

S4 GROUPE n

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Exercice 1:

1. $y' = \sin(y)$

2. $x^2 y' = e^y$

$$x^2 y' = e^y$$

$$x^2 \frac{dy}{dx} = e^y$$

$$\frac{dy}{e^y} = \frac{dx}{x^2}$$

$$\int \frac{1}{e^y} dy = \int \frac{1}{x^2} dx$$

$$\int e^{-y} dy = \int x^{-2} dx$$

$$-e^{-y} = -\frac{1}{x} + C, \quad C \in \mathbb{R}$$

$$\ln(e^{-y}) = \ln\left|\frac{1}{x} + C\right|$$

$$-y = \ln\left|\frac{1}{x} + C\right|$$

$$y = -\ln\left|\frac{1}{x} + C\right|$$

$y = -\ln\left|\frac{1}{x} + C\right|$

3. $(x^2 + 1)y' + 3xy = x^2$

Etape 1: ESSM

$$(x^2 + 1)y' + 3xy = 0$$

$$(x^2 + 1)y' = -3xy$$

$$(x^2 + 1)\frac{dy}{dx} = -3xy$$

$$(x^2 - 1)\frac{dy}{y} = -3x dx$$

$$\int \frac{1}{y} dy = \int -\frac{3x}{x^2 + 1} dx$$

Posons

$$\begin{cases} u = x^2 + 1 \\ u' = 2x \end{cases}$$

$$\int -\frac{3x}{u} dx = -3 \int \frac{x}{u} dx = -3 \int \frac{du}{2u} = -\frac{3}{2} \int \frac{du}{u}$$

$$\frac{du}{dx} = 2x \implies du = 2x dx \implies x dx = \frac{du}{2}$$

$$\frac{-3}{2} \int \frac{du}{u} = -\frac{3}{2} \ln|u| + C, \quad C \in \mathbb{R}$$

$$\ln|y| = -\frac{3}{2} \ln|x^2 + 1| + C$$

$$y = k e^{-\frac{3}{2} \ln|x^2 + 1|} \text{ avec } e^C = k$$

4. $y' + y = 2e^x + 4 \sin(x)$

$$\frac{d\lambda}{dx} e^{-x} = 2e^x + 4 \sin(x)$$

$$\int d\lambda = \int (2e^{2x} + 4e^x \sin(x)) dx$$

$$\int d\lambda = \int 2e^{2x} dx + 4 \int e^x \sin(x) dx$$

$$\int d\lambda = e^{2x} + C + 4[-\cos(x)e^x + \int e^x \cos(x) dx]$$

Soit $I = \int e^x \cos(x)$

$$\begin{cases} u = e^x \implies u' = e^x \\ v = -\cos(x) \implies v' = \sin(x) \end{cases}$$

$$-\cos(x)e^x + \int e^x \cos(x) dx$$

$$\begin{cases} u = e^x \implies v' = \cos(x) \\ u' = e^x \implies v = \sin(x) \end{cases}$$

$$-e^x \cos(x) + -e^x \sin(x) - \int e^x \sin(x) dx$$

$$-e^x \cos(x) + -e^x \sin(x) - I$$

$$I = e^x (\sin(x) - \cos(x)) - I$$

$$2I = e^x (\sin(x) - \cos(x))$$

$$I = \frac{e^x}{2} (\sin(x) - \cos(x))$$

$$\int d\lambda = e^{2x} + c + 2e^x(\sin(x) - \cos(x)) + k$$

$$\lambda x = e^{2x} + e^{2x}(\sin(x) - \cos(x)) + C$$

$$\lambda x = e^x(e^x + 2(\sin(x) - \cos(x))) + C$$

$$y = e^x \cdot e^{-x}(e^x + 2(\sin(x) - \cos(x))) + Ce^{-x}$$

$$y = e^x + 2\sin(x) - 2\cos(x) + Ce^{-x}$$

$$\boxed{y = e^x + 2\sin(x) - 2\cos(x) + Ce^{-x}}$$

$$5. \quad y' - 2y = 2x^3 + x \quad \text{ou} \quad y(3) = 1$$

$$6. \quad y' + 2xy = e^{x-x^2}$$

$$7. \quad y' \cos(2y) - \sin(y) = 0$$

$$8. \quad y' - 2y = 2x^3 + x \quad \text{ou} \quad y(3) = 1$$

Exercise 2:

$$1. \quad xy' = y + 3xy^2$$

$$2. \quad y' + \frac{y}{x+1} = \frac{1}{2}(x+1)^3y^3$$

$$3. \quad y'(1 - \sin(s) \cos(x)) + y^2 \cos(x) - y' + \sin(x) = 0$$

Si $y = \cos(x)$ est une solution particuliere

$$4. \quad y = xy' + (y')^3$$