

Cryptographic Primitives and Algorithms in L^AT_EX

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Abstract

This document is a collection of simple sample of cryptographic primitives and algorithms.

1 Diffie-Hellman Key Exchange

Diffie-Hellman Key Exchange [1] preceeds several cryptography mechanisms we use today.

1. Choose Public Parameters

Two parties, *Alice* and *Bob*, agree on:

A prime number $p = 17$

A primitive root modulo of $p = 17$, called the generator $g = 3$

2. Generate Private Keys *Alice* and *Bob* picks a private key each (a secret number):

Alice picks private key $a = 6$ and only *Alice* knows about it

Bob picks private key $b = 11$ and only *Bob* knows about it

These private keys are kept secret.

3. Compute Public Keys

Each party computes a public key using the formula:

$$A = g^a \mod p \tag{1}$$

$$B = g^b \mod p \tag{2}$$

Alice computes her public key A

$$\begin{aligned} A &= 3^6 \mod 17 \\ A &= 15 \end{aligned}$$

Bob computes his public key B

$$\begin{aligned} B &= 3^{11} \mod 17 \\ B &= 7 \end{aligned}$$

These public keys are exchanged over the insecure channel

4. Compute the Shared Secret

Now each party computes the shared secret $s = s_A = s_B$

At *Alice*

$$s_A = B^a \mod p \tag{3}$$

At *Bob*

$$s_B = A^b \mod p \tag{4}$$

Alice computes s_A

$$\begin{aligned} s_A &= 7^6 \mod 17 \\ s_A &= 2 \end{aligned}$$

Bob computes s_B

$$s_B = 15^{11} \mod 17$$
$$s_B = 2$$

Thus,

$$s_A = s_B$$

Verify

Since

$$(g^a)^b = g^{ab}$$

And multiplication is commutative $ab = ba$

References

- [1] Whitfield Diffie and Martin E Hellman. New directions in cryptography. In Democratizing Cryptography: The Work of Whitfield Diffie and Martin Hellman, pages 365–390. 2022.