Comparative Strategic Asset Allocation

Divya Patel, Tarun Kumar, Giga Nozadze

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Professor: James Conklin
NYU Tandon School of Engineering

Project Overview and Modeling Framework

Fund and Investment Context

- Defined contribution pension plan with monthly salary-based contributions
- Strategic investment horizon: 10 years — focused on long-term growth and capital preservation under uncertainty

Asset Universe

- 15 benchmark indices including cash, public/private equities, public/private fixed income, and alternatives (real estate, infrastructure)
- Covers a range of liquidity, duration, and transparency characteristics

Return and Risk Estimation

- Expected returns and volatilities derived from JPM and BlackRock capital market assumptions, as well as historical time series.
- Blending forward-looking assumptions with backward-looking statistics to ensure robust input quality

Optimization Objective

- Apply robust optimization based on the multi-prior framework (Garlappi et al., 2004)
- Combine mean-variance and minimum-variance portfolios to reduce sensitivity to estimation error.
- Construct portfolios tailored to long-horizon institutional investors under return uncertainty

Long-Term Capital Market Assumptions: JPM and BlackRock

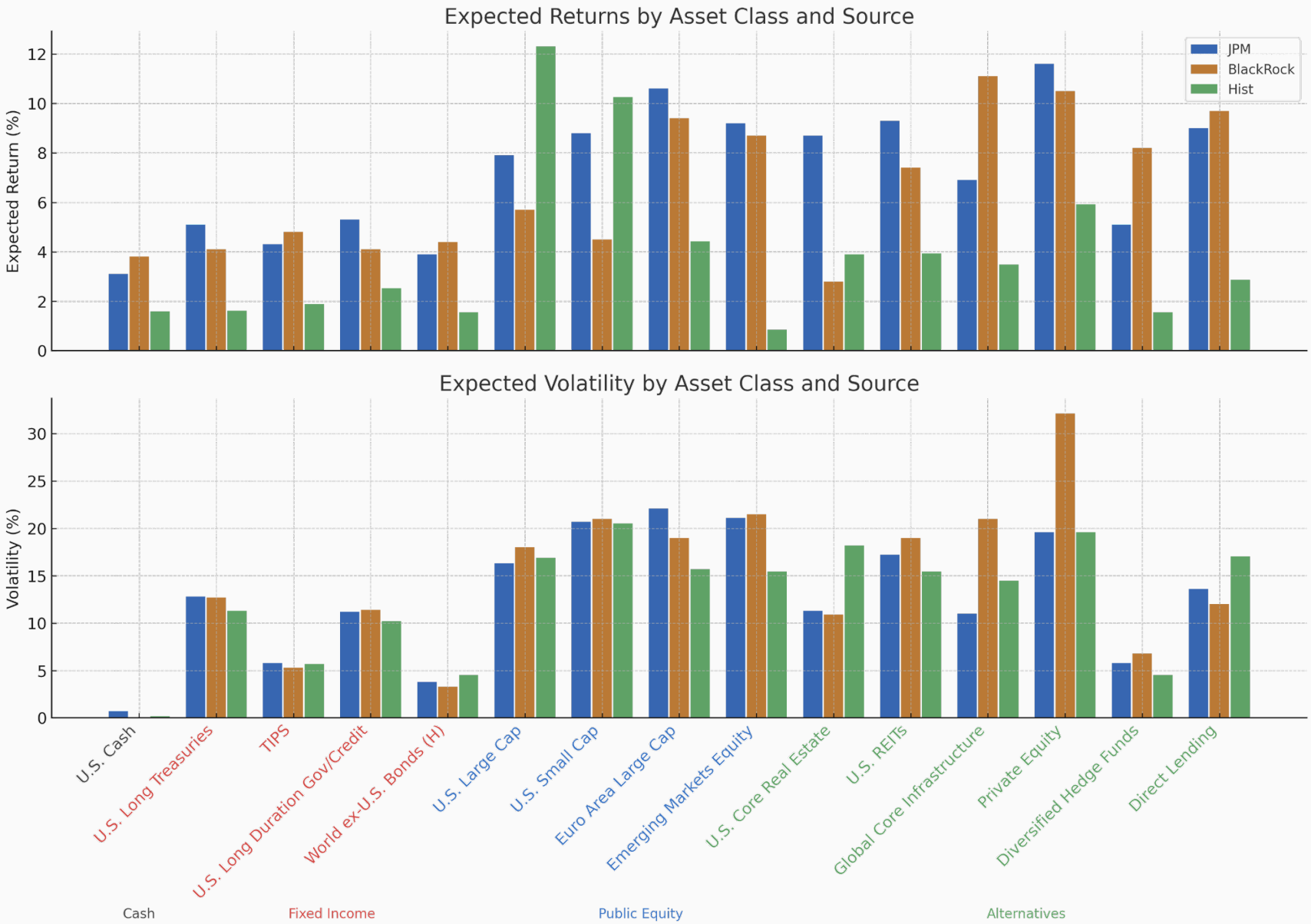
J.P. Morgan (2025 LTCMAs)

- Transitioning from the low-rate, low-growth regime of the 2010s to a higher-growth, higher-rate environment.
- Projected 6.4% return for a USD 60/40 portfolio; slightly below last year but in line with long-term averages.
- Higher rates support bond returns; capital investment and AI support long-run equity returns.
- Inflation volatility remains elevated due to geopolitical fragmentation and fiscal activism.
- AI expected to add 20bps to developed market growth and improve productivity.
- Emphasis on real assets and alternatives for diversification and inflation protection.

BlackRock (2025 CMAs)

- We are in a transformational era: AI, deglobalization, and fiscal changes reshape economic trajectories.
- Rising bond yields lead to overweight in short/mid-duration developed market bonds, underweight long Treasuries.
- Private credit and infrastructure equity seen as key beneficiaries of AI-driven capex boom.
- Long-term inflation and rates expected to remain above pre-pandemic norms.
- Rethink of portfolio construction: emphasize themes over asset classes and favor dynamic, granular positioning.

Expected Return and Volatility Comparison



Summary of Observations by Asset Class

Asset Class	Expected Return	Volatility
Public Equities	Forward-looking estimates are generally higher than historical averages, except for US where LTMCA assume moderation of AI boom.	Historical and forward volatilities are broadly aligned, though forward estimates slightly higher due to macro uncertainty.
Fixed Income	Modest uplift in expected return in capital market assumptions, reflecting higher yield environment.	Volatility higher in long-duration bonds; historical vol estimates tend to be slightly lower.
Alternatives	Forward-looking returns significantly higher than historical averages, especially for private credit and PE.	Historical volatilities often understated due to infrequent pricing and smoothing effects.
Real Estate	Historical returns are notably lower than forward-looking estimates due to COVID impact.	Volatility appears lower in historical data; forward estimates reflect valuation and liquidity risk.

From Classical to Robust Optimization

Motivation: Why Robust Optimization?

- Classical mean-variance optimization assumes perfectly known expected returns μ.
- In reality, μ must be estimated, often imprecisely — especially for private and illiquid assets.
- Even small estimation errors can lead to extreme, unstable, and unrealistic portfolios.

Idea: Incorporate Uncertainty Aversion

- Use a *multi-prior* framework (Garlappi, Uppal, Wang, 2004) to model ambiguity in μ.
- Investor does not trust a single point estimate $\hat{\mu}$; instead considers a set of plausible values within a confidence region.
- The worst-case (min) expected return in this set is used in optimization — leading to more conservative portfolios.

Approach Summary

- Optimization becomes:

$$\max_{w} \min_{\mu \in \mathcal{U}} \left(w^\top \mu - \frac{\gamma}{2} w^\top \Sigma w \right)$$

- $\mathcal{U}$: uncertainty set defined by confidence intervals or ellipsoids around $\hat{\mu}$
- Robust solution behaves like a blend of mean-variance and minimum-variance portfolios
- Degree of conservatism controlled by size of the uncertainty region (parameter ε)

Robust Optimization: Final Formulation and Constraints

Robust Reformulation (Ellipsoidal Uncertainty Set):

$$\max_{w} \left(w^\top \hat{\mu} - \frac{\gamma}{2} w^\top \Sigma w - \sqrt{\varepsilon \cdot w^\top \Sigma w} \right)$$

- Uncertainty Penalty Parameter ε reflects the investor's aversion to return estimation error:

$$\varepsilon = F^{-1}_{N,T-N}(1-p) \cdot \frac{(T-1)N}{T(T-N)}$$

- $F^{-1}_{N,T-N}(1-p)$: quantile from the F-distribution.

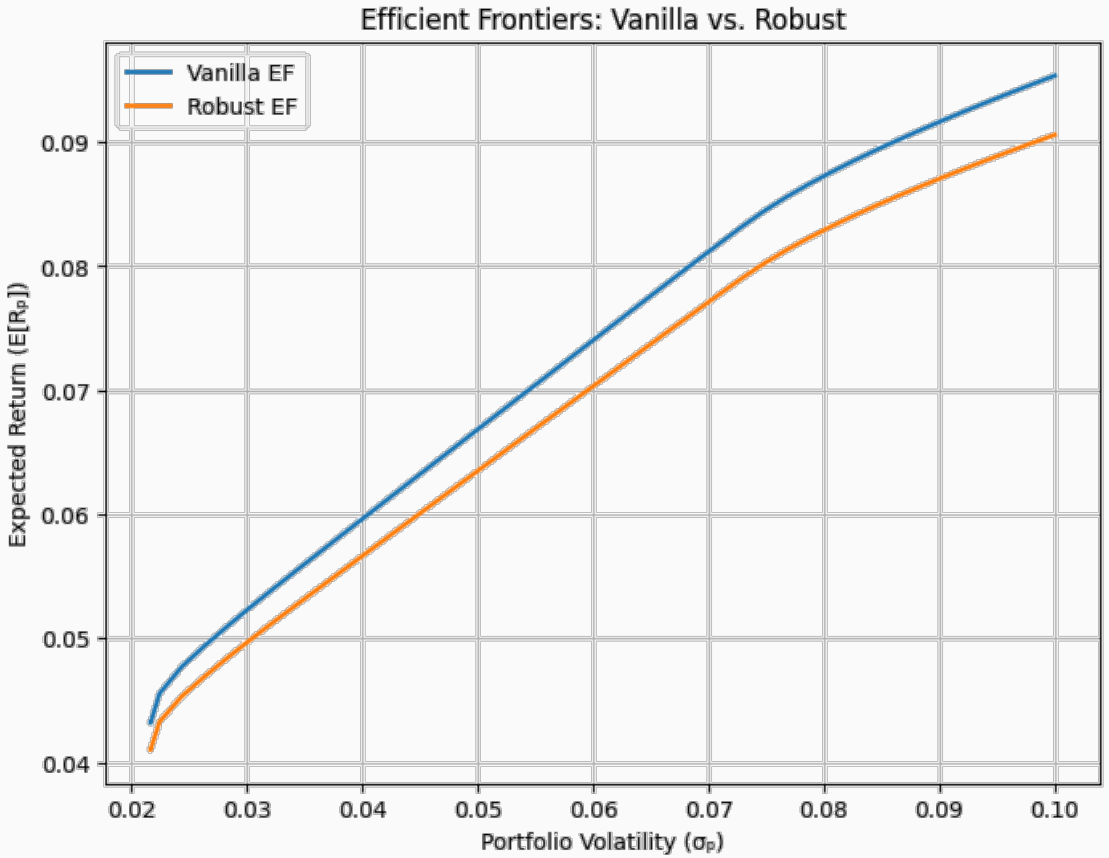
Constraints:

- Fully invested: $\sum w_i = 1$, long-only: $0 \leq w_i \leq 50\%$
- Minimum return: $w^\top \hat{\mu} \geq r_f + 2\%$ (spread) $+ 0.5\%$ (management fee)
- Duration floor: $\sum w_i \cdot \text{Duration}i \geq D\textbf{min}$
- Asset class bands: pension-specific policy limits:

Asset Class	Minimum Allocation	Maximum Allocation
Public Equity	40%	60%
Fixed Income	20%	40%
Cash	1%	3%
Alternatives	5%	15%

Comparison of Optimization Approaches

- Robust optimization accounts for parameter uncertainty, leading to more conservative portfolios with lower risk exposure.
- As a result, the robust efficient frontier shifts inward compared to the standard mean-variance frontier, reflecting a trade-off between robustness and return.



Portfolio Allocation by Risk Aversion Parameter (Historical)

Asset	Exp. Return	Volatility	$\gamma = 2$	$\gamma = 5$	$\gamma = 8$
Cash					
3-Month T-Bill	1.59%	0.17%	1.00%	1.00%	1.59%
Fixed Income					
U.S. Long Treasuries	1.62%	11.31%	1.00%	1.00%	1.00%
TIPS	1.88%	5.69%	8.33%	15.54%	15.48%
U.S. Long Duration Government/Credit	2.52%	10.21%	19.67%	12.46%	9.13%
World ex-U.S. Government Bonds hedged	1.55%	4.51%	1.00%	1.00%	4.40%
Public Equities					
U.S. Large Cap	12.30%	16.89%	50.00%	48.54%	40.28%
U.S. Small Cap	10.26%	20.52%	7.62%	1.00%	1.00%
Euro Area Large Cap	4.42%	15.68%	1.00%	6.46%	14.13%
Emerging Markets Equity	0.85%	15.44%	1.00%	1.00%	1.00%
Alternatives					
U.S. Core Real Estate	1.59%	0.17%	1.00%	1.00%	1.00%
U.S. REITs	3.93%	15.44%	1.00%	1.00%	1.00%
Global Core Infrastructure	3.49%	14.48%	1.00%	1.00%	1.00%
Private Equity	5.92%	19.57%	4.38%	7.00%	7.00%
Diversified Hedge Funds	1.55%	4.51%	1.00%	1.00%	1.00%
Direct Lending	2.87%	17.04%	1.00%	1.00%	1.00%
		Portfolio $E[r]$	**8.08%**	**7.57%**	**6.87%**
		Portfolio Volatility	**10.00%**	**8.72%**	**7.56%**
		Portfolio Sharpe Ratio	**0.81**	**0.87**	**0.91**

Portfolio Allocation by Risk Aversion Parameter (JP Morgan)

Asset	Exp. Return	Volatility	$\gamma = 2$	$\gamma = 5$	$\gamma = 8$
Cash					
3-Month T-Bill	3.10%	0.70%	2.00%	2.00%	2.00%
Fixed Income					
U.S. Long Treasuries	5.10%	12.80%	21.68%	21.68%	14.77%
TIPS	4.30%	5.80%	1.00%	1.00%	1.00%
U.S. Long Duration Government/Credit	5.30%	11.20%	1.00%	1.00%	1.00%
World ex-U.S. Government Bonds hedged	3.90%	3.80%	12.32%	12.32%	19.23%
Public Equities					
U.S. Large Cap	7.90%	16.30%	28.97%	28.97%	39.21%
U.S. Small Cap	8.80%	20.70%	1.00%	1.00%	1.00%
Euro Area Large Cap	10.60%	22.10%	13.80%	13.80%	1.65%
Emerging Markets Equity	9.20%	21.10%	6.23%	6.23%	8.14%
Alternatives					
U.S. Core Real Estate	8.70%	11.30%	7.00%	7.00%	7.00%
U.S. REITs	9.30%	17.20%	1.00%	1.00%	1.00%
Global Core Infrastructure	6.90%	11.00%	1.00%	1.00%	1.00%
Private Equity	11.60%	19.60%	1.00%	1.00%	1.00%
Diversified Hedge Funds	5.10%	5.80%	1.00%	1.00%	1.00%
Direct Lending	9.00%	13.60%	1.00%	1.00%	1.00%
		Portfolio $E[r]$	**7.18%**	**7.18%**	**6.79%**
		Portfolio Volatility	**10.00%**	**10.00%**	**9.44%**
		Portfolio Sharpe Ratio	**0.72**	**0.72**	**0.72**

Portfolio Allocation by Risk Aversion Parameter (BlackRock)

Asset	Exp. Return	Volatility	$\gamma = 2$	$\gamma = 5$	$\gamma = 8$
Cash					
U.S. Cash	3.8%	0.0%	1.00%	1.00%	1.00%
Fixed Income					
U.S. Long Treasuries	4.1%	12.7%	1.00%	1.00%	1.00%
TIPS	4.8%	5.3%	27.00%	27.37%	23.40%
U.S. Long Duration Government/Credit	4.1%	11.4%	1.00%	1.00%	1.00%
World ex-U.S. Government Bonds hedged	4.4%	3.3%	1.00%	7.63%	11.60%
Public Equities					
US large cap equities	5.7%	18.0%	1.00%	8.62%	12.08%
US small cap equities	4.5%	21.0%	1.00%	1.00%	1.00%
Europe large cap equities	9.4%	19.0%	37.72%	25.43%	22.10%
Emerging Markets Equity	8.7%	21.5%	17.28%	14.95%	14.82%
Alternatives					
US core real estate	2.8%	10.9%	1.00%	1.00%	1.00%
Listed REITs	7.4%	19.0%	1.00%	1.00%	1.00%
Global infrastructure equity	11.1%	21.0%	7.00%	7.00%	6.60%
US private equity (buyout)	10.5%	32.1%	1.00%	1.00%	1.40%
Hedge funds (global)	8.2%	6.8%	1.00%	1.00%	1.00%
Global direct lending	9.7%	12.0%	1.00%	1.00%	1.00%
		Portfolio $E[r]$	**7.78%**	**7.17%**	**7.02%**
		Portfolio Volatility	**8.00%**	**6.15%**	**5.84%**
		Portfolio Sharpe Ratio	**0.97**	**1.17**	**1.20**

Conclusion

Strategic Insights from Robust Optimization Framework

- Robust optimization provides a realistic and resilient portfolio construction method, particularly valuable under model uncertainty and estimation risk.
- By explicitly modeling ambiguity in expected returns using the multi-prior framework (Garlappi et al., 2007), the optimizer penalizes estimation risk—leading to more stable, diversified, and implementable allocations.
- This approach reduces sensitivity to noisy or misspecified inputs and avoids the extreme allocations commonly produced by classical mean-variance optimization.

Key Findings Across Capital Market Assumptions

- **J.P. Morgan CMAs** support balanced portfolios across varying risk preferences, benefiting from structured, forward-looking macro assumptions.
- **BlackRock CMAs** tilt more heavily toward equities and alternatives, reflecting thematic confidence in long-term structural shifts such as AI and deglobalization.
- **Historical estimates** result in more return-seeking and aggressive portfolios under low risk aversion, driven by favorable past performance. However, Historical estimates may lead to less robust portfolios when used in naive models, as they reflect past regimes and contain estimation noise. Forward-looking CMAs, while also uncertain, can enable more context-aware allocations if incorporated through robust modeling.

Appendix: References and Attachments

Papers Referred

- J.P. Morgan (2025). *Long-Term Capital Market Assumptions*.
- BlackRock (2025). *Long-Term Capital Market Assumptions*.
- Garlappi, L., Uppal, R., Wang, T. (2007). *Portfolio Selection with Parameter and Model Uncertainty*, Review of Financial Studies.
- Ambachtsheer, K. (2021). *Canadian Pensions: Past, Present and Future*, Journal of Portfolio Management.
- Dyck, A., Pomorski, L. (2012). *Can Large Pension Funds Beat the Market? Asset Allocation, Market Timing, Security Selection, and the Limits of Liquidity*.

Attachments Provided

- **Code:** 3 Python files implementing robust optimization and asset allocation.
- **Data:** 5 excel input data files, including capital market assumptions and historical returns.