

S4 - 17B. Devoir 2

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Exercice 2:

Resolito pm des equations differentielles non lineraires de 2nd ordre

1. (E): $y' = xy' + \sqrt{1 + (y')^2}$

2. (E): $x = yy' + (y')^3$

3. (E): $2xy' = y + y'$

Exercice 3:

Resolution des equations differentielles du second ordre:

1. (E): $y'' + y = xe^x + 2e^{-x}$ ou $y(0) = y'(0) = 1$

2. (E): $y'' - 4y' + y = xe^{2x}$

3. (E): $y'' + 2y' + 5y = \cos x$ ou $y(0) = y'(0) = 0$

Etape 1 : ESSM:

$$y'' + 2y' + 5y = 0$$

Equation caracteristique:

$$r^2 + 2r + 5 = 0$$

$$\Delta = 2^2 - 4 \times 1 \times 5 = 4 - 20 = -16$$

$$r = \frac{-2 \pm \sqrt{-16}}{2 \times 1} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

D'ou on a:

$$y = e^{-x}(E \cos(2x) + F \sin(2x))$$

Etape 2 : EASM:

$$y'' + 2y' + 5y = \cos(x) \quad (E)$$

$$y_p = A \cos(x) + B \sin(x) \quad (1)$$

$$y'_p = -A \sin(x) + B \cos(x) \quad (2)$$

$$y''_p = -A \cos(x) - B \sin(x) \quad (3)$$

(1), (2), (3) dans (E):

$$(-A \cos(x) - B \sin(x)) + 2(-A \sin(x) + B \cos(x)) + 5(A \cos(x) + B \sin(x)) = \cos(x)$$

$$(-A + 2B + 5A) \cos(x) + (-B - 2A + 5B) \sin(x) = \cos(x)$$

$$(4A + 2B) \cos(x) + (4B - 2A) \sin(x) = \cos(x)$$

Systeme:

$$4A + 2B = 1$$

$$-2A + 4B = 0$$

Resolution:

$$\begin{aligned}
-2A + 4B &= 0 \Rightarrow A = 2B \\
4(2B) + 2B &= 1 \Rightarrow 8B + 2B = 1 \\
10B &= 1 \Rightarrow B = \frac{1}{10} \\
A &= 2B = \frac{2}{10} = \frac{1}{5}
\end{aligned}$$

D'ou:

$$\begin{aligned}
y &= e^{-x}(E \cos(2x) + F \sin(2x)) + \frac{1}{5} \cos(x) + \frac{1}{10} \sin(x) \\
y(0) &= e^0(E \cos(0) + F \sin(0)) + \frac{1}{5} \cos(0) + \frac{1}{10} \sin(0) = 0 \\
E + \frac{1}{5} &= 0 \Rightarrow E = -\frac{1}{5}
\end{aligned}$$

Derivee:

$$\begin{aligned}
y'(0) &= 0 \\
y' &= e^{-x}(E \cos(2x) + F \sin(2x)) - e^{-x}(-2E \sin(2x) + 2F \cos(2x)) - \frac{1}{5} \sin(x) \\
y'(0) &= e^0(E \cos(0) + F \sin(0)) - e^0(-2E \sin(0) + 2F \cos(0)) - \frac{1}{5} \sin(0) + \frac{1}{10} \\
-E + 2F + \frac{1}{10} &= 10 \\
-(-\frac{1}{5}) + 2F + \frac{1}{10} &= 0 \Rightarrow \frac{1}{5} + 2F + \frac{1}{10} = 0 \\
2F &= -\frac{1}{5} - \frac{1}{10} \\
2F &= -\frac{3}{10} \\
F &= -\frac{3}{20}
\end{aligned}$$

$$y = e^{-x}\left(-\frac{1}{5} \cos(2x) - \frac{3}{20} \sin(2x)\right) + \frac{1}{5} \cos(x) + \frac{1}{10} \sin(x)$$

4. (E): $y'' - 4y + 13y = \cos(3x)$

Etape 1 : ESSM

$$y'' - 4y + 13y = 0$$

Equation caracteristique :

$$r^2 - 4r + 13 = 0$$

$$\Delta = (-4)^2 - 4 \times 1 \times 13 = 16 - 52$$

$$r = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

$$\Delta < 0, \text{ donc } y = e^{2x}(A \cos 3x + B \sin 3x)$$

Etape 2 : EASM

$$y'' - 4y' + 13y = \cos(3x)$$

$$f(x) = \cos(3x)$$

$$y_p = m \cos(3x) + n \sin(3x)$$

$$y'_p = -3m \sin(3x) + 3n \cos(3x)$$

$$y''_p = -9m \cos(3x) - 9n \sin(3x)$$

Substituons dans l'equation :

$$(-9m \cos(3x) - 9n \sin(3x)) - 4(-3m \sin(3x) + 3n \cos(3x)) + 13(m \cos(3x) + n \sin(3x)) = \cos(3x)$$

$$(-9m + 12n + 13m) \cos(3x) + (-9n - 12m + 13n) \sin(3x) = \cos(3x)$$

$$(4m + 12n) \cos(3x) + (4n - 12m) \sin(3x) = \cos(3x)$$

On obtient le systeme :

$$4m + 12n = 1$$

$$-12m + 4n = 0$$

Resolution :

$$\begin{aligned}
-12m + 4n &= 0 \\
4n &= 12m \\
n &= 3m
\end{aligned}$$

Substitution dans la premiere equation :

$$\begin{aligned}
4m + 12(3m) &= 1 \\
4m + 36m &= 1 \\
40m &= 1 \\
m &= \frac{1}{40}
\end{aligned}$$

Donc, on a solution :

$$y = e^{2x}(A \cos(3x) + B \sin(3x)) + \frac{1}{40} \cos(3x) + \frac{3}{40} \sin(3x)$$