0.02	0.29	-0.01	0.36	0.59	0.67	0.64	0.26	0.34
	UK Large Cap	7.80	9.19	17.60	8.60	0.04	0.00	-0.14	-0.14	0.25	0.19	0.20	0.11	0.18	0.45	0.43	0.72	0.61	-0.05	0.31	0.01	0.40	0.61	0.66	0.60	0.20	0.34
	EAFE Equity	8.10	9.49	17.61	9.20	-0.02	0.03	-0.04	-0.04	0.32	0.30	0.29	0.21	0.28	0.54	0.52	0.77	0.58	0.06	0.41	0.11	0.49	0.70	0.75	0.66	0.26	0.33
	Chinese Domestic Equity	7.80	11.36	28.85	10.80	-0.05	0.06	-0.05	-0.02	0.11	0.16	0.15	0.10	0.14	0.29	0.26	0.34	0.29	0.02	0.15	0.02	0.18	0.34	0.36	0.40	0.11	0.18
	Emerging Markets Equity	7.20	9.18	21.08	8.80	-0.01	0.03	-0.05	-0.02	0.33	0.29	0.27	0.21	0.27	0.53	0.49	0.72	0.57	0.04	0.39	0.08	0.47	0.70	0.80	0.70	0.24	0.37
	AC Asia ex-Japan Equity	7.20	9.15	20.89	8.90	-0.06	0.02	-0.02	0.02	0.32	0.31	0.29	0.23	0.30	0.54	0.51	0.69	0.53	0.06	0.38	0.09	0.45	0.68	0.75	0.69	0.26	0.37
	AC World Equity	7.10	8.37	16.71	7.80	-0.01	0.01	-0.06	-0.04	0.34	0.29	0.28	0.18	0.28	0.54	0.51	0.79	0.61	0.07	0.37	0.14	0.45	0.69	0.71	0.65	0.25	0.34
	U.S. Equity Value Factor	7.70	9.08	17.52	8.40	-0.03	-0.04	-0.10	-0.09	0.26	0.22	0.24	0.10	0.20	0.44	0.42	0.74	0.57	0.03	0.25	0.12	0.33	0.58	0.60	0.54	0.20	0.27
	U.S. Equity Momentum Factor	7.60	8.86	16.74	7.90	0.01	0.00	-0.07	-0.02	0.35	0.27	0.26	0.11	0.27	0.50	0.48	0.76	0.61	0.08	0.27	0.16	0.34	0.62	0.56	0.57	0.27	0.37
	U.S. Equity Quality Factor	6.70	7.71	14.89	7.00	-0.02	0.00	-0.04	-0.01	0.32	0.28	0.27	0.14	0.27	0.49	0.48	0.73	0.55	0.11	0.30	0.18	0.37	0.63	0.61	0.56	0.26	0.32
	U.S. Equity Minimum Volatility Factor	7.00	7.77	12.99	7.40	-0.01	-0.07	-0.03	0.02	0.32	0.28	0.27	0.13	0.29	0.49	0.48	0.72	0.53	0.12	0.31	0.20	0.38	0.62	0.62	0.54	0.27	0.32
	U.S. Equity Dividend Yield Factor	7.70	8.89	16.24	8.00	-0.01	-0.07	-0.07	-0.05	0.30	0.25	0.25	0.12	0.24	0.47	0.46	0.74	0.57	0.05	0.28	0.13	0.36	0.61	0.62	0.55	0.25	0.30
	Global Convertible Bonds hedged	6.70	7.34	11.81	7.90	-0.09	-0.03	-0.07	-0.01	0.35	0.33	0.30	0.20	0.31	0.60	0.56	0.82	0.69	0.09	0.29	0.16	0.36	0.70	0.62	0.70	0.31	0.39

Week 6: Expected Returns

JPM LTCMAs

		Compound Return 2025 (%)	Arithmetic Return 2025 (%)	Annualized Volatility (%)	Compound Return 2024 (%)																						
Alternatives	U.S. Core Real Estate	8.10	8.68	11.32	7.50	0.33	-0.18	-0.27	-0.19	0.08	-0.13	-0.09	-0.23	-0.13	0.00	-0.02	0.35	0.44	-0.19	-0.16	-0.16	-0.13	0.11	0.12	0.22	-0.20	0.34
	U.S. Value-Added Real Estate	10.10	11.70	19.11	9.70	0.33	-0.18	-0.27	-0.19	0.08	-0.13	-0.09	-0.23	-0.13	0.00	-0.02	0.35	0.44	-0.19	-0.16	-0.16	-0.13	0.11	0.12	0.22	-0.20	0.34
	European Core Real Estate	7.60	8.44	13.58	7.30	0.37	-0.09	-0.30	-0.27	0.20	-0.08	-0.04	-0.12	-0.11	0.15	0.08	0.51	0.53	-0.21	0.05	-0.15	0.12	0.30	0.37	0.41	-0.06	0.36
	Asia Pacific Core Real Estate	8.10	9.25	15.94	8.70	0.20	-0.09	-0.25	-0.23	0.25	0.04	0.07	-0.03	0.02	0.33	0.26	0.66	0.64	-0.19	0.15	-0.13	0.23	0.48	0.51	0.55	0.15	0.49
	U.S. REITs	8.00	9.33	17.22	8.20	-0.02	-0.06	0.10	0.19	0.38	0.39	0.38	0.18	0.40	0.53	0.54	0.67	0.42	0.25	0.38	0.30	0.41	0.61	0.58	0.51	0.32	0.35
	Commercial Mortgage Loans	6.40	6.68	7.69	6.30	0.08	0.02	0.24	0.24	0.55	0.48	0.50	0.32	0.38	0.51	0.45	0.49	0.43	0.35	0.30	0.37	0.31	0.57	0.41	0.53	0.51	0.59
	Global Core Infrastructure	6.30	6.86	11.01	6.80	0.19	0.05	-0.22	-0.24	0.24	0.04	0.07	0.02	-0.02	0.26	0.20	0.57	0.59	-0.17	0.18	-0.11	0.27	0.48	0.52	0.49	0.13	0.34
	Global Core Transport	7.80	8.63	13.54	7.70	0.22	0.15	0.04	0.06	-0.06	-0.09	-0.05	-0.09	-0.08	-0.27	-0.22	-0.20	-0.12	0.00	-0.07	-0.04	-0.11	-0.25	-0.10	-0.25	-0.18	-0.04
	Global Timberland	5.30	5.78	10.14	6.20	-0.05	0.18	-0.08	-0.17	0.17	0.09	0.11	0.13	0.04	0.26	0.22	0.38	0.30	-0.07	0.24	-0.02	0.31	0.41	0.51	0.36	0.13	0.14
	Commodities	3.80	5.32	18.10	3.80	0.27	-0.03	-0.17	-0.23	0.26	-0.01	-0.02	0.02	-0.02	0.19	0.16	0.44	0.41	-0.22	0.20	-0.18	0.28	0.33	0.45	0.34	-0.05	0.19
	Gold	4.00	5.31	16.76	4.10	-0.01	0.10	0.36	0.30	0.47	0.39	0.33	0.37	0.34	0.36	0.32	0.13	-0.04	0.28	0.51	0.22	0.50	0.33	0.39	0.31	0.24	0.16
	Private Equity	9.90	11.59	19.62	9.70	0.09	0.00	-0.33	-0.37	0.18	0.00	0.02	-0.02	-0.06	0.36	0.26	0.72	0.68	-0.24	0.09	-0.15	0.20	0.59	0.60	0.62	0.13	0.38
	Venture Capital	8.80	10.94	22.08	9.20	-0.06	-0.07	-0.24	-0.24	0.16	0.03	0.05	-0.03	-0.02	0.27	0.20	0.54	0.51	-0.11	0.02	-0.03	0.10	0.45	0.40	0.47	0.12	0.33
	Diversified Hedge Funds	4.90	5.06	5.80	5.00	0.08	0.04	-0.30	-0.21	0.20	0.04	0.02	-0.05	0.06	0.35	0.31	0.61	0.67	-0.17	0.04	-0.09	0.12	0.46	0.41	0.50	0.10	0.41
	Event Driven Hedge Funds	4.90	5.24	8.50	5.00	0.07	-0.02	-0.23	-0.21	0.24	0.12	0.13	0.06	0.11	0.42	0.38	0.77	0.76	-0.12	0.15	-0.03	0.24	0.56	0.54	0.58	0.16	0.40
	Long Bias Hedge Funds	5.00	5.59	11.20	4.70	0.00	0.00	-0.16	-0.14	0.28	0.20	0.20	0.13	0.19	0.48	0.44	0.77	0.68	-0.05	0.26	0.02	0.35	0.62	0.62	0.62	0.19	0.34
	Relative Value Hedge Funds	5.00	5.15	5.60	4.90	0.13	-0.04	-0.27	-0.22	0.26	0.13	0.13	0.07	0.11	0.45	0.39	0.82	0.86	-0.14	0.09	-0.06	0.18	0.59	0.55	0.64	0.19	0.47
	Macro Hedge Funds	3.80	4.03	7.00	3.60	0.00	0.05	-0.11	-0.09	0.07	-0.08	-0.16	-0.01	-0.02	0.04	0.05	0.10	0.07	-0.11	0.09	-0.08	0.11	0.08	0.16	0.05	-0.10	-0.06
	Direct Lending	8.20	9.04	13.60	8.50	0.21	0.03	-0.29	-0.27	0.18	0.04	0.13	-0.06	0.00	0.28	0.24	0.67	0.67	-0.15	-0.02	-0.05	0.07	0.48	0.43	0.50	0.17	0.46

Week 6: Expected Returns

Econometric Factor Approach

Four factors: public equities, rates, US credit, Structured credit ("complexity")

Run sequential orthogonalization on excess returns; factors will be mean zero

$$f_{eq,t} = r_{eq,t} - \overline{r_{eq}}$$

$$r_{ty,t} = \alpha_1 + \beta_{1,1} f_{eq,t} + \varepsilon_{1,t}, \qquad \varepsilon_{1,t} \perp f_{eq,t}$$

$$f_{ty,t} = \widehat{\varepsilon_{1,t}}$$

$$r_{cr,t} = \alpha_2 + \beta_{2,1} f_{eq,t} + \beta_{2,2} f_{ty,t} + \varepsilon_{2,t}, \ \ \varepsilon_{2,t} \perp f_{eq,t}, f_{ty,t}$$

$$f_{cr,t} = \widehat{\varepsilon_{2,t}}$$

$$r_{strPr,t} = \alpha_3 + \beta_{3,1} f_{eq,t} + \beta_{3,2} f_{ty,t} + \beta_{3,3} f_{cr,t} + \varepsilon_{3,t}$$

$$f_{strPr,t} = \widehat{\varepsilon_{3,t}}$$

Week 6: Expected Returns

Estimate the following factor model on excess returns:

$$r_{ty,t} = \alpha_1 + \beta_{1,1} f_{eq,t} + \beta_{1,2} f_{ty,t} + \beta_{1,3} f_{kr,t} + \beta_{1,4} f_{strPr,t} + \varepsilon_{1,t}$$

$$r_{tip,t} = \alpha_2 + \beta_{2,1} f_{eq,t} + \beta_{2,2} f_{ty,t} + \beta_{2,3} f_{kr,t} + \beta_{1,4} f_{strPr,t} + \varepsilon_{2,t}$$

$$r_{ig,t} = \alpha_3 + \beta_{3,1} f_{eq,t} + \beta_{3,2} f_{ty,t} + \beta_{3,3} f_{kr,t} + \beta_{1,4} f_{strPr,t} + \varepsilon_{3,t}$$

$$r_{hy,t} = \alpha_4 + \beta_{4,1} f_{eq,t} + \beta_{4,2} f_{ty,t} + \beta_{4,3} f_{kr,t} + \beta_{4,4} f_{strPr,t} + \varepsilon_{4,t}$$

$$r_{em\_ig,t} = \alpha_5 + \beta_{5,1} f_{eq,t} + \beta_{5,2} f_{ty,t} + \beta_{5,3} f_{kr,t} + \beta_{5,4} f_{strPr,t} + \varepsilon_{5,t}$$

$$r_{em\_hy,t} = \alpha_6 + \beta_{6,1} f_{eq,t} + \beta_{3,2} f_{ty,t} + \beta_{3,3} f_{kr,t} + \beta_{6,4} f_{strPr,t} + \varepsilon_{6,t}$$

$$r_{mbs,t} = \alpha_7 + \beta_{7,1} f_{eq,t} + \beta_{7,2} f_{ty,t} + \beta_{7,3} f_{kr,t} + \beta_{7,4} f_{strPr,t} + \varepsilon_{7,t}$$

$$r_{cmbs,t} = \alpha_8 + \beta_{8,1} f_{eq,t} + \beta_{8,2} f_{ty,t} + \beta_{8,3} f_{kr,t} + \beta_{8,4} f_{strPr,t} + \varepsilon_{8,t}$$

$$r_{abs,t} = \alpha_9 + \beta_{9,1} f_{eq,t} + \beta_{9,2} f_{ty,t} + \beta_{9,3} f_{kr,t} + \beta_{9,4} f_{strPr,t} + \varepsilon_{9,t}$$

Week 6: Expected Returns

$$r_{eq,t} = \alpha_{10} + \beta_{10,1} f_{eq,t} + \beta_{10,2} f_{ty,t} + \beta_{10,3} f_{kr,t} + \beta_{10,4} f_{strPr,t} + \varepsilon_{10,t}$$

$$r_{pe,t} = \alpha_{11} + \beta_{11,1} f_{eq,t} + \beta_{11,2} f_{ty,t} + \beta_{11,3} f_{kr,t} + \beta_{11,4} f_{strPr,t} + \varepsilon_{11,t}$$

$$r_{repe,t} = \alpha_{12} + \beta_{12,1} f_{eq,t} + \beta_{12,2} f_{ty,t} + \beta_{12,3} f_{kr,t} + \beta_{12,4} f_{strPr,t} + \varepsilon_{12,t}$$

$$r_{hf,t} = \alpha_{13} + \beta_{13,1} f_{eq,t} + \beta_{13,2} f_{ty,t} + \beta_{13,3} f_{kr,t} + \beta_{13,4} f_{strPr,t} + \varepsilon_{13,t}$$

$$r_{arhf,t} = \alpha_{14} + \beta_{14,1} f_{eq,t} + \beta_{14,2} f_{ty,t} + \beta_{14,3} f_{kr,t} + \beta_{14,4} f_{strPr,t} + \varepsilon_{14,t}$$

$$r_{pvtKr,t} = \alpha_{15} + \beta_{15,1} f_{eq,t} + \beta_{15,2} f_{ty,t} + \beta_{15,3} f_{kr,t} + \beta_{15,4} f_{strPr,t} + \varepsilon_{15,t}$$

In vector form, where $\boldsymbol{r}_t$, α and ε_t are 15x1, f_t is 4x1 and β is 15x4

$$\boldsymbol{r}_t = \alpha + \beta \boldsymbol{f}_t + \boldsymbol{\varepsilon}_t$$

The private returns we will handle as outlined in sections on private equity and private real estate, using smoothed or de-smoothed, as the case dictates

Week 6: Expected Returns

Remember APT?

$$E[r_i] = \lambda_0 + b_{i,1}\lambda_1 + b_{i,2}\lambda_{2,} + \cdots + b_{i,K}\lambda_K$$

- First, we are deriving the $b_{i,k}$s (just like in APT) but denote them β
- Our terms $\beta \boldsymbol{f}_t$ are zero on average so we can't get a meaningful equation the factor return contributions on these asset classes by just running the expectations operator on the regression

$$\mathrm{E}\left[r_{i,t}\right] = \mathrm{E}\left[\alpha_i + \beta_{i,1} f_{eq,t} + \beta_{i,2} f_{ty,t} + \beta_{i,3} f_{kr,t} + \beta_{i,4} f_{str,t} + \varepsilon_{i,t}\right] = \alpha_i + \beta_{i,1}\mathrm{E}\left[f_{eq,t}\right] + \beta_{i,2}\mathrm{E}\left[f_{ty,t}\right] + \beta_{i,3}\mathrm{E}\left[f_{kr,t}\right] + \beta_{i,4}\mathrm{E}\left[f_{str,t}\right] = \alpha_i$$

- We can use APT to derive λ_k (remember? We run a cross-sectional regression on our coefficients)
- But we don't really have a very big cross-sectional sample…
- Another alternative is to use historical econometric studies (rather than cross sectional regression) to estimate our λ_ks

Week 6: Expected Returns [update]

Regression Results

- LT RFR is the 1 year SOFR rate 5 years forward; ERPB estimated on R3K through Q3
- Credit, Structured Product premium are after other factor attributes (i.e., most of credit spread is loss allowance + equity beta)

				Factor E[xsRtns]			
LT RFR	3.80%			4.00%	0.50%	0.25%	0.15%
			Factor Coefficients				
Asset	**E[rtn]**	**Vol**	**Intercept**	**PubEqty**	**Rates**	**Credit**	**StrctProd**
'USTs_BbgBarc'	4.3%	5.3%	0.00%	0.000	1.000	0.000	0.000
'TIPS_BbgBarc'	5.0%	12.9%	0.00%	0.029	2.331	-0.158	0.000
'UScorpCreditIG_BbgBarc'	5.4%	5.0%	0.41%	0.174	0.659	0.633	0.000
'UScorpCreditHY_BbgBarc'	7.7%	10.5%	1.71%	0.459	0.066	1.367	0.000
'EM_IG_BbgBarc'	5.8%	7.8%	0.00%	0.317	1.097	0.718	0.000
'EMBroad_BbgBarc'	7.3%	9.1%	1.27%	0.413	0.774	0.924	0.000
'US_MBS_Agency_BbgBarc'	4.3%	3.5%	0.00%	0.043	0.604	0.144	0.243
'US_CMBS_BbgBarc'	5.9%	6.9%	0.55%	0.204	0.579	0.835	1.821
'US_ABS_BbgBarc'	5.1%	4.0%	0.63%	0.067	0.254	0.603	0.936
'US Public Equity'	7.8%	17.6%	0.00%	1.000	0.000	0.000	0.000
'USPrivateEquityUnSmthd'	12.6%	16.2%	6.65%	0.494	0.000	0.608	0.000
'US RealEstate_UnSmthd'	4.8%	20.3%	0.00%	0.140	0.000	0.725	1.553
'GlobalHedgeFunds_BbgBarc'	9.2%	6.3%	4.11%	0.295	0.000	0.356	0.000
'AbsRtnHedgeFunds_BbgBarc'	9.8%	5.9%	5.39%	0.096	0.234	0.286	0.000

Coefficient T-Stats					
Intercept	**PubEqty**	**Rates**	**Credit**	**StrctProd**	**R^2**
1.52	-2.71	9999	-2.72	-4.53	1.00
-0.35	1.03	25.13	-1.70	-0.84	0.65
1.11	15.84	18.01	17.25	0.74	0.89
4.66	41.78	1.81	37.26	-0.74	0.97
0.40	14.88	15.45	10.08	0.41	0.83
1.40	15.19	8.54	10.18	-0.64	0.79
0.80	4.14	17.42	4.15	3.08	0.79
1.37	17.00	14.48	20.81	20.02	0.93
2.23	7.87	8.95	21.22	14.52	0.89
-1.57	9999	0.69	-1.20	-2.06	1.00
2.37	5.88	-0.94	2.17	0.79	0.37
0.94	1.10	-0.28	1.70	1.61	0.08
5.57	13.31	0.86	4.82	0.71	0.71
4.53	2.71	1.97	2.40	0.07	0.13

Week 6: Expected Risk

$\boldsymbol{r}_t^* = \alpha + \beta \boldsymbol{f}_t + \boldsymbol{\varepsilon}_t$

$\mathrm{E}[\boldsymbol{r} \cdot \boldsymbol{r}'] = [\alpha + \beta \boldsymbol{f} + \boldsymbol{\varepsilon}] \cdot [\alpha + \beta \boldsymbol{f} + \boldsymbol{\varepsilon}]'$

$\mathrm{E}[\alpha \cdot \boldsymbol{f}'] = 0,\ \mathrm{E}[\boldsymbol{\varepsilon} \cdot \boldsymbol{f}'] = 0.$ Why?

$\mathrm{E}[\boldsymbol{r} \cdot \boldsymbol{r}'] = \mathrm{E}[\beta \boldsymbol{f} \cdot \boldsymbol{f}' \beta'] + \mathrm{E}[\boldsymbol{\varepsilon} \cdot \boldsymbol{\varepsilon}']$

$\Sigma = \beta \Omega \beta' + \Theta$

- Ω is 4 x 4
- β is 14 x 4
- Θ, the idiosyncratic covariance matrix, is 14 x 14
- Σ, the asset covariance matrix, is 14 x 14
- If this is a good factor model, what do we expected of Θ?
- We will examine the correlation matrices of Σ, Ω and Θ before “jumping into” the expected return estimates

Week 6: Expected Returns [update]

P_Ω:

Factor Correlation			
Equity	Treasuries	Credit	products
1	-0.398	0	0
-0.398	1	0	0
0	0	1	0
0	0	0	1

P_Θ:

	Residual Correlation													
Asset	'USTs_Bbg	'TIPS_Bbg	'UScorpCr	'UScorpCr	'EM_IG_B	'EMBroad_	'US_MBS_	'US_CMBS	'US_ABS_	'USPrivate	'US RealEs	'USRealEs	'GlobalHe	'AbsRtnHe
'USTs_BbgBarc'	1	0.113	-0.035	0.035	0.063	0.073	-0.021	0.044	-0.036	-0.121	0.018	0.049	-0.066	-0.020
'TIPS_BbgBarc'	0.113	1	-0.212	0.212	-0.033	0.005	-0.154	0.033	0.142	0.037	-0.022	0.072	-0.216	-0.134
'UScorpCreditIG_BbgBarc'	-0.035	-0.212	1	-1.000	0.214	0.077	0.026	-0.023	0.000	-0.066	-0.096	-0.177	0.251	0.158
'UScorpCreditHY_BbgBarc'	0.035	0.212	-1.000	1	-0.214	-0.077	-0.026	0.023	0.000	0.066	0.096	0.177	-0.251	-0.158
'EM_IG_BbgBarc'	0.063	-0.033	0.214	-0.214	1	0.761	0.210	-0.105	-0.109	0.059	-0.190	0.009	0.049	-0.041
'EMBroad_BbgBarc'	0.073	0.005	0.077	-0.077	0.761	1	0.208	-0.173	-0.010	0.054	-0.031	0.066	0.169	0.033
'US_MBS_Agency_BbgBarc'	-0.021	-0.154	0.026	-0.026	0.210	0.208	1	-0.720	-0.206	-0.017	-0.147	-0.120	0.057	-0.046
'US_CMBS_BbgBarc'	0.044	0.033	-0.023	0.023	-0.105	-0.173	-0.720	1	-0.530	0.007	0.079	0.158	-0.114	0.078
'US_ABS_BbgBarc'	-0.036	0.142	0.000	0.000	-0.109	-0.010	-0.206	-0.530	1	0.012	0.068	-0.075	0.091	-0.054
'US Public Equity'	-0.121	0.037	-0.066	0.066	0.059	0.054	-0.017	0.007	0.012	1	-0.026	-0.004	-0.004	0.003
'USPrivateEquityUnSmthd'	0.018	-0.022	-0.096	0.096	-0.190	-0.031	-0.147	0.079	0.068	-0.026	1	0.492	0.320	0.249
'US RealEstate_UnSmthd'	0.049	0.072	-0.177	0.177	0.009	0.066	-0.120	0.158	-0.075	-0.004	0.492	1	0.022	0.039
'GlobalHedgeFunds_BbgBarc'	-0.066	-0.216	0.251	-0.251	0.049	0.169	0.057	-0.114	0.091	-0.004	0.320	0.022	1	0.728
'AbsRtnHedgeFunds_BbgBarc'	-0.020	-0.134	0.158	-0.158	-0.041	0.033	-0.046	0.078	-0.054	0.003	0.249	0.039	0.728	1

Week 6: Expected Returns [update]

P_{Σ}:

Asset	avgRtn	compoundI	vol	correlation															
'USTs_BbgBarc'	3.56%	1.51%	5.31%	1	0.61	0.44	-0.32	0.45	0.12	0.83	0.22	0.21	-0.44	-0.33	-0.09	-0.28	0.10	-0.33	-0.09
'TIPS_BbgBarc'	4.80%	2.74%	5.32%	0.61	1	0.64	0.27	0.70	0.54	0.69	0.55	0.59	-0.01	0.09	0.16	0.23	0.38	0.10	0.26
'UScorpCreditIG_BbgBarc'	4.52%	2.46%	4.86%	0.44	0.64	1	0.57	0.87	0.78	0.59	0.73	0.68	0.30	0.13	-0.12	0.49	0.37	0.20	0.05
'UScorpCreditHY_BbgBarc'	6.67%	4.61%	10.23%	-0.32	0.27	0.57	1	0.58	0.78	-0.03	0.61	0.54	0.75	0.48	0.04	0.74	0.25	0.54	0.23
'EM_IG_BbgBarc'	5.93%	3.87%	7.77%	0.45	0.70	0.87	0.58	1	0.89	0.62	0.66	0.58	0.39	0.16	-0.02	0.49	0.30	0.20	0.08
'EMBroad_BbgBarc'	7.02%	4.96%	9.04%	0.12	0.54	0.78	0.78	0.89	1	0.37	0.62	0.55	0.60	0.35	0.03	0.68	0.33	0.39	0.16
'US_MBS_Agency_BbgBarc'	3.51%	1.45%	3.48%	0.83	0.69	0.59	-0.03	0.62	0.37	1	0.37	0.41	-0.21	-0.21	-0.10	-0.02	0.17	-0.21	0.00
'US_CMBS_BbgBarc'	4.72%	2.67%	6.81%	0.22	0.55	0.73	0.61	0.66	0.62	0.37	1	0.82	0.31	0.27	0.09	0.44	0.29	0.28	0.20
'US_ABS_BbgBarc'	3.48%	1.43%	3.93%	0.21	0.59	0.68	0.54	0.58	0.55	0.41	0.82	1	0.11	0.14	-0.02	0.36	0.25	0.23	0.23
'USPublicEquity_BbgBarc'	8.16%	6.10%	17.37%	-0.44	-0.01	0.30	0.75	0.39	0.60	-0.21	0.31	0.11	1	0.62	0.14	0.78	0.17	0.56	0.12
'USPrivateEquity_BbgBarc'	11.89%	9.83%	10.09%	-0.33	0.09	0.13	0.48	0.16	0.35	-0.21	0.27	0.14	0.62	1	0.48	0.63	0.28	0.87	0.44
'USRealEstate_BbgBarc'	8.36%	6.30%	9.54%	-0.09	0.16	-0.12	0.04	-0.02	0.03	-0.10	0.09	-0.02	0.14	0.48	1	0.11	0.04	0.35	0.80
'GlobalHedgeFunds_BbgBarc'	8.07%	6.01%	6.28%	-0.28	0.23	0.49	0.74	0.49	0.68	-0.02	0.44	0.36	0.78	0.63	0.11	1	0.60	0.59	0.14
'AbsRtnHedgeFunds_BbgBarc'	8.39%	6.33%	5.88%	0.10	0.38	0.37	0.25	0.30	0.33	0.17	0.29	0.25	0.17	0.28	0.04	0.60	1	0.27	0.02
'USPrivateEquityUnSmthd'	11.32%	9.27%	16.01%	-0.33	0.10	0.20	0.54	0.20	0.39	-0.21	0.28	0.23	0.56	0.87	0.35	0.59	0.27	1	0.43
'US RealEstate_UnSmthd'	6.75%	4.70%	20.21%	-0.09	0.26	0.05	0.23	0.08	0.16	0.00	0.20	0.23	0.12	0.44	0.80	0.14	0.02	0.43	1

Week 6: Expected Returns [update]

Regression Results

- Fixed income factor expected returns are generally higher than using straight historical estimates; this is driven by LT FRF and E[fctr_rtns]
- Note how TIPs have a fairly complex factor profile… interpretations?
- Corp credit coefficients are what Merton would have predicted… can anyone explain?
- EM is in-line with US Corporate Credit

				Factor E[xsRtns]									
LT RFR	3.80%			4.00%	0.50%	0.25%	0.15%						
			Factor Coefficients					**Coefficient T-Stats**					
Asset	**E[rtn]**	**Vol**	**Intercept**	**PubEqty**	**Rates**	**Credit**	**StrctProd**	**Intercept**	**PubEqty**	**Rates**	**Credit**	**StrctProd**	**R^2**
'USTs_BbgBarc'	4.3%	5.3%	0.00%	0.000	1.000	0.000	0.000	1.52	-2.71	9999	-2.72	-4.53	1.00
'TIPS_BbgBarc'	5.0%	12.9%	0.00%	0.029	2.331	-0.158	0.000	-0.35	1.03	25.13	-1.70	-0.84	0.65
'UScorpCreditIG_BbgBarc'	5.4%	5.0%	0.41%	0.174	0.659	0.633	0.000	1.11	15.84	18.01	17.25	0.74	0.89
'UScorpCreditHY_BbgBarc'	7.7%	10.5%	1.71%	0.459	0.066	1.367	0.000	4.66	41.78	1.81	37.26	-0.74	0.97
'EM_IG_BbgBarc'	5.8%	7.8%	0.00%	0.317	1.097	0.718	0.000	0.40	14.88	15.45	10.08	0.41	0.83
'EMBroad_BbgBarc'	7.3%	9.1%	1.27%	0.413	0.774	0.924	0.000	1.40	15.19	8.54	10.18	-0.64	0.79
'US_MBS_Agency_BbgBarc'	4.3%	3.5%	0.00%	0.043	0.604	0.144	0.243	0.80	4.14	17.42	4.15	3.08	0.79
'US_CMBS_BbgBarc'	5.9%	6.9%	0.55%	0.204	0.579	0.835	1.821	1.37	17.00	14.48	20.81	20.02	0.93
'US_ABS_BbgBarc'	5.1%	4.0%	0.63%	0.067	0.254	0.603	0.936	2.23	7.87	8.95	21.22	14.52	0.89
'US Public Equity'	7.8%	17.6%	0.00%	1.000	0.000	0.000	0.000	-1.57	9999	0.69	-1.20	-2.06	1.00
'USPrivateEquityUnSmthd'	12.6%	16.2%	6.65%	0.494	0.000	0.608	0.000	2.37	5.88	-0.94	2.17	0.79	0.37
'US RealEstate_UnSmthd'	4.8%	20.3%	0.00%	0.140	0.000	0.725	1.553	0.94	1.10	-0.28	1.70	1.61	0.08
'GlobalHedgeFunds_BbgBarc'	9.2%	6.3%	4.11%	0.295	0.000	0.356	0.000	5.57	13.31	0.86	4.82	0.71	0.71
'AbsRtnHedgeFunds_BbgBarc'	9.8%	5.9%	5.39%	0.096	0.234	0.286	0.000	4.53	2.71	1.97	2.40	0.07	0.13

Week 6: Expected Returns [RE-DO]

Regression Results

- Structured Product… why are MBS E[rtns] so low? Are they still attractive relative to USTs?
- CMBS clearly beat other IG assets from a straight E[rtn], E[vol] perspective
- ABS lie in between MBS, CMBS from an E[rtn], E[vol] perspective

			Factor E[xsRtns]			
LT RFR	3.80%		4.00%	0.50%	0.25%	0.15%

			Factor Coefficients					**Coefficient T-Stats**					
Asset	**E[rtn]**	**Vol**	**Intercept**	**PubEqty**	**Rates**	**Credit**	**StrctProd**	**Intercept**	**PubEqty**	**Rates**	**Credit**	**StrctProd**	**R^2**
'USTs_BbgBarc'	4.3%	5.3%	0.00%	0.000	1.000	0.000	0.000	1.52	-2.71	9999	-2.72	-4.53	1.00
'TIPS_BbgBarc'	5.0%	12.9%	0.00%	0.029	2.331	-0.158	0.000	-0.35	1.03	25.13	-1.70	-0.84	0.65
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'GlobalHedgeFunds_BbgBarc'	9.2%	6.3%	4.11%	0.295	0.000	0.356	0.000	5.57	13.31	0.86	4.82	0.71	0.71
'AbsRtnHedgeFunds_BbgBarc'	9.8%	5.9%	5.39%	0.096	0.234	0.286	0.000	4.53	2.71	1.97	2.40	0.07	0.13

Week 6: Expected Returns [RE-DO]

Regression Results

- Equities load on equity with a unit coefficient… as do Trsys on Term Premium
- The regression is telling us that we’re not picking up the majority of returns from our 4 factors… is that alpha? Illiquidity premium?
- What was Amit’s thesis? Sector + factor + leverage. Wouldn’t we pick up leverage here?
- Real Estate PE not looking great… but that loading on Struct Product is interesting (CMBS connection)
- HFs show a lot of alpha… AbsRtn show low factor loadings… as one would expect

				Factor E[xsRtns]									
LT RFR	3.80%			4.00%	0.50%	0.25%	0.15%						
			Factor Coefficients					**Coefficient T-Stats**					
Asset	**E[rtn]**	**Vol**	**Intercept**	**PubEqty**	**Rates**	**Credit**	**StrctProd**	**Intercept**	**PubEqty**	**Rates**	**Credit**	**StrctProd**	**R^2**
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'US RealEstate_UnSmthd'	4.8%	20.3%	0.00%	0.140	0.000	0.725	1.553	0.94	1.10	-0.28	1.70	1.61	0.08
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Week 6: Expected Returns

Fundamental Model of Returns

$$r_{ty,t} = r^* + \pi^e + \tau$$

Term premium estimates can be found at NY Fed ACM (https://www.newyorkfed.org/research/data_indicators/term-premia-tabs#/overview)

$$r_{tip,t} = r^* + \tau\ (+\pi_t^T)$$

$$r_{ig,t} = r_{ty,t} + sprd_{ig,t} - \mathrm{E}[Cost\ Of\ Default\ IG]$$

$$r_{hy,t} = r_{ty,t} + sprd_{hy,t} - \mathrm{E}[Cost\ Of\ Default\ HY]$$

$$r_{em\_ig,t} = r_{ty,t} + sprd_{em\_ig,t} - \mathrm{E}[Cost\ Of\ Default\ EM\_IG]$$

$$r_{em\_hy,t} = r_{ty,t} + sprd_{em\_hy,t} - \mathrm{E}[Cost\ Of\ Default\ EM\_HY]$$

$$r_{pvt\_kr,t} = r_{ty,t} + sprd_{pvt\_kr,t} + prem_{pvt\_kr,t} - \mathrm{E}[Cost\ Of\ Default\ EM\_HY]$$

Week 6: Expected Returns

Fundamental Model of Returns

$$r_{mbs,t} = r_{ty,t} + sprd_{mbs,t} - \mathrm{E}[Cost\ Of\ Default\ MBS]$$

$$r_{cmbs,t} = r_{ty,t} + sprd_{cmbs,t} - \mathrm{E}[Cost\ Of\ Default\ CMBS]$$

$$r_{abs,t} = r_{ty,t} + sprd_{abs,t} - \mathrm{E}[Cost\ Of\ Default\ ABS]$$

$$r_{pubEq,t} = r_{ty,t} + \xi_t$$

$$r_{pvtEq,t} = r_{ty,t} + \xi_t + prem_{PE} + \frac{D}{E}(\xi_t + prem_{PE} - spread_{PE}) - fees_{PE}$$

$$r_{pvtRE,t} = r_{ty,t} + \zeta_t + prem_{RE} + \frac{D}{E}\left(\xi_{RE,t} + prem_{RE} - spread_{RE}\right) - fees_{RE}$$

$$r_{HF,t} = rfr_t + \alpha_{HF,t} + \beta_{hf,eq}\xi_t + \beta_{hf,ty}\tau_t - fees_{HF}$$

Week 6: The Fundamental Factor Model [update]

And the numbers… Inputs

Fundamental Factor	Estimate	Estimation Method
'USFwdBEinflation'	1.87%	Current data, UST & TIPS fwd curves
'US5yrFwd1Yrate'	4.06%	Current data, 5y forward SOFR rate (as of 9/30/2023)
'USTermPrem'	0.77%	NY Fed, AWS est., FRBNY website, 20-yr average
'US_Rstar'	2.15%	Computed from BE infl & fwd funding
'IG Credit Risk Premium'	1.25%	BlackRock estimate (net of default, mgmt costs)
'HY Credit Risk Premium'	1.60%	BlackRock estimate (net of default, mgmt costs)
'MBS Spread Premium'	1.10%	BlackRock estimate (net of default, mgmt costs)
'CMBS Spread Premium'	1.60%	BlackRock estimate (net of default, mgmt costs)
'ABS Spread Premium'	1.40%	BlackRock estimate (net of default, mgmt costs)
'Equity Risk Prem 2 Bonds'	4.00%	Use bbg data on Russell 3000, Damodaran DDM method
'Real Estate Risk Prem 2 Bonds'	2.50%	Use bbg data on , Damodaran DDM method
'PE Liquidity Factor'	1.7	Sinha estimate
'Real Estate Liquidity Factor'	0.9	Discussions w/ CRBG internal RE group
'PE Funding Cost'	6.95%	BLK Leverage Loans Estimate, adjusted for tax
'Real Estate Funding Cost'	5.70%	BLK CML Estimate, adjusted for tax

Week 6: The Fundamental Factor Model [update]

And the numbers… Estimates

Asset Class	E[return]	E[vol]	E[gross return]	E[dflt cost]	E[other fees]	E[mkt adj]	Raw Alpha	Rtn 2 Leverage	Equity Beta	Term Prem Beta
'USTs_BbgBarc'	4.82%	5.3%	4.82%	0%	0%	0%	0%			
'TIPS_BbgBarc'	4.82%	12.9%	4.82%	0%	0%	0%	0%			
'UScorpCreditIG_BbgBarc'	6.17%	5.0%	6.07%	0.25%	0.10%	0%	0%			
'UScorpCreditHY_BbgBarc'	6.52%	10.5%	6.42%	1.00%	0.10%	0%	0%			
'EM_IG_BbgBarc'	6.17%	7.8%	6.07%	0.20%	0.10%	0%	0%			
'EMBroad_BbgBarc'	6.92%	9.1%	6.42%	1.00%	0.50%	0%	0%			
'US_MBS_Agency_BbgBarc'	6.02%	3.5%	5.92%	0.03%	0.10%	0%	0%			
'US_CMBS_BbgBarc'	6.52%	6.9%	6.42%	0.10%	0.10%	0%	0%			
'US_ABS_BbgBarc'	6.32%	4.0%	6.22%	0.10%	0.10%	0%	0%			
'USPublicEquity_BbgBarc'	8.85%	17.6%	8.82%	0%	0.03%	0%	0%			
'USPrivateEquity_BbgBarc'	7.46%	16.2%	11.8%	0%	-4.37%	0%	3.0%	0.77%		
'USRealEstate_BbgBarc'	5.47%	20.3%	7.47%	0%	-1.00%	-1.00%	1.0%	-0.09%		
'GlobalHedgeFunds_BbgBarc'	4.60%	6.3%	8.26%	0%	-3.65%	0%	3.0%		30%	0%
'AbsRtnHedgeFunds_BbgBarc'	5.49%	5.9%	9.36%	0%	-3.87%	0%	8.0%		10%	25%

Week 6: The Fundamental Factor Model

And the numbers… Estimates

Fundamental Factor	Estimate	Estimation Method
'USFwdBEInflation'	1.87%	Current data, UST & TIPS fwd curves
'US5yrFwd1Yrate'	4.06%	Current data, 5y forward SOFR rate
'USTermPrem'	0.77%	NY Fed, AWS est., FRBNY website, 20-yr average
'US_Rstar'	2.15%	Computed from BE infl & fwd funding
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'HY Credit Risk Premium'	1.60%	BlackRock estimate (net of default, mgmt costs)
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'CMBS Spread Premium'	1.60%	BlackRock estimate (net of default, mgmt costs)
'ABS Spread Premium'	1.40%	BlackRock estimate (net of default, mgmt costs)
'Equity Risk Prem 2 Bonds'	4.00%	Use bbg data on Russell 3000, Damodaran DDM method
'Real Estate Risk Prem 2 Bonds'	2.50%	Use bbg data on , Damodaran DDM method
'PE Liquidity Factor'	1.7	Sinha estimate
'Real Estate Liquidity Factor'	0.9	Discussions w/ CRBG internal RE group
'PE Funding Cost'	6.95%	BLK Leverage Loans Estimate, adjusted for tax
'Real Estate Funding Cost'	5.70%	BLK CML Estimate, adjusted for tax

Asset Class	E[return]	E[vol]	E[gross return]	E[dflt cost]	E[other fees]	E[mkt adj]	Raw Alpha	Rtn 2 Leverage	Equity Beta	Term Prem Beta
'USTs_BbgBarc'	4.82%	5.3%	4.82%	0%	0%	0%	0%			
'TIPS_BbgBarc'	4.82%	12.9%	4.82%	0%	0%	0%	0%			
'UScorpCreditIG_BbgBarc'	6.17%	5.0%	6.07%	0.25%	0.10%	0%	0%			
'UScorpCreditHY_BbgBarc'	6.52%	10.5%	6.42%	1.00%	0.10%	0%	0%			
'EM_IG_BbgBarc'	6.17%	7.8%	6.07%	0.20%	0.10%	0%	0%			
'EMBroad_BbgBarc'	6.92%	9.1%	6.42%	1.00%	0.50%	0%	0%			
'US_MBS_Agency_BbgBarc'	6.02%	3.5%	5.92%	0.03%	0.10%	0%	0%			
'US_CMBS_BbgBarc'	6.52%	6.9%	6.42%	0.10%	0.10%	0%	0%			
'US_ABS_BbgBarc'	6.32%	4.0%	6.22%	0.10%	0.10%	0%	0%			
'USPublicEquity_BbgBarc'	8.85%	17.6%	8.82%	0%	0.03%	0%	0%			
'USPrivateEquity_BbgBarc'	7.46%	16.2%	=C$6+C14+M15+N15		-4.37%	0%	3.0%	0.77%		
'USRealEstate_BbgBarc'	5.47%	20.3%	7.47%	0%	-1.00%	-1.00%	1.0%	-0.09%		
'GlobalHedgeFunds_BbgBarc'	4.60%	6.3%	8.26%	0%	-3.65%	0%	3.0%		30%	0
'AbsRtnHedgeFunds_BbgBarc'	5.49%	5.9%	9.36%	0%	-3.87%	0%	8.0%		10%	25

Asset Class	E[return]	E[vol]	return]	E[dflt cost]	E[other fees]	E[mkt adj]	Raw Alpha	Leverage	Equity Beta	Beta
'USTs_BbgBarc'	4.82%	5.3%	4.82%	0%	0%	0%	0%			
'TIPS_BbgBarc'	4.82%	12.9%	4.82%	0%	0%	0%	0%			
'UScorpCreditIG_BbgBarc'	6.17%	5.0%	6.07%	0.25%	0.10%	0%	0%			
'UScorpCreditHY_BbgBarc'	6.52%	10.5%	6.42%	1.00%	0.10%	0%	0%			
'EM_IG_BbgBarc'	6.17%	7.8%	6.07%	0.20%	0.10%	0%	0%			
'EMBroad_BbgBarc'	6.92%	9.1%	6.42%	1.00%	0.50%	0%	0%			
'US_MBS_Agency_BbgBarc'	6.02%	3.5%	5.92%	0.03%	0.10%	0%	0%			
'US_CMBS_BbgBarc'	6.52%	6.9%	6.42%	0.10%	0.10%	0%	0%			
'US_ABS_BbgBarc'	6.32%	4.0%	6.22%	0.10%	0.10%	0%	0%			
'USPublicEquity_BbgBarc'	8.85%	17.6%	8.82%	0%	0.03%	0%	0%			
'USPrivateEquity_BbgBarc'	=I15+K15+L15		11.8%	0%	-4.37%	0%	3.0%	0.77%		
'USRealEstate_BbgBarc'	5.47%	20.3%	7.47%	0%	-1.00%	-1.00%	1.0%	-0.09%		
'GlobalHedgeFunds_BbgBarc'	4.60%	6.3%	8.26%	0%	-3.65%	0%	3.0%		30%	0%
'AbsRtnHedgeFunds_BbgBarc'	5.49%	5.9%	9.36%	0%	-3.87%	0%	8.0%		10%	25%

Week 6: Problems with Optimizers

Michaud, “Error Maximization”

- Asset pricing equilibrium forces ratios of covariances to make investors close to indifferent
- E[rtn], E[risk] estimates will have errors
- MVO will significantly overweight positions with positive errors because relative to the risk matrix, the assets look like bargains
- MVO will significantly underweight with negative errors for the same reason
- How can we tell, ex ante, if our optimized positions are due to error maximization or just good allocation?
- We can’t always
- Like with so many things, ‘regularization’ (e.g., Ridge Regression) or ‘shrinkage’ (Black-Litterman) is how we avoid this
- 2 Solutions: Cheng, Murphy and Kolanovic (JPM, 2019), and Garlacci (2007)

Week 6: Problems with Optimizers

The Cheng, Murphy and Kolanovic (JPM, 2019) framework

1. Risk expression: penalty or constraint?
2. Risk estimation: distribution of assets vs. distribution of parameter estimates
3. Specification constraints: Some are fine, some because we don't "trust" the optimizer
4. A known problem: high correlations with noisy E[rtn] estimates

Canonical problem:

$$\max \omega \cdot E[rtn] - \lambda_1 \cdot \omega\Sigma\omega'$$

$s.t.$

$$\omega \cdot \iota' = 1$$
$$\omega_n \geq lb_n, \forall\, n$$
$$\omega_n \leq ub_n, \forall\, n$$

CMK (2019) problem:

$$\max\ U(\omega \cdot \mu') - \lambda \cdot (\omega - \omega^{ECR}) \cdot (\omega - \omega^{ECR})'$$

$s.t.$

$$\omega \cdot \iota' = 1$$
$$\omega \cdot \Sigma \cdot \omega' \leq \overline{\sigma_p^2}$$
$$\omega_n \geq lb_n, \forall\, n$$
$$\omega_n \leq ub_n, \forall\, n$$

Week 6: Problems with Optimizers

The Cheng, Murphy and Kolanovic (JPM, 2019) framework

From Mean Variance to Utility Maximization

Mean variance optimization (MVO) is a special case of the CRRA utility maximization function [3] below,[3] where r is vector of the expected asset returns for the period and γ is the risk aversion parameter ($\gamma \neq 1$). In a previous report we have demonstrated applying utility maximization function to the optimization of portfolios containing options.

$$\max_{w} E[U(w)] = \frac{1}{1-\gamma} E[(1 + w^T r)^{1-\gamma}] \quad [3]$$

To see why MVO is a special case of [3], we do a Taylor expansion on [3] around $r = 0$. The results is shown in [4]. Note the first term is a constant and therefore does not enter into the optimization. The second and third terms coincide with the mean and variance of the distribution. Given the values of $U'(0)$ and $U''(0)$, the function will maximize mean and minimize variance, at a tradeoff rate of γ.

Furthermore, the function expands into higher moments such as third (skewness), fourth (kurtosis), up to infinity. Given the nature of the function and its derivatives, by maximizing [3], we prefer all the odd moments (i.e., expected returns, positive skewness, etc), and dislike all the even moments (i.e., standard deviation, kurtosis, etc). It is important to note that specifying the exact return distribution is not necessary.

$$E[U(r)] = U(0) + U'(0) \cdot E[r] + \frac{1}{2!} U''(0) \cdot E[r^2] + \frac{1}{3!} U^{(3)}(0) \cdot E[r^3] + \frac{1}{4!} U^{(4)}(0) \cdot E[r^4] + \cdots \quad [4]$$

$$U(0) = \frac{1}{1-\gamma}$$

$$U'(0) = 1$$

$$U''(0) = -\gamma$$

$$U^{(3)}(0) = \gamma^2 - \gamma$$

$$U^{(4)}(0) = -\gamma^3 - \gamma^2 + 2\gamma$$

- Expressing the utility function as a Taylor expansion allows us to incorporate higher moments
- A single optimization that captures non-normality of asset return distributions
- Allows us to get away from certainty equivalence, but simpler than Monte Carlo, boot-strapping, other robust methods

Week 6: Problems with Optimizers

Garlappi Et. Al. (2004)

Classic problem:

$$\max_w \; w^\top \mu - \frac{\gamma}{2} w^\top \Sigma w,$$

Classic solution:

$$w = \frac{1}{\gamma} \Sigma^{-1} \mu,$$

Actual problem:

$$\max_w \; w^\top \hat{\mu} - \frac{\gamma}{2} w^\top \Sigma w$$

Proposed fix: allow there to be a large universe of potential expected returns (mixing), $\mu \in \mathrm{M}$, and *f*() is a vector function and the vector ϵ reflects both the investor's uncertainty and his aversion to uncertainty:

$$\max_w \min_\mu \; w^\top \mu - \frac{\gamma}{2} w^\top \Sigma w$$

subject to

$$\begin{aligned} f(\mu, \hat{\mu}, \Sigma) &\leq \epsilon, \\ w^\top \mathbf{1}_N &= 1, \end{aligned}$$

$$f_j(\mu, \hat{\mu}, \Sigma) = \frac{(\mu_j - \hat{\mu}_j)^2}{\sigma_j^2 / T_j}, \quad j = 1, \ldots, N$$

Week 6: Problems with Optimizers

Garlappi Et. Al. (2004).

How to implement fix:

Proposition 1 *The max-min problem (6) subject to (8) and (10) is equivalent to the following maximization problem*

$$\max_{w} \left\{ w^\top(\hat{\mu} - \mu^{adj}) - \frac{\gamma}{2} w^\top \Sigma w \right\}, \tag{12}$$

subject to (8), where μ^{adj} is the N-vector of adjustments to be made to the estimated expected return:

$$\mu^{adj} \equiv \left\{ sign(w_1)\frac{\sigma_1}{\sqrt{T}}\sqrt{\epsilon_1}, \ldots, sign(w_N)\frac{\sigma_N}{\sqrt{T}}\sqrt{\epsilon_N} \right\}. \tag{13}$$

Proposition 2 *The max-min problem (6) subject to (8) and (15) is equivalent to the following maximization problem*

$$\max_{w} w^\top \hat{\mu} - \frac{\gamma}{2} w^\top \Sigma w - \sqrt{\varepsilon w^\top \Sigma w}, \tag{16}$$

subject to $w^\top \mathbf{1}_N = 1$, where $\varepsilon \equiv \epsilon \frac{(T-1)N}{T(T-N)}$. Moreover, the expression for the optimal portfolio weights can be written as:

$$w^* = \frac{1}{\gamma}\Sigma^{-1}\left(\frac{1}{1+\frac{\sqrt{\varepsilon}}{\gamma\sigma_P^*}}\right)\left(\hat{\mu} - \frac{B - \gamma\left(1+\frac{\sqrt{\varepsilon}}{\gamma\sigma_P^*}\right)}{A}\mathbf{1}_N\right), \tag{17}$$

where $A = \mathbf{1}_N^\top \Sigma^{-1} \mathbf{1}_N$, $B = \hat{\mu}^\top \Sigma^{-1} \mathbf{1}_N$, and σ_P^* is the variance of the optimal portfolio that can be obtained from solving the polynomial equation (A11) in Appendix A.

Week 6: Practical Optimization

Rules-of-thumb for Asset Allocation (what humans and optimizers do)

- We use the Sharpe ratio and its components, expected portfolio return and portfolio vol, as our metrics of performance for the portfolio as a whole; these summary measures clearly omit important portfolio attributes
- Maximize diversification (e.g., Sharpe ratio, minimize vol), subject to meeting a minimum E[rtn] criterion; note, this incorporates considering various contingent scenarios
- We can use E[rtn] neutral concepts for portfolio construction (Risk Parity)… though that seems to throw away important information, e.g., equity risk premium
- The key to a high Sharpe ratio is diversification; in statistical terms, this means combining assets with positive returns and low correlations
- Low volatility, high Sharpe ratio portfolios are not enough: return objectives will likely require concentration in assets with high returns and high correlations
- The slope of the Efficient Frontier in MVO analysis illustrates these dynamics
- Other constraints – driven by liquidity, regulatory, other concerns, drive allocations below the EF
- Estimation of E[return], E[risk] inputs become the focus of analysis: these drive optimization outputs

Week 6: Practical Optimization

E[rtns], E[risk]

- We examined 4 approaches: historical, financial consultant, econometric factor, fundamental factors
- Fundamental aligns w/ JPM on credit, HFs
- Factor aligns w/ JPM on rates, structured credit
- Privates are “all over the map”
- Remember: small differences in inputs can result in large differences in final results

	E[rtns]				E[vol]	
Asset Class	**Historical**	**Factor**	**Fundamental**	**JPM_2024**	**Factor**	**JPM**
USTs	3.56%	4.3%	4.8%	3.9%	5.3%	3.3%
TIPS	4.80%	5.0%	4.8%	4.6%	12.9%	12.4%
US Corp Credit, IG	4.52%	5.4%	6.2%	5.8%	5.0%	7.1%
US Corp Credit, HY	6.67%	7.7%	6.5%	6.5%	10.5%	8.4%
EM IG	5.93%	5.8%	6.2%	6.7%	7.8%	9.0%
EM Broad	7.02%	7.3%	6.9%	6.8%	9.1%	9.6%
US MBS Agency	3.51%	4.3%	6.0%	5.3%	3.5%	3.3%
US CMBS	4.72%	5.9%	6.5%	5.7%	6.9%	3.3%
US ABS	3.48%	5.1%	6.3%	5.3%	4.0%	3.3%
US Public Equity	8.16%	7.8%	8.9%	7.2%	17.6%	17.5%
US Private Equity	11.89%	12.6%	7.5%	9.7%	16.2%	20.1%
US Real Estate	8.36%	4.8%	5.5%	9.7%	20.3%	17.7%
Global Hedge Funds	8.07%	9.2%	4.6%	5.0%	6.3%	5.8%
AbsRtn Funds	8.39%	9.8%	5.5%	4.3%	5.9%	6.4%

Week 6 [update]

E[rtns], E[risk]

- We will simply to average over our 3 types of estimates
- The result, to the right, looks reasonable
- Beware human bias… it is prevalent at large organizations and senior management's instincts predominate in decision making
- "Unless you have a Sharpe ratio higher than 2, you depend upon consensus". Friend at GSAM
- Absent a strong argument that any one method for optimizer inputs is superior, it is not a bad idea to run with an average

Asset Class	**E[rtn]**	**E[vol]**
USTs	4.3%	4.3%
TIPS	4.8%	12.6%
US Corp Credit, IG	5.8%	6.1%
US Corp Credit, HY	6.9%	9.4%
EM IG	6.2%	8.4%
EM Broad	7.0%	9.4%
US MBS Agency	5.2%	3.4%
US CMBS	6.1%	5.1%
US ABS	5.6%	3.7%
US Public Equity	8.0%	17.5%
US Private Equity	9.9%	18.1%
US Real Estate	6.6%	19.0%
Global Hedge Funds	6.3%	6.1%
AbsRtn Funds	6.5%	6.1%

Week 6: Modifying the MVO framework to fit institutions

Endowments and SWFs:

$$\max_{\omega} E\left[rtn_{port}(\omega)\right] - \lambda \cdot Var(rtn_{port}(\omega))$$

$s.t.$

$$\sum_{n=1}^{N} \omega_n = 1$$
$$\omega_n \geq 0, \forall\, n$$
$$\omega_n \leq 0.5, \forall\, n$$
$$\omega_{TY} + \omega_{IG} \geq 0.06$$

Asset Class	**E[rtn]**	**E[vol]**
USTs	4.3%	4.3%
TIPS	4.8%	12.6%
US Corp Credit, IG	5.8%	6.1%
US Corp Credit, HY	6.9%	9.4%
EM IG	6.2%	8.4%
EM Broad	7.0%	9.4%
US MBS Agency	5.2%	3.4%
US CMBS	6.1%	5.1%
US ABS	5.6%	3.7%
US Public Equity	8.0%	17.5%
US Private Equity	9.9%	18.1%
US Real Estate	6.6%	19.0%
Global Hedge Funds	6.3%	6.1%
AbsRtn Funds	6.5%	6.1%

- Endowments have the longest time-horizon and fewest regulatory / board restrictions, hence larger/longer draw-downs are tolerable
- Maximizing wealth growth does require high risk adjusted returns… we want to maximize compound returns, which require moderate draw-downs, so $\lambda > 0$
- We still have to worry about the problem of private assets… do we use smooth returns, or de-smoothed returns? In the slide below we use de-smoothed (“instrinsic”) risk estimates for privates

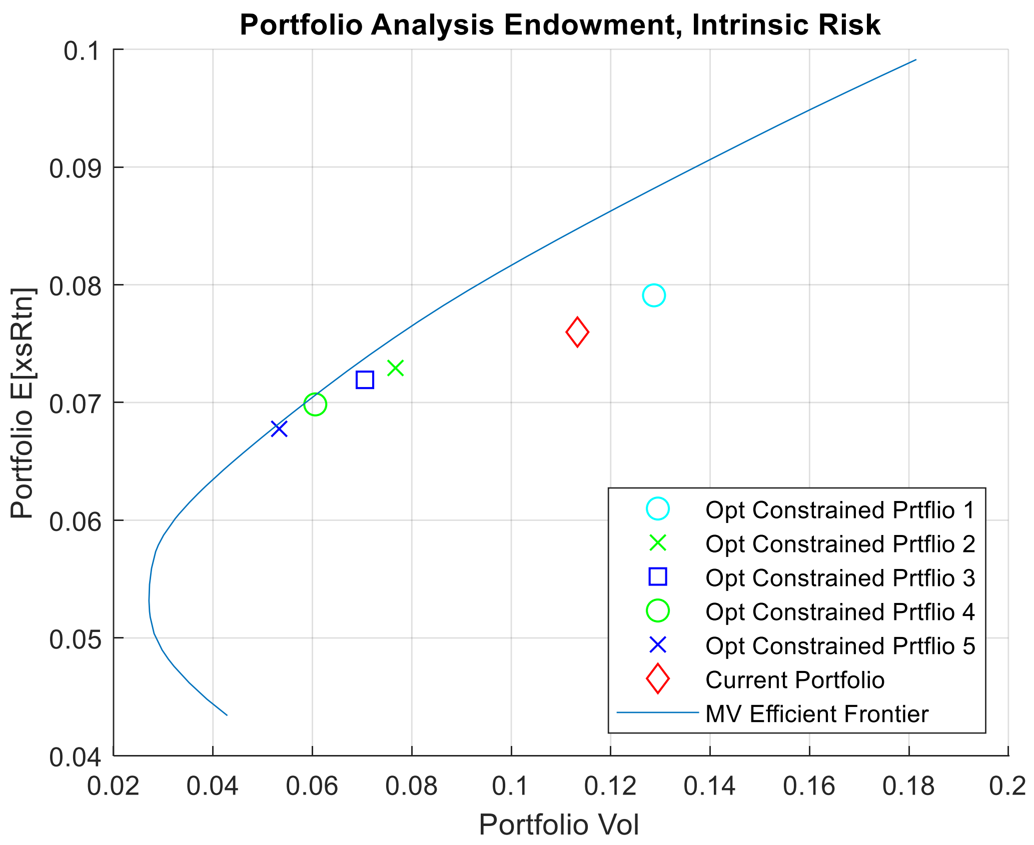
Week 6: Modifying the MVO framework to fit institutions

Intrinsic risk in an Endowment SAA

- Averaged across E[rtns], E[risk] derived in our class
- Endowments Average allocations shown are from data shown in the first lecture
- Liquidity, asset specific constraints “force” allocations off the Efficient Frontier
- Note the large impact different input assumptions make on allocations

Endowments

Simplified Asset Class	Endowments	OptWts_Avg	OptWts_JPM
USTs	4.65%	8.6%	4.5%
Credit, IG	5.70%	0.0%	20.3%
Credit, HY	5.70%	21.6%	34.4%
Public Equity	40.5%	31.6%	26.0%
PE	19.70%	18.6%	19.3%
Real Estate / Real Assets	6.70%	0.0%	0.0%
Hedge Funds	17.10%	19.6%	0.0%



Week 6: Modifying the MVO framework to fit institutions

Pensions:

$$\max_{\omega} E\left[rtn_{port}(\omega)\right] - \lambda \cdot Var(rtn_{port}(\omega))$$

$s.t.$

$$\sum_{n=1}^{N} \omega_n = 1$$
$$A_{duration} \cdot \omega' \geq d$$
$$A_{liquidity} \cdot \omega' \leq \underline{\varphi}$$
$$\omega \cdot \Sigma \cdot \omega' \leq \overline{\sigma_p^2}$$
$$\omega_n \geq 0, \forall\, n$$
$$\omega_n \leq 0.5, \forall\, n$$

Asset Class	E[rtn]	E[vol]
USTs	4.3%	4.3%
TIPS	4.8%	12.6%
US Corp Credit, IG	5.8%	6.1%
US Corp Credit, HY	6.9%	9.4%
EM IG	6.2%	8.4%
EM Broad	7.0%	9.4%
US MBS Agency	5.2%	3.4%
US CMBS	6.1%	5.1%
US ABS	5.6%	3.7%
US Public Equity	8.0%	17.5%
US Private Equity	9.9%	18.1%
US Real Estate	6.6%	19.0%
Global Hedge Funds	6.3%	6.1%
AbsRtn Funds	6.5%	6.1%

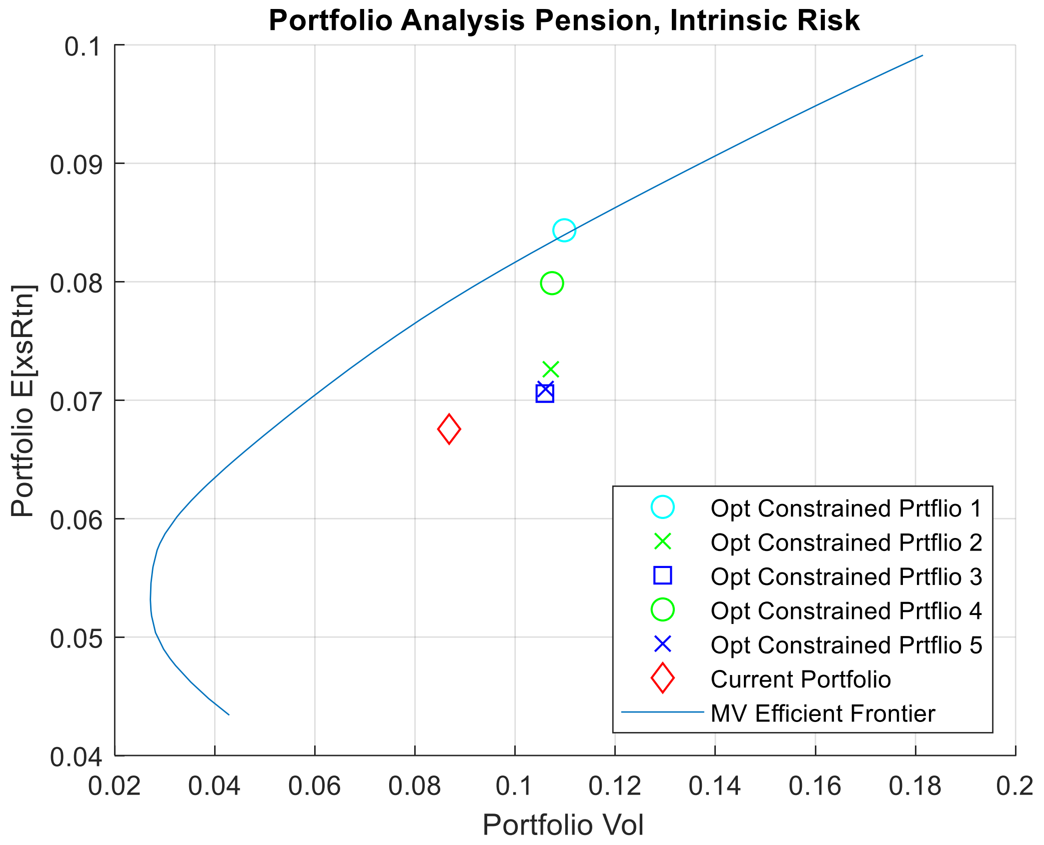
- Due to regulatory restrictions / industry practices, pensions are constrained by somewhat contradicting restrictions: duration must be long (to match expected mortality), liquidity must be abundant, yet risks have to be contained to avoid large drawdowns
- Low risk we do two ways: we select a somewhat large value of lambda $\lambda >> 0$, or through the constraints
- We constrain portfolio average liquidity horizon (units: years) to be below 4 yrs, duration > 8 yrs

Week 6: Modifying the MVO framework to fit institutions

Intrinsic risk in a Pension SAA

- Average across E[rtns], E[risk] derived in our class
- Endowments Average allocations shown are from data shown in the first lecture
- Liquidity, asset specific constraints “force” allocations off the Efficient Frontier
- Note the large impact different input assumptions make on allocations

Pensions			
Simplified Asset Class	**Pensions**	**OptWts_Avg**	**OptWts_JPM**
USTs	22.20%	39.79%	5.70%
Credit, IG	11.50%	0.11%	50.0%
Credit, HY	11.50%	28.7%	4.7%
Public Equity	36.80%	3.81%	20.40%
PE	6.10%	20.00%	15.50%
Real Estate / Real Assets	6.60%	3.86%	3.80%
Hedge Funds	5.30%	3.73%	0%



Week 6: Modifying the MVO framework to fit institutions

L&R Insurers:

$$\max_{\omega} E\left[rtn_{port}(\omega)\right] - \lambda_1 \cdot Var(rtn_{port}(\omega)) - \lambda_2 \cdot penalty(\omega, A_{rbc})$$

$s.t.$

$$\sum_{n=1}^{N} \omega_n = 1$$
$$A_{rbc} \cdot \omega' \leq 0.04$$
$$\omega \cdot \Sigma \cdot \omega' \leq \overline{\sigma_p^2}$$
$$\omega_n \geq 0, \forall\, n$$
$$\omega_n \leq 0.5, \forall\, n$$

Asset Class	E[rtn]	E[vol]
USTs	4.3%	4.3%
TIPS	4.8%	12.6%
US Corp Credit, IG	5.8%	6.1%
US Corp Credit, HY	6.9%	9.4%
EM IG	6.2%	8.4%
EM Broad	7.0%	9.4%
US MBS Agency	5.2%	3.4%
US CMBS	6.1%	5.1%
US ABS	5.6%	3.7%
US Public Equity	8.0%	17.5%
US Private Equity	9.9%	18.1%
US Real Estate	6.6%	19.0%
Global Hedge Funds	6.3%	6.1%
AbsRtn Funds	6.5%	6.1%

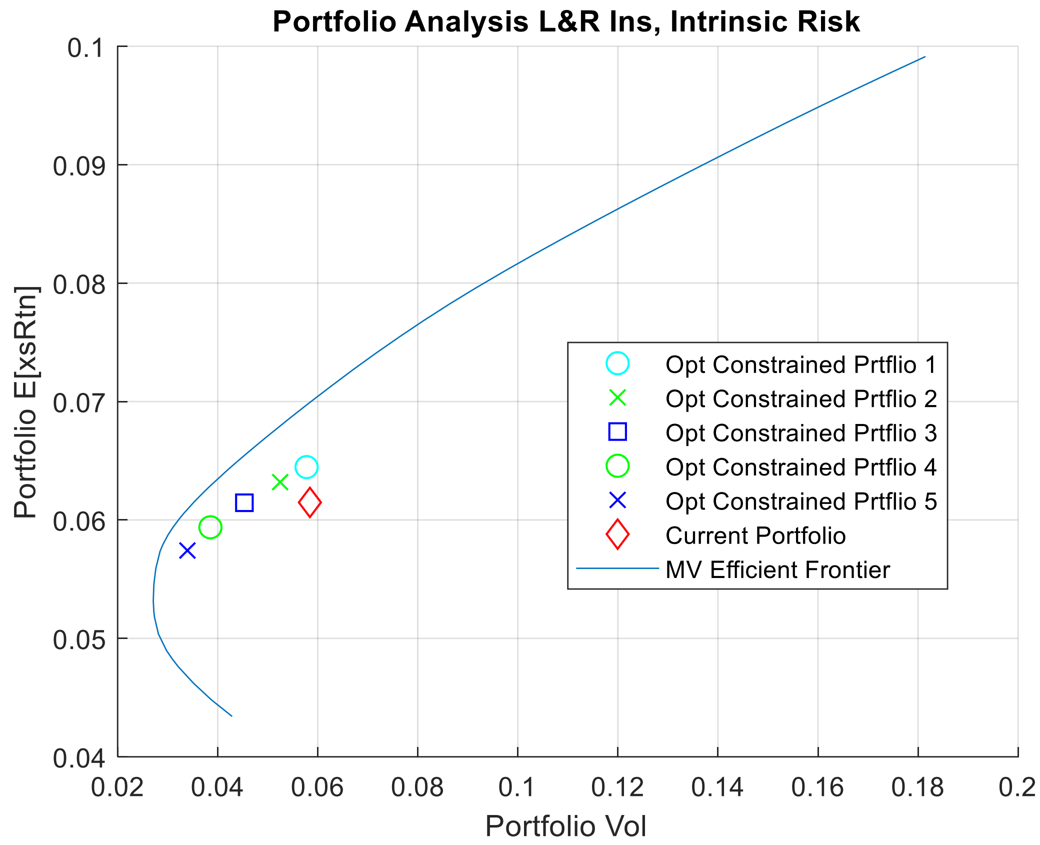
- Similarly to pensions, L&R insurers face close regulation on their portfolio risk through the NAIC in the form of “RBC” or risk-based capital charges
- Both in the objective function, through a quadratic penalty function, and a linear constraint, we set up the problem to control risk
- The table below shows RBC risk charges by asset class

Week 6: Modifying the MVO framework to fit institutions

Intrinsic risk in an L&R SAA

- Averaged across E[rtns], E[risk] derived in our class
- Endowments Average allocations shown are from data shown in the first lecture
- Liquidity, asset specific constraints "force" allocations off the Efficient Frontier
- Note the large impact different input assumptions make on allocations

L&R			
Simplified Asset Class	**L&R**	**OptWts_Avg**	**OptWts_JPM**
USTs	13.20%	0.00%	28.3%
Credit, IG	50.0%	75.3%	40.0%
Credit, HY	18.10%	12.80%	5.50%
Public Equity	8.70%	2.23%	2.50%
PE	9.60%	5.25%	7.40%
Real Estate / Real Assets	0.40%	1.69%	9.70%
Hedge Funds	0%	2.78%	6.70%



Quiz Readings

- Ilmanen, Chandra and McQuinn (2019). "Demystifying Illiquid Assets: Expected Returns for Private Real Estate." (Week 4)
- Fender and Mitchell (2005). Structured finance. Complexity and Ratings. (Week 4)
- Jobst (2006). A Primer on Structured Finance. (Week 4)
- Guggenheim. ABCs of Structured Finance (Week 4)
- Fama. Cambridge Associates: Private Credit Introduction (Week 5)
- Jacobs and Kobor (2021). SAA at the NYU Endowment. (Week 6)