A Factor Approach to Strategic Asset Allocation for Portfolios with Alternatives

FRE-GY 6921

NYU Tandon Program in Financial Engineering

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Week 2: Review & Outline

Last week:

- Intro to the large end-investors; similarities and differences in their objectives
- Utility theory – risk aversion and consumption smoothing
- Multi-period optimization and Dynamic Programming: how should Endowments time consumption?
- Pensions: DB vs DC
- Strategic (long-term) time horizon
- Run-down of the asset Classes

Key takeaway: these portfolios are **large** → Turnover must be low → Investing in “betas”

- Long only, highly diversified portfolios within each asset class
- We will seek true factor returns (only non-diversifiable, systemic risk will pay you a risk premium)

Agenda today: Tool review; CAPM/APT/MVO; Bond term premium; Equity risk premium.

Week 2: Review of Analytical Tools

Portfolio Variance:

Let: z_i be the USD value of an allocation to security or strategy i (the currency value of holdings)

$z_{port} = \sum_{i=1}^{N} z_i$ is the USD value of a portfolio of strategies or securities

$\omega_i = z_i / z_{port}$ is the corresponding allocation weight in strategy i

r_i is the return to strategy i

$$r_{port} = \omega_1 r_1 + \omega_2 r_2$$

$$Var(r_{port}) = Var(\omega_1 r_1 + \omega_2 r_2) = {\omega_1}^2 Var(r_1) + {\omega_2}^2 Var(r_2) + 2\omega_1\omega_2 \cdot Cov(r_1, r_2)$$

$${\sigma_{port}}^2 = {\omega_1}^2{\sigma_1}^2 + {\omega_2}^2{\sigma_2}^2 + 2\omega_1\omega_2 \cdot \rho_{1,2}\sigma_1\sigma_2$$

With N assets or strategies

$$r_{port} = \sum_{i=1}^{N} \omega_i r_i$$

$Var(r_{port}) = \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i\omega_j\,\sigma_{ij}$ where $\sigma_{ij} = {\sigma_i}^2$ if $i = j$

Week 2: Review of Analytical Tools

Portfolio Variance:

With linear algebra

$$r_{port} = [r_1\ r_2] \cdot [\omega_1\ \omega_2]' = r \cdot \omega'$$

$$\sigma_{port}{}^2 = [\omega_1\ \omega_2] \cdot \begin{bmatrix} \sigma_1{}^2 & \sigma_{1,2} \\ \sigma_{1,2} & \sigma_2{}^2 \end{bmatrix} \cdot [\omega_1\ \omega_2]'$$

$$= \omega \Sigma \omega'$$

where

$$\Sigma = \begin{bmatrix} 1\sigma_1\sigma_1 & \rho_{1,2}\sigma_1\sigma_2 \\ \rho_{1,2}\sigma_1\sigma_2 & 1\sigma_2\sigma_2 \end{bmatrix}$$

Week 2: Review of Analytical Tools

Sharpe ratio:

$$E[SharpeRatio] = \frac{E[r_i - \mu]}{E[\sigma_i]}$$

Sortino ratio:

$$Downside\ Deviation\ = DD_i = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(min(0, r_i - \mu)\right)^2}$$

$$E[SortinoRatio] = \frac{E[r_i - \mu]}{E[DD_i]}$$

- Half of the returns distribution is above the mean (right-tail) which we ***want***. Why have that dispersion generate a larger magnitude denominator? Well, statistics says you don’t get one w/o the other when skew is zero.
- So the Sortino Ratio seeks to incorporate benefits of skew – it will differentiate among strategies / assets with different 3rd moment characteristics

Week 2: Expected Returns, Expected Risk

OLS: $y = \alpha + X\beta + \varepsilon$

Practical matters

1. Always plot your residuals
2. Outliers mean you have misspecification. Most practitioners just remove them. Variable transformation is a better approach

   Example: $q_{i,t+1} = \alpha + X_t\beta + \varepsilon_t$ where $q_{i,t+1}$ is an equity price and X_ts are your factors of interest. When do your outliers occur?

$$\frac{q_{i,t+1}}{VIX_{t+1}} = \hat{\alpha} + \frac{X_t}{\boldsymbol{VIX}_{t+1}}\beta + \hat{\varepsilon}_t$$

3. Auto-correlation: misspecification. Diagnose with Durbin – Watson stat; you can use Cochrane Orcutt procedure, though you are probably misspecified still
4. If elements of X are not linearly independent:
   1. Pre-orthogonalization
   2. Sequential regression

Week 2: Expected Returns, Expected Risk

CAPM, Ross (1976, APT), Merton (1974) review: What is a risk premium?

Theory-free linear algebra:

$$(r_{i,t} - r_{rfr,t}) = \alpha_i + \beta_i(r_{bm,t} - r_{rfr,t}) + \varepsilon_{i,t}$$

Or rearranging terms:

$$r_{i,t} - r_{rfr,t} = \beta_i\left(r_{bm,t} - r_{rfr,t}\right) + \left(\alpha_i + \varepsilon_{i,t}\right)$$

$$= systemic\ risk\ return + idiosyncratic\ return$$

Importantly, $\beta_i r_{bm,t} \perp \left(\alpha_i + \varepsilon_{i,t}\right)$

Is $\left(r_{bm,t} - r_{rfr,t}\right) > 0$? Can $\alpha_i > 0$ as a general proposition?

Week 2: Expected Returns, Expected Risk

CAPM:

$$\mathrm{E}\left[\left(\alpha_i + \varepsilon_{i,t}\right)\right] = 0$$

Implications are consistent with efficient markets theory:

- the market will not compensate you for taking diversifiable (idiosyncratic) risk
- Non-diversifiable (systemic) risk does earn a risk premium
- The theory is equilibrium-based: how much will the market compensate you for systematic risk in a way that all other investors and capital users are also content to accept?
- The magnitude of the equity risk premium not addressed; see Mehra and Prescott (1982), Damodaran (2020), Consumption CAPM Breeden (1979), etc.

Week 2: Expected Returns, Expected Risk

CAPM:

- The market will not compensate you for taking diversifiable (idiosyncratic) risk
- Non-diversifiable (systemic) risk does earn a risk premium
- So you get compensated as an investor for bearing undiversifiable risks
- Is there just one risk? Or do stocks have lots of risks that we just bundled together?
- Are factor risks related to assets' attributes?
- Potential factor return-driving risks:
  - Flight to quality and lender of last resort… CHF, USD, USTs, gold…
  - Resource disruption… commodities, industrial companies
  - Long-run decline versus a transitory set-back… the risk of buying cheap companies ("value")
  - Small companies versus large companies…

Week 2: Expected Returns, Expected Risk

Ross (1976, 1977) Arbitrage Pricing Theory (APT): modify CAPM to account for granular risks

- CAPM does require different assets to have different rates of return in equilibrium; different required returns correspond to risks inherent in each asset
- However differing risks must be priced consistently across assets or there will be arbitrage opportunities: every investor has to be rationale / optimal
- Ross posits that there merely need to be some (sophisticated) arbitrageurs to exploit such opportunities and push prices back in line to achieve consistent pricing of risk in the market (not every investor holding an optimized portfolio on the EF)
- The approach is plausible if there are enough such arbitrageurs, with deep enough pockets, and relatively few limits to arbitrage
- APT will allow multiple risk/pricing factors; for investors able to identify and evaluate the right factors, APT can be view as a framework for identifying profitable trades
- The (deceptively simple) summary of the APT, where λ is the expected return the corresponds to bearing the risk of a given factor k:

$$E[r_i] = \lambda_0 + b_{i,1}\lambda_1 + b_{i,2}\lambda_{2,} + \cdots + b_{i,K}\lambda_K$$

Week 2: Expected Returns, Expected Risk

Ross (1976, 1977) Arbitrage Pricing Theory (APT)

Starting point: run a reduced form factor model regression for each asset *i*. Transform your factors so that they have zero-means. Recall from regression theory that it doesn't matter what your RHS variables means are for the purposes of estimating "betas" (though de-meaning does impact your estimate of "alpha"). So $\mathrm{E}\left[f_{k,i}\right] = 0, \forall\, i, k$:

$$r_{i,t} = a_i + b_{i,1} f_{1,t} + b_{i,2} f_{2,t} + \cdots + b_{i,K} f_{K,t} + \varepsilon_{i,t}$$

Now, define an arbitrage portfolio where weights sum to zero

$$\iota \cdot w' = 0$$

and choose that portfolio's weights to have zero exposure to systematic risk to the f_ks:

$$b_{*,k} \cdot w' = 0, \forall\, k, where\ b_{*,k}\ is\ a\ 1\ by\ N\ vector$$

Basic linear algebra demonstrates it is possible to create weights that fulfill these conditions if N > K. <u>Exercise</u>: show that this portfolio is zero arbitrage if our set of factors is complete.

Lecture 2: Expected Returns

Ross (1976, 1977) Arbitrage Pricing Theory (APT)

For individual assets we have:

$$r_i = \mathrm{E}[r_i] + b_{i,1} f_{1,t} + b_{i,2} f_{2,t} + \cdots + b_{i,K} f_{K,t} + \varepsilon_{i,t}$$

Going with the APT assumptions, we get

$$\mathrm{E}[r_i] - r_{rfr} = \sum_{k=1}^{K} b_{i,k} \lambda_k$$

Combining, we get

$$r_{i,t} - r_{rfr} = \sum_{k=1}^{K} b_{i,k} \left(\lambda_k + f_{k,t} \right) + \varepsilon_{i,t}$$

Lecture 2: Expected Returns

Ross (1976, 1977) Arbitrage Pricing Theory (APT)

Consider the arbitrage portfolio $\{w_1, w_2, \dots, w_N\}$ just defined. What is our expected return on it if we imposed the assumption of 0 arbitrage?

$$\mathrm{E}\left[r_{portolio}\right] = \mathrm{E}[r_1 w_1 + r_2 w_2 + \cdots + r_N w_N] = \mathrm{E}[a_1 w_1 + a_2 w_2 + \cdots + a_N w_N]$$

From this, plus a bit of linear algebra, we get

$$a_i = E[r_i] = \lambda_0 + b_{i,1}\lambda_1 + b_{i,2}\lambda_{2,} + \cdots + b_{i,K}\lambda_K$$

How do we compute lambdas? Stack:

$$\begin{bmatrix} a_1 & \cdots & b_{1,K} \\ \vdots & \ddots & \vdots \\ a_N & \cdots & b_{N,K} \end{bmatrix}$$

Now run the cross-sectional regression:

$$a_i = r_{rfr} + \lambda_1 b_{i,1} + \cdots + \lambda_k b_{i,k} + \cdots + \lambda_K b_{i,K}, i = 1 \dots N$$

Lecture 2: Expected Returns

Ross (1976, 1977) Arbitrage Pricing Theory (APT)

In the cross-sectional regression $a_i = r_{rfr} + \lambda_1 b_{i,1} + \cdots + \lambda_k b_{i,k} + \cdots + \lambda_K b_{i,K}$, the lambas are your derived coefficients. Suspending belief in efficient markets (you only get compensated for bearing systematic factor risk), what is alpha in the Ross framework?

$$\alpha_i = a_i - \left(r_{rfr} + \lambda_1 b_{i,1} + \cdots + \lambda_k b_{i,k} + \cdots + \lambda_K b_{i,K}\right)$$

Recall a_i is an asset's (empirical) average return.

Lecture 2: Expected Returns

Ross (1976, 1977) Arbitrage Pricing Theory (APT)

$$E[r_i] = \lambda_{i,0} + b_{i,1}\lambda_1 + b_{i,2}\lambda_{2,} + \cdots + b_{i,K}\lambda_K$$

For “risk assets”, what are some common factors?

- Cyclical indicators: Economic “nowcasts”, recession predictors (2s-10s, etc.), monetary indicators (MPCs, changes in forward B-Es, inflation realization surprises)
- Financial crisis / deleveraging indicators: VIX volatility, GS FCI, MOVE, home-bias currency (JPY, CHF, USD) appreciation, credit spreads
- Uncertainty (asset-specific risk): e.g., common stock implied volatility, sector-specific spreads or stock returns, country-specific risk measures
- Ross (1980), Ross and Roll (1982), Chen, Ross and Roll (1986) early empirical explorations:
    - Inflation: changes in expected inflation, inflation realizations - π^e
    - Changes in T-bill rate
    - IP –E[IP]
    - BBB bond returns
    - Equity returns
    - Oil prices, etc.

A GOOD DEAL OF SYSTEMATIC HEDGE FUND INVESTING IS SEARCHING FOR FACTORS λ_K

Week 2: Review of MVO

The canonical problem:

$$\max \omega \cdot \mu' - \lambda_1 \cdot \omega\Sigma\omega'$$
$$s.t.$$
$$\omega \cdot \iota' = 1$$
$$\omega\Sigma\omega' \leq \bar{\sigma}^2$$
$$\omega_n \geq lb_n, \forall\, n$$
$$\omega_n \leq ub_n, \forall\, n$$

μ = $E[rtn]$ = vector of N asset-specific expected returns

$\omega \cdot \mu'$= $\sum_{n=1}^{N} \omega_n \cdot E[rtn_n]$ = expected portfolio return

Σ= the N x N variance-covariance matrix of the underlying assets

$\omega\Sigma\omega'$= the expected variance of the portfolio

Week 2: Review of MVO

Relationship between CAPM and MVO:

- CAPM assumptions basically imply all households use MVO to choose their portfolios (full information, no TCs, no taxes, all HHs have same preferences, etc.)
- MVO gives us the efficient frontier, and CAPM selects the “market portfolio” as the optimal choice when all investors face the same opportunity set
- CAPM portfolio is on the EF
- Homogeneous expectations
- Assumption of risk-free asset
- No arbitrage (under MVO, EF > all other portfolios; CAPM: all assets lie on the Security Market Line)

In the Finance literature, MVO is a general equilibrium concept (all households hold same portfolio, due to assumptions).

At asset managers, portfolio optimization is a “partial equilibrium” tool, used to build the best portfolio

Week 2: Expected Returns, Expected Risk: CAPM vs. APT

	CAPM (Market Factor)	Multifactor Models
Lesson 1	Diversification works. The market diversifies away idiosyncratic risk.	Diversification works. The tradeable version of a factor diversifies away idiosyncratic risk.
Lesson 2	Each investor has her own optimal exposure of the market portfolio.	Each investor has her own optimal exposure of each factor risk.
Lesson 3	The average investor holds the market.	The average investor holds the market.
Lesson 4	The market factor is priced in equilibrium under the CAPM assumptions.	Risk premiums exist for each factor assuming no arbitrage or equilibrium.
Lesson 5	Risk of an asset is measured by the CAPM beta.	Risk of an asset is measured in terms of the factor exposures (factor betas) of that asset.
Lesson 6	Assets paying off in bad times when the market return is low are attractive, and these assets have low risk premiums.	Assets paying off in bad times are attractive, and these assets have low risk premiums.

Week 2: Expected Returns, Risk

Single Security analysis vs. Asset Class analysis

- For single securities, the analyst is looking for idiosyncrasies in a company relative to a peer group
  - Geographic footprint (less or more saturated market)
  - Product mix
  - Margins
  - Tax assets, accounting anomalies
  - Niche or comparative advantage (Buffett’s moats)…

$$E[r_i] = \beta_{mkt} \cdot \mu_{mkt} + \beta_{sctr} \cdot \mu_{sctr} + \cdots + \alpha_i$$

- For asset classes, those idiosyncratic elements all cancel out.
  - For a county’s equity markets as a whole, how fast will sales grow? The rate of growth of nominal GDP.
  - What will happen to margins? Depends on labor market tightness, real interest rates, corporate taxes, aggregate capex, industry composition of the economy, etc.

$$E\left[\sum_i \omega_i \cdot r_i\right] = \sum_i \omega_i \cdot \beta_{mkt} \cdot \mu_{mkt} + \cancelto{0}{\sum_i \omega_i \cdot \beta_{sctr} \cdot \mu_{sctr}} + \cdots + \cancelto{0}{\sum_i \omega_i \cdot \alpha_i} = \beta_{mkt} \cdot \mu_{mkt}$$

Week 2: Expected Returns, Risk

We will use finance theory to motivate estimation methods for return forecasts for a basis of factor risk premia. Then we will build up our expected returns using OLS.

The 5 risk premium factors:

1. Term (rates) premium
2. Equity risk premium (more in the CAPM spirit than the Ross APT spirit)
3. Credit risk premium
4. Structured security (complexity) premium
5. Illiquidity premium

Prior to reviewing data sources, a quick run-through of related literature that provides a basis for building expected returns methodology around this

Week 2: Expected Returns, Expected Risk

Term premium

Contributors: Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005), Kim and Wright (2005), Adrian Crump and Moench (2013)

Expectations theory:

$$r_t^{0,10} = \left(\left(1 + E[rfr_t^{0,1}]\right) \cdot \left(1 + E[rfr_t^{1,2}]\right) \cdot \ \cdots \ \cdot \left(1 + E[rfr_t^{9,10}]\right)\right)^{1/10} - 1$$

Interpretation: If expectations of future rates are unbiased 10-year rates are the arithmetic average of short-term rates and the yield curve should be flat on average.

Term premium: at each tenor the market compensates you a bit more than the expected *rfr*:

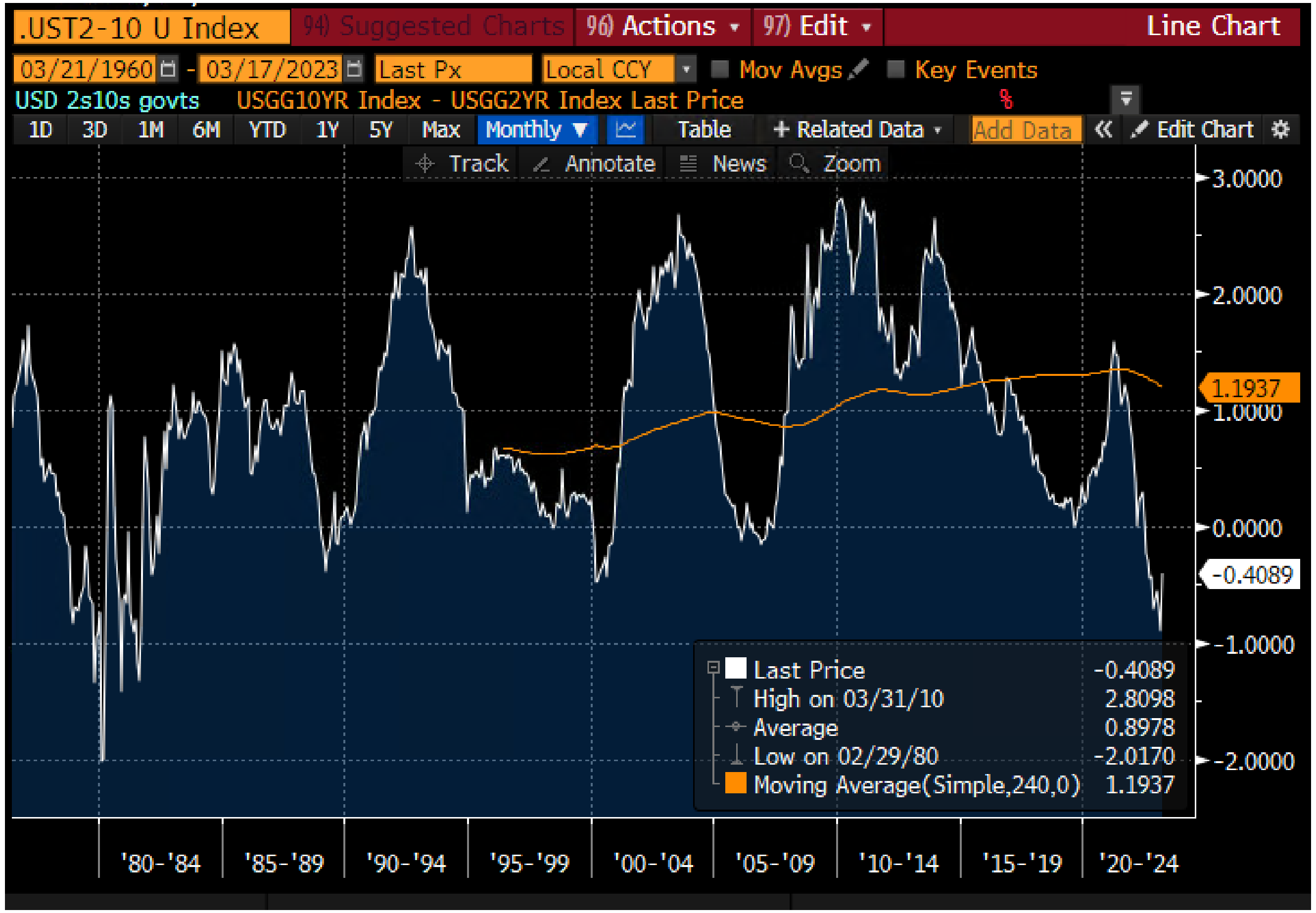
$$r_t^{0,10} = \left(\left(1 + E[rfr_t^{0,1}] + \tau_t^{0,1}\right) \cdot \left(1 + E[rfr_t^{1,2}] + \tau_t^{1,2}\right) \cdot \ \cdots \ \cdot \left(1 + E[rfr_t^{9,10}] + \tau_t^{9,10}\right)\right)^{1/10} - 1$$

Week 2: Expected Returns, Expected Risk

Term premium

$$r_t^{0,10} = \left(\left(1 + E[rfr_t^{0,1}] + \tau_t^{0,1}\right) \cdot \left(1 + E\left[rfr_t^{1,2}\right] + \tau_t^{1,2}\right) \cdot \cdots \cdot \left(1 + E[rfr_t^{9,10}] + \tau_t^{9,10}\right)\right)^{1/10} - 1$$

- Long term average of 2s-10s is 80-120 bps, violating a key assumption of Expectations Theory over 20 years
- For most investors, short term investments are less risky than long-term investments
- Investors who match maturities and bear reinvestment risk may not find long-term more risky, but are a minority



Week 2: Expected Returns, Expected Risk

Equity Risk Premium to Bonds

Gordon and Shapiro had the insight that fundamental asset pricing methods could be applied to stocks. Updating this insight for a world where share buy-backs are an important part of corporate treasury capital structure management, let:

- $P_t = \textit{price of a common stock or stock index}$
- $PO_t = payout_t = div_t + buybacks_t$
- $r = \textit{yield to maturity of risk free bonds}$
- $\xi = ERPB = \textit{Equity Risk Premium to Bonds}$
- $g = E[\Delta PO_t / PO_t] \; \forall \; t$

Applying standard cash-flow discounting in the presence of non-payment risk we get:

$$P_t = PO_t + \left(\frac{1}{1+r+\xi}\right) \cdot E[PO_{t+1}] + \left(\frac{1}{1+r+\xi}\right)^2 \cdot E[PO_{t+2}] + \left(\frac{1}{1+r+\xi}\right)^3 \cdot E[PO_{t+3}] + \ldots$$

Week 2: Expected Returns, Expected Risk

Buybacks

Why are buy-backs pay-outs?

- There 15.11bn shares outstanding, worth $3,377.6bn at a price of $223.45 / share
- Suppose you own 500mm shares; you own 3.3% of AAPL
- Suppose Apple buy back 250mm shares; now, you own 3.4% of AAPL. So you have a bigger share of their forward earnings…
- What should you do to get back to your earlier ownership stake? You need to sell 8.25mm shares, or receive almost $2bn in dollars
- You pay capital gains on your $2bn, whereas you pay ordinary income on the $19.6mm in dividends you receive every year.

Week 2: Expected Returns, Expected Risk

Equity Risk Premium to Bonds

Deploying the summation operator:

$$P_t = \sum_{k=0}^{\infty} \left(\frac{1}{1 + r + \xi} \right)^k E[PO_{t+k}]$$

Assuming $E[PO_{t+k}] = (1 + g)^k PO_t$, we can write

$$P_t = \sum_{k=0}^{\infty} \left(\frac{(1 + g)}{(1 + r + \xi)} \right)^k PO_t$$

Since $\sum_{k=0}^{\infty} \beta^k = \frac{1}{(1-\beta)}$ we have:

$$\xi_t = ERPB_t = \frac{(1 + g)}{\left(1 - \left(\frac{PO_t}{P_t} \right) \right)} - (1 + r)$$

Week 2: Expected Returns, Expected Risk

Equity Risk Premium to Bonds

Damodaron (2022), following Gordon (1959), uses a different timing convention:

$$P_t = \sum_{k=1}^{\infty} \left(\frac{1}{(1 + r + \xi)} \right)^k E[PO_{t+k+1}]$$

Substituting $E[PO_{t+k}] = (1 + g)^k PO_t$ we get

$$P_t = \sum_{k=1}^{\infty} \left(\frac{(1 + g)}{(1 + r + \xi)} \right)^k E[PO_{t+1}]$$

Which simplifies to

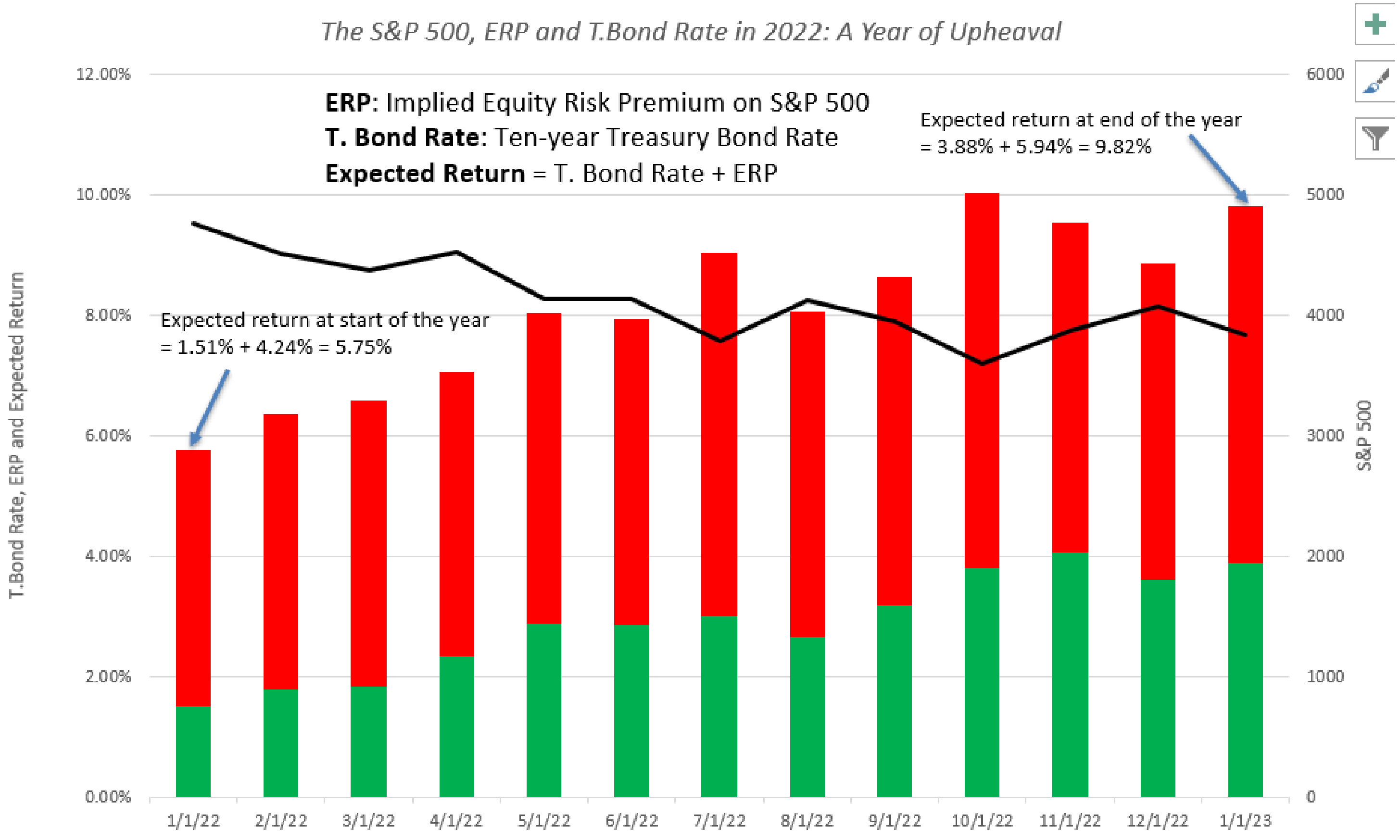
$$P_t = \frac{E[PO_{t+1}]}{(r+\xi-g)} \text{ or } \xi = \frac{E[PO_{t+1}]}{P_t} + g - r$$

Week 2: Expected Returns, Expected Risk

Equity Risk Premium to Bonds

An estimate from Damodaran:

- Higher than many Street estimates which do not include Buy-backs or Growth (i.e., earnings yield – US 10yr)
- His projection methodology for Payouts(t+i) is detailed…



	1/1/22	2/1/22	3/1/22	4/1/22	5/1/22	6/1/22	7/1/22	8/1/22	9/1/22	10/1/22	11/1/22	12/1/22	1/1/23
ERP (T12m)	4.24%	4.56%	4.75%	4.73%	5.14%	5.07%	6.01%	5.42%	5.45%	6.21%	5.48%	5.26%	5.94%
T.Bond Rate	1.51%	1.79%	1.83%	2.33%	2.89%	2.86%	3.02%	2.65%	3.19%	3.82%	4.05%	3.61%	3.88%
S&P 500	4766	4515	4374	4530	4132	4132	3785	4130	3955	3596	3872	4080	3840

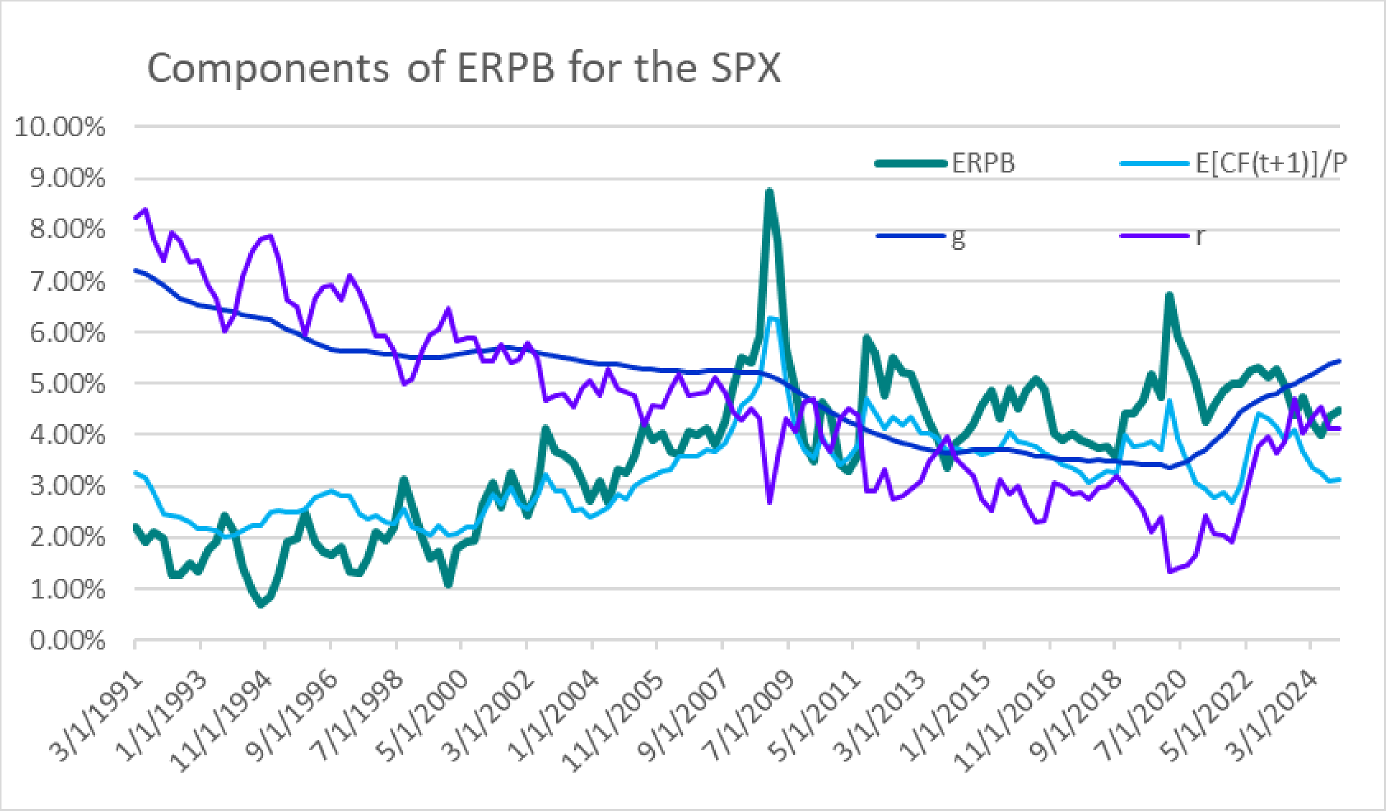
T.Bond Rate ERP (T12m) S&P 500

Week 2: Expected Returns, Expected Risk

Equity Risk Premium to Bonds

An estimate from Bloomberg Data:

- g from CBO
- r = UST30yr
- CF/P = Cyclically Adjusted CF
- CF = dividends + buybacks



Week 2: Expected Returns, Expected Risk

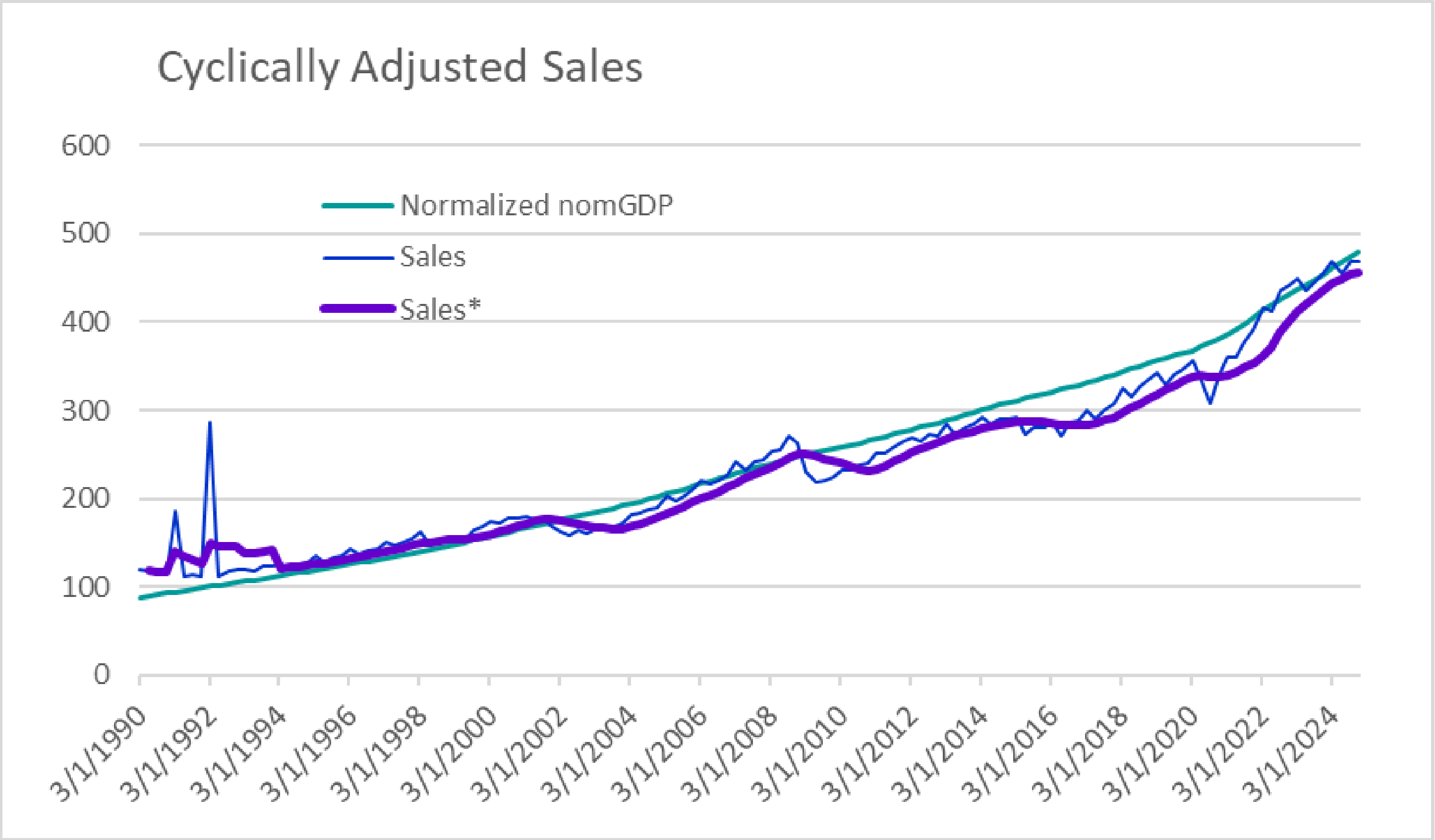
Equity Risk Premium to Bonds

An estimate from Bloomberg Data:

- CF/P = div2earnRatio*Earn_sm +

  bb2earnRatio* Earn_sm

- Earn_sm = earn2salesRatio*Sales_sm



Week 2: Resources and References

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Week 2: Resources and References

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