S4 GROUPE n

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Exercice 1:

1. $y' = \sin(y)$

2. $x^2 y' = e^y$

$$
\begin{aligned}
x^2 y' &= e^y \\
x^2 \frac{dy}{dx} &= e^y \\
\frac{dy}{e^y} &= \frac{dx}{x^2} \\
\int \frac{1}{e^y}\, dy &= \int \frac{1}{x^2}\, dx \\
\int e^{-y}\, dy &= \int x^{-2}\, dx \\
-e^{-y} &= -\frac{1}{x} + C, \quad C \in \mathbb{R} \\
\ln(e^{-y}) &= \ln|\frac{1}{x} + C| \\
-y &= \ln|\frac{1}{x} + C| \\
y &= -\ln|\frac{1}{x} + C|
\end{aligned}
$$

$$\boxed{y = -\ln|\frac{1}{x} + C|}$$

3. $(x^2+1)y' + 3xy = x^2$
   Etape 1: ESSM

$$
\begin{aligned}
(x^2+1)y' + 3xy &= 0 \\
(x^2+1)y' &= -3xy \\
(x^2+1)\frac{dy}{dx} &= -3xy \\
(x^2-1)\frac{dy}{y} &= -3x\, dx \\
\int \frac{1}{y}\, dy &= \int -\frac{3x}{x^2+1}\, dx
\end{aligned}
$$

Posons

$$\begin{cases} u = x^2 + 1 \\ u' = 2x \end{cases}$$

$\int -\frac{3x}{u}\,dx = -3\int \frac{x}{u}\,dx = -3\int \frac{du}{2u} = \frac{-3}{2}\int \frac{du}{u}$

$\frac{du}{dx} = 2x \implies du = 2xdx \implies xdx = \frac{du}{2}$
$\frac{-3}{2}\int \frac{du}{u}$ = -$\frac{3}{/2}ln|u|$+$C, \quad C \in \mathbb{R}$

$ln|y| = -\frac{3}{2}ln|x^2+1|+C$
$y = ke^{-\frac{3}{2}ln|x^2+1|}$ avec $e^C = k$
$\boxed{y = ke^{-\frac{3}{2}ln|x^2+1|} \text{ avec } e^C = k}$

4. $y' + y = 2e^x + 4\sin(x)$

$$\begin{aligned} \frac{d\lambda}{dx}e^{-x} &= 2e^x + 4sin(x) \\ \int d\lambda &= \int (2e^{2x} + 4e^x \sin(x))\,dx \\ \int d\lambda &= \int 2e^{2x}\,dx + 4\int e^x \sin(x))\,dx \\ \int d\lambda &= e^{2x} + C + 4[-\cos(x)e^x + \int e^x \cos(x)\,dx] \end{aligned}$$

Soit $I = \int e^x \cos(x)$

$$\begin{cases} u = e^x \implies u' = e^x \\ v = -\cos(x) \implies v' = \sin(x) \end{cases}$$

$-\cos(x)e^x + \int e * x\cos(x)\,dx$

$$\begin{cases} u = e^x \implies v' = \cos(x) \\ u' = e^x \implies v' = \sin(x) \end{cases}$$

$$-e^x cos(x) + -e^x sin(x) - \int e^x \sin(x))\, dx$$

$$-e^x cos(x) + -e^x sin(x) - I$$

$$I = e^x(sin(x) - cos(x)) - I$$

$$2I = e^x(sin(x) - cos(x))$$

$$I = \frac{e^x}{2}(sin(x) - cos(x))$$

$$\int d\lambda = e^{2x} + c + 2e^x(sin(x) - cos(x)) + k$$

$$\lambda x = e^{2x} + e^{2x}(\sin(x) - \cos(x)) + C$$

$$\lambda x = e^x(e^x + 2(\sin(x) - \cos(x)) + C$$

$$y = e^x.e^{-x}(e^x + 2(\sin(x) - \cos(x)) + Ce^{-x}$$

$$y = e^x + 2\sin(x) - 2\cos(x) + Ce^{-x}$$

$$\boxed{y = e^x + 2\sin(x) - 2\cos(x) + Ce^{-x}}$$

5. $y' - 2y = 2x^3 + x \quad ou \quad y(3) = 1$
6. $y' + 2xy = e^{x-x^2}$
7. $y'\cos(2y) - \sin(y) = 0$
8. $y' - 2y = 2x^3 + x \quad \text{ou} \quad y(3) = 1$

Exercice 2:

1. $xy' = y + 3xy^2$
2. $y' + \frac{y}{x+1} = \frac{1}{2}(x+1)^3 y^3$
3. $y'(1 - \sin(s)\cos(x)) + y^2\cos(x) - y' + \sin(x) = 0$
   si $y = \cos(x)$ est une solution particuliere
4. $y = xy' + (y')^3$