S4-B, Devoir 1

ETU003291 - Raharijaona Tehina Fanekena

Mars 2025

Exercice 2

Résolution des équations différentielles non linéaires du 1^{er} ordre

3. $y'(1-\sin(x)\cos(x)) + y^2\cos(x) - y + \sin(x) = 0$
si $y_p = \cos(x)$ est une solution particulière

$y'(1-\sin(x)\cos(x)) + y^2\cos(x) - y + \sin(x) = 0$

$(1-\sin(x)\cos(x))y' - y + \cos(x)y^2 = -\sin(x)$**(E)**

Posons $y = \frac{1}{z} + \cos(x)$**(1)** , $y' = -\frac{z'}{z^2} - \sin(x)$**(2)**

(1) et **(2)** dans **(E)**

$(1-\sin(x)\cos(x))(-\frac{z'}{z^2} - \sin(x)) - (\frac{1}{z} + \cos(x)) + \cos(x)(\frac{1}{z} + \cos(x))^2 = -\sin(x)$

$-\frac{z'}{z^2} - \sin(x) + \frac{z'}{z^2}\sin(x)\cos(x) + \sin^2(x)\cos(x) - \frac{1}{z} - \cos(x) + \cos(x)(\frac{1}{z} + \cos(x))^2 = -\sin(x)$

$-\frac{z'}{z^2} + \frac{z'}{z^2}\sin(x)\cos(x) + \sin^2(x)\cos(x) - \frac{1}{z} - \cos(x) + \cos(x)(\frac{1}{z} + \cos(x))^2 = 0$

Développons $\cos(x)(\frac{1}{z} + \cos(x))^2$

$\cos(x)(\frac{1}{z} + \cos(x))^2 = \cos(x)(\frac{1}{z^2} + \frac{2\cos(x)}{z} + \cos^2(x)) = \frac{\cos(x)}{z^2} + \frac{2\cos^2(x)}{z} + \cos^3(x)$

$\cos^3(x) = \cos^2(x)\cos(x)$, $\cos^2(x) = 1 - \sin^2(x)$

$\cos^3(x) = (1-\sin^2(x))\cos(x) = \cos(x) - \sin^2(x)\cos(x)$

$\cos(x)(\frac{1}{z} + \cos(x))^2 = \frac{\cos(x)}{z^2} + \frac{2\cos^2(x)}{z} + \cos(x) - \sin^2(x)\cos(x)$

Ainsi

$-\frac{z'}{z^2} + \frac{z'}{z^2}\sin(x)\cos(x) + \sin^2(x)\cos(x) - \frac{1}{z} - \cos(x) + \frac{\cos(x)}{z^2} + \frac{2\cos^2(x)}{z} + \cos(x) - \sin^2(x)\cos(x) = 0$

$-\frac{z'}{z^2} + \frac{z'}{z^2}\sin(x)\cos(x) - \frac{1}{z} + \frac{\cos(x)}{z^2} + \frac{2\cos^2(x)}{z} = 0$

$(\sin(x)\cos(x) - 1)\frac{z'}{z^2} + (2\cos^2(x) - 1)\frac{1}{z} + \cos(x)\frac{1}{z^2} = 0$

Multiplions chaque membre par z^2

$(\sin(x)\cos(x) - 1)z' + (2\cos^2(x) - 1)z + \cos(x) = 0$

$(\sin(x)\cos(x) - 1)z' + (2\cos^2(x) - 1)z = -\cos(x)$

<u>ESSM:</u> $(\sin(x)\cos(x) - 1)z' + (2\cos^2(x) - 1)z = 0$

$z_0 = \lambda e^{\phi(x)}$, avec $\phi(x) = \int -\frac{(2\cos^2(x)-1)}{(\sin(x)\cos(x)-1)}dx = -\int \frac{2\cos^2(x)-1}{\sin(x)\cos(x)-1}dx$

Calculons $I = \int \frac{2\cos^2(x)-1}{\sin(x)\cos(x)-1}dx$

$2\cos^2(x) - 1 = 2\cos^2(x) - \sin^2(x) - \cos^2(x) = \cos^2(x) - \sin^2(x) = \cos(2x)$

$2\cos^2(x) - 1 = \cos(2x)$ **(3.1)**

$\sin(b)\cos(a) = \frac{\sin(a+b)}{2} - \frac{\sin(a-b)}{2}$

$\sin(x)\cos(x) = \frac{\sin(2x)}{2} - \frac{\sin(0)}{2} = \frac{\sin(2x)}{2}$

$\sin(x)\cos(x) = \frac{\sin(2x)}{2}$ **(3.2)**

$I = \int \frac{\cos(2x)}{\frac{\sin(2x)}{2}-1}dx = \int \frac{\cos(2x)}{\frac{\sin(2x)-2}{2}}dx = \int \frac{2\cos(2x)}{\sin(2x)-2}dx$

Posons $u = \sin(2x) - 2$, $u' = 2\cos(2x)$

$I = \int \frac{u'}{u}dx = \ln(|u|) + c = \ln(|\sin(2x) - 2|) + c$

$z_0 = \lambda e^{-\ln(|\sin(2x)-2|)+c} = \lambda e^{\ln(|\sin(2x)-2|)^{-1}} e^c = \lambda e^c |\sin(2x) - 2|^{-1}, K = \pm\lambda e^c$

$z_0 = \frac{K}{\sin(2x)-2}$

<u>EASM:</u> $(\sin(x)\cos(x) - 1)z' + (2\cos^2(x) - 1)z = -\cos(x)$ **(E1)**

<u>MVC:</u> $z_0 = \frac{K}{\sin(2x)-2}$ **(3)** , $z_0' = \frac{K'(\sin(2x)-2)-(2\cos(2x))K}{(\sin(2x)-2)^2}$ **(4)**

(3) et **(4)** dans **(E1)**

$(\sin(x)\cos(x) - 1)\frac{K^{'}(\sin(2x)-2)-(2\cos(2x))K}{(\sin(2x)-2)^2} + (2\cos^2(x) - 1)\frac{K}{\sin(2x)-2} = -\cos(x)$

$(\sin(x)\cos(x) - 1)\frac{K'(\sin(2x)-2)-(2\cos(2x))K}{(\sin(2x)-2)^2} + (2\cos^2(x) - 1)\frac{K}{\sin(2x)-2} = -\cos(x)$

$(\frac{\sin(2x)}{2} - 1)[\frac{K'}{\sin(2x)-2} - \frac{2K\cos(2x)}{(\sin(2x)-2)^2}] + (\cos(2x))\frac{K}{\sin(2x)-2} = -\cos(x)$ **(3.1)(3.2)**

$(\frac{\sin(2x)}{2} - 1)(2K\cos(2x)) = (\sin(2x) - 2)cos(2x)K$

$\frac{K'(\frac{\sin(2x)}{2}-1)}{\sin(2x)-2} - \frac{(\sin(2x)-2)K\cos(2x)}{(\sin(2x)-2)^2} + (\cos(2x))\frac{K}{\sin(2x)-2} = -\cos(x)$

$\frac{K'(\frac{\sin(2x)}{2}-1)}{\sin(2x)-2} - \frac{K\cos(2x)}{\sin(2x)-2} + \frac{K\cos(2x)}{\sin(2x)-2} = -\cos(x)$

$\frac{K'(\frac{\sin(2x)}{2}-1)}{\sin(2x)-2} = -\cos(x)$, $\frac{K'(\frac{\sin(2x)-2}{2})}{\sin(2x)-2} = -\cos(x)$, $\frac{K'(\sin(2x)-2)}{2(\sin(2x)-2)} = -\cos(x)$

$\frac{K'}{2} = -\cos(x)$, $K' = -2\cos(x)$

$\frac{dK}{dx} = -2\cos(x)$, $K = \int -2\cos(x)dx$, $K = -2\sin(x) + \epsilon$

$z_0 = \frac{-2\sin(x)+\epsilon}{\sin(2x)-2}$ or $y = \frac{1}{z} + \cos(x)$

$y = \frac{\sin(2x)-2}{-2\sin(x)+\epsilon} + \cos(x)$