S4 - 17B. Devoir 2

ETU003235: ANDERSON Soamiavaka Vanille
ETU003247: ANDRIANAJA Onja Fanilo
ETU003286: RABETOKOTANY Yvan Noaaaaaaaaaaaaaah
ETU003298: RAJAONARIVONY Tandrifiniaina Dylan
ETU003305: RAKOTOARIVONY Loïc Dylan
ETU003331: RANAIVOSON Miora Randie
ETU003335: RANDRIAMAHEFA Liantsoa Alicia
ETU003348: RANDRIANIRINA Niriela Andraina
ETU003363: RATSITO Oelirivo Mitia
ETU003378: RAZAKANDISA Sariaka Niaina

Exercice 2:

Resolitopm des equations differentielles non lineraires de 2nd ordre

1. (E): $y' = xy' + \sqrt{(}1 + (y')^2)$

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2. (E): $x = yy' + (y')^3$

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3. (E): $2xy' = y + y'$

Exercice 3:

Resolution des equations differentielles du second ordre:

1. (E): $y'' + y = xe^x + 2e^{-x}$ ou $y(0) = y'(0) = 1$

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2. (E): $y'' - 4y' + y = xe^{2x}$

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3. (E): $y'' + 2y' + 5y = cosx$ ou $y(0) = y'(0) = 0$
   Etape 1 : ESSM:

$$y'' + 2y^t + 5y = 0$$

Equation caracteristique:

$$r^2 + 2r + 5 = 0$$
$$\Delta = 2^2 - 4 \times 1 \times 5 = 4 - 20 = -16$$
$$r = \frac{-2 \pm \sqrt{-16}}{2 \times 1} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

D'ou on a:

$$y = e^{-x}(E\cos(2x) + F\sin(2x))$$

Etape 2 : EASM:
$y'' + 2y' + 5y = \cos(x) \quad (E)$

$$y_p = A\cos(x) + B\sin(x) \quad (1)$$
$$y_p' = -A\sin(x) + B\cos(x) \quad (2)$$
$$y_p'' = -A\cos(x) - B\sin(x) \quad (3)$$

(1), (2), (3) dans (E):

$$(-A\cos(x) - B\sin(x)) + 2(-A\sin(x) + B\cos(x)) + 5(A\cos(x) + B\sin(x)) = \cos(x)$$
$$(-A + 2B + 5A)\cos(x) + (-B - 2A + 5B)\sin(x) = \cos(x)$$
$$(4A + 2B)\cos(x) + (4B - 2A)\sin(x) = \cos(x)$$

Systeme:

$$4A + 2B = 1$$
$$-2A + 4B = 0$$

Resolution:

$$-2A + 4B = 0 \Rightarrow A = 2B$$
$$4(2B) + 2B = 1 \Rightarrow 8B + 2B = 1$$
$$10B = 1 \Rightarrow B = \frac{1}{10}$$
$$A = 2B = \frac{2}{10} = \frac{1}{5}$$

D'ou:

$$y = e^{-x}(E\cos(2x) + F\sin(2x)) + \frac{1}{5}\cos(x) + \frac{1}{10}\sin(x)$$
$$y(0) = e^{0}(E\cos(0) + F\sin(0)) + \frac{1}{5}\cos(0) + \frac{1}{10}\sin(0) = 0$$
$$E + \frac{1}{5} = 0 \Rightarrow E = -\frac{1}{5}$$

Derivee:

$$y'(0) = 0$$
$$y' = e^{-x}(E\cos(2x) + F\sin(2x)) - e^{-x}(-2E\sin(2x) + 2F\cos(2x)) - \frac{1}{5}\sin($$
$$y'(0) = e^{0}(E\cos(0) + F\sin(0)) - e^{0}(-2E\sin(0) + 2F\cos(0)) - \frac{1}{5}\sin(0) + \frac{1}{10}$$
$$-E + 2F + \frac{1}{10} = 10$$
$$-(-\frac{1}{5}) + 2F + \frac{1}{10} = 0 \implies \frac{1}{5} + 2F + \frac{1}{10} = 0$$
$$2F = -\frac{1}{5} - \frac{1}{10}$$
$$2F = -\frac{3}{10}$$
$$F = -\frac{3}{20}$$

$$\boxed{y = e^{-x}(-\frac{1}{5}\cos(2x) - \frac{3}{20}\sin(2x)) + \frac{1}{5}\cos(x) + \frac{1}{10}\sin(x)}$$

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4. (E): $y'' - 4y + 13y = cos(3x)$