

## S4 GROUPE n

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## Exercice 1:

1.  $y' = \sin(y)$

2.  $x^2 y' = e^y$

$$x^2 y' = e^y$$

$$x^2 \frac{dy}{dx} = e^y$$

$$\frac{dy}{e^y} = \frac{dx}{x^2}$$

$$\int \frac{1}{e^y} dy = \int \frac{1}{x^2} dx$$

$$\int e^{-y} dy = \int x^{-2} dx$$

$$-e^{-y} = -\frac{1}{x} + C, \quad C \in \mathbb{R}$$

$$\ln(e^{-y}) = \ln\left|\frac{1}{x} + C\right|$$

$$-y = \ln\left|\frac{1}{x} + C\right|$$

$$y = -\ln\left|\frac{1}{x} + C\right|$$

$y = -\ln\left|\frac{1}{x} + C\right|$

3.  $(x^2 + 1)y' + 3xy = x^2$

Etape 1: ESSM

$$(x^2 + 1)y' + 3xy = 0$$

$$(x^2 + 1)y' = -3xy$$

$$(x^2 + 1)\frac{dy}{dx} = -3xy$$

$$(x^2 - 1)\frac{dy}{y} = -3x dx$$

$$\int \frac{1}{y} dy = \int -\frac{3x}{x^2 + 1} dx$$

Posons

$$\begin{cases} u = x^2 + 1 \\ u' = 2x \end{cases}$$

$$\int -\frac{3x}{u} dx = -3 \int \frac{x}{u} dx = -3 \int \frac{du}{2u} = \frac{-3}{2} \int \frac{du}{u}$$

$$\begin{aligned} \frac{du}{dx} = 2x &\implies du = 2x dx \implies x dx = \frac{du}{2} \\ \frac{-3}{2} \int \frac{du}{u} &= -\frac{3}{2} \ln|u| + C, \quad C \in \mathbb{R} \end{aligned}$$

$$\begin{aligned} \ln|y| &= -\frac{3}{2} \ln|x^2 + 1| + C \\ y &= k e^{-\frac{3}{2} \ln|x^2 + 1|} \text{ avec } e^C = k \end{aligned}$$

$$\begin{aligned} 4. \quad y' + y &= 2e^x + 4 \sin(x) \\ y' + y &= 2e^x + 4 \sin(x) \end{aligned}$$

$$\begin{aligned} \frac{d\lambda}{dx} e^{-x} &= 2e^x + 4 \sin(x) \\ \int d\lambda &= \int (2e^{2x} + 4e^x \sin(x)) dx \\ \int d\lambda &= \int 2e^{2x} dx + 4 \int e^x \sin(x) dx \\ \int d\lambda &= e^{2x} + C + 4[-\cos(x)e^x + \int e^x \cos(x) dx] \end{aligned}$$

Soit  $I = \int e^x \cos(x)$

$$\begin{cases} u = e^x \implies u' = e^x \\ v = -\cos(x) \implies v' = \sin(x) \end{cases}$$

$$-\cos(x)e^x + \int e^x \cos(x) dx$$

$$\begin{cases} u = e^x \implies u' = e^x \\ v' = \cos(x) \implies v = \sin(x) \end{cases}$$

$$\begin{aligned}
& -e^x \cos(x) + -e^x \sin(x) - \int e^x \sin(x) dx \\
& -e^x \cos(x) + -e^x \sin(x) - I \\
& I = e^x (\sin(x) - \cos(x)) - I \\
& 2I = e^x (\sin(x) - \cos(x)) \\
& I = \frac{e^x}{2} (\sin(x) - \cos(x))
\end{aligned}$$

$$\begin{aligned}
\int d\lambda &= e^{2x} + c + 2e^x (\sin(x) - \cos(x)) + k \\
\lambda x &= e^{2x} + e^{2x} (\sin(x) - \cos(x)) + C \\
\lambda x &= e^x (e^x + 2(\sin(x) - \cos(x))) + C
\end{aligned}$$

$$\begin{aligned}
y &= e^x \cdot e^{-x} (e^x + 2(\sin(x) - \cos(x))) + C e^{-x} \\
y &= e^x + 2 \sin(x) - 2 \cos(x) + C e^{-x}
\end{aligned}$$

$$\boxed{y = e^x + 2 \sin(x) - 2 \cos(x) + C e^{-x}}$$

5.  $y' - 2y = 2x^3 + x$  ou  $y(3) = 1$
6.  $y' + 2xy = e^{x-x^2}$
7.  $y' \cos(2y) - \sin(y) = 0$
8.  $y' - 2y = 2x^3 + x$  ou  $y(3) = 1$

## Exercice 2:

1.  $xy' = y + 3xy^2$
2.  $y' + \frac{y}{x+1} = \frac{1}{2}(x+1)^3 y^3$
3.  $y'(1 - \sin(s) \cos(x)) + y^2 \cos(x) - y' + \sin(x) = 0$   
si  $y = \cos(x)$  est une solution particuliere
4.  $y = xy' + (y')^3$