S4 - 17B. Devoir 2

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Exercice 2:

Resolution des equations differentielles non lineraires de 2nd ordre

5. (E): $y' = xy' + \sqrt(1 + (y')^2)$

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6. (E): $x = yy' + (y')^3$

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7. (E): $2xy' = y + y'$

$$\begin{aligned} 2xy' &= y + y' \\ -y' + 2xy' &= y \\ y'(-1 + 2x) &= y \\ y' \frac{(-1 + 2x)}{(-1 + 2x)} &= \frac{1}{(-1 + 2x)} \cdot y \\ y' &= \frac{1}{(-1 + 2x)} \cdot y \end{aligned}$$

<u>Calcul de la solution generale :</u>

$$\begin{aligned} y &= \lambda e^{-\int \frac{1}{\frac{1}{(-1+2x)}}}, \quad \lambda \in \mathbb{R} \\ &= \lambda e^{-\int (-1+2x)}, \quad \lambda \in \mathbb{R} \end{aligned}$$

<u>Calculons d'abord l'integrale :</u>

$$\begin{aligned} -\int (-1 + 2x) &= -(-x + x^2) \\ &= x - x^2 \end{aligned}$$

Alors :

$$y = \lambda e^{x-x^2}, \quad \lambda \in \mathbb{R}$$

$\boxed{y = \lambda e^{x-x^2}, \quad \lambda \in \mathbb{R}}$

Exercice 3:

Resolution des equations differentielles du second ordre:

5. (E): $y'' + y = xe^x + 2e^{-x}$ ou $y(0) = y'(0) = 1$

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6. (E): $y'' - 4y' + y = xe^{2x}$

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7. (E): $y'' + 2y' + 5y = cosx$ ou $y(0) = y'(0) = 0$
   Etape 1 : ESSM:

$$y'' + 2y^t + 5y = 0$$

Equation caracteristique:

$$r^2 + 2r + 5 = 0$$

$$\Delta = 2^2 - 4 \times 1 \times 5 = 4 - 20 = -16$$

$$r = \frac{-2 \pm \sqrt{-16}}{2 \times 1} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

D'ou on a:

$$y = e^{-x}(E\cos(2x) + F\sin(2x))$$

<u>Etape 2</u> : EASM:

$y'' + 2y' + 5y = \cos(x) \quad (E)$

$$\begin{aligned}
y_p &= A\cos(x) + B\sin(x) \quad (1)\\
y_p' &= -A\sin(x) + B\cos(x) \quad (2)\\
y_p'' &= -A\cos(x) - B\sin(x) \quad (3)
\end{aligned}$$

(1), (2), (3) dans (E):

$$\begin{aligned}
(-A\cos(x) - B\sin(x)) + 2(-A\sin(x) + B\cos(x)) + 5(A\cos(x) + B\sin(x)) &= \cos(x)\\
(-A + 2B + 5A)\cos(x) + (-B - 2A + 5B)\sin(x) &= \cos(x)\\
(4A + 2B)\cos(x) + (4B - 2A)\sin(x) &= \cos(x)
\end{aligned}$$

Systeme:

$$\begin{aligned}
4A + 2B &= 1\\
-2A + 4B &= 0
\end{aligned}$$

Resolution:

$$\begin{aligned}
-2A + 4B = 0 &\Rightarrow A = 2B\\
4(2B) + 2B = 1 &\Rightarrow 8B + 2B = 1\\
10B = 1 &\Rightarrow B = \frac{1}{10}\\
A = 2B &= \frac{2}{10} = \frac{1}{5}
\end{aligned}$$

D'ou:

$$\begin{aligned}
y &= e^{-x}(E\cos(2x) + F\sin(2x)) + \frac{1}{5}\cos(x) + \frac{1}{10}\sin(x)\\
y(0) &= e^{0}(E\cos(0) + F\sin(0)) + \frac{1}{5}\cos(0) + \frac{1}{10}\sin(0) = 0
\end{aligned}$$

$$E + \frac{1}{5} = 0 \Rightarrow E = -\frac{1}{5}$$

Derivee:

$$y'(0) = 0$$

$$y' = e^{-x}(E\cos(2x) + F\sin(2x)) - e^{-x}(-2E\sin(2x) + 2F\cos(2x)) - \frac{1}{5}\sin($$

$$y'(0) = e^{0}(E\cos(0) + F\sin(0)) - e^{0}(-2E\sin(0) + 2F\cos(0)) - \frac{1}{5}\sin(0) + \frac{1}{10}$$

$$-E + 2F + \frac{1}{10} = 10$$

$$-(-\frac{1}{5}) + 2F + \frac{1}{10} = 0 \implies \frac{1}{5} + 2F + \frac{1}{10} = 0$$

$$2F = -\frac{1}{5} - \frac{1}{10}$$

$$2F = -\frac{3}{10}$$

$$F = -\frac{3}{20}$$

$$\boxed{y = e^{-x}(-\frac{1}{5}\cos(2x) - \frac{3}{20}\sin(2x)) + \frac{1}{5}\cos(x) + \frac{1}{10}\sin(x)}$$

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8. (E): $y'' - 4y + 13y = cos(3x)$

<u>Etape 1 : ESSM</u>

$y'' - 4y + 13y = 0$

<u>Equation caracteristique :</u>

$$r^2 - 4r + 13 = 0$$

$$\Delta = (-4)^2 - 4 \times 1 \times 13 = 16 - 52$$

$$r = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

$\Delta < 0$, donc $y = e^{2x}(A\cos 3x + B\sin 3x)$

Etape 2 : EASM

$$\begin{aligned}
y'' - 4y' + 13y &= \cos(3x) \\
f(x) &= \cos(3x) \\
y_p &= m\cos(3x) + n\sin(3x) \\
y_p' &= -3m\sin(3x) + 3n\cos(3x) \\
y_p'' &= -9m\cos(3x) - 9n\sin(3x)
\end{aligned}$$

Substituons dans l'equation :

$$\begin{aligned}
(-9m\cos(3x) - 9n\sin(3x)) - 4(-3m\sin(3x) + 3n\cos(3x)) + 13(m\cos(3x) + n\sin(3x)) &= \mathrm{c} \\
(-9m + 12n + 13m)\cos(3x) + (-9n - 12m + 13n)\sin(3x) &= \mathrm{c} \\
(4m + 12n)\cos(3x) + (4n - 12m)\sin(3x) &= \mathrm{c}
\end{aligned}$$

On obtient le systeme :

$$\begin{aligned}
4m + 12n &= 1 \\
-12m + 4n &= 0
\end{aligned}$$

Resolution :

$$\begin{aligned}
-12m + 4n &= 0 \\
4n &= 12m \\
n &= 3m
\end{aligned}$$

Substitution dans la premiere equation :

$$\begin{aligned}
4m + 12(3m) &= 1 \\
4m + 36m &= 1 \\
40m &= 1 \\
m &= \frac{1}{40}
\end{aligned}$$

Donc, on a solution :

$$\boxed{y = e^{2x}(A\cos(3x) + B\sin(3x)) + \frac{1}{40}\cos(3x) + \frac{3}{40}\sin(3x)}$$