S4 - 17B. Devoir 2

ETU003235: ANDERSON Soamiavaka Vanille
ETU003247: ANDRIANAJA Onja Fanilo
ETU003286: RABETOKOTANY Yvan Noaaaaaaaaaaaaaaaah
ETU003298: RAJAONARIVONY Tandrifiniaina Dylan
ETU003305: RAKOTOARIVONY Loïc Dylan
ETU003331: RANAIVOSON Miora Randie
ETU003335: RANDRIAMAHEFA Liantsoa Alicia
ETU003348: RANDRIANIRINA Niriela Andraina
ETU003363: RATSITO Oelirivo Mitia
ETU003378: RAZAKANDISA Sariaka Niaina

Exercice 2:

Resolution des equations differentielles non lineraires de 2nd ordre

1. (E): $y' = xy' + \sqrt(1 + (y')^2)$

---

2. (E): $x = yy' + (y')^3$

---

3. (E): $2xy' = y + y'$

Exercice 3:

Resolution des equations differentielles du second ordre:

1. (E): $y'' + y = xe^x + 2e^{-x}$ ou $y(0) = y'(0) = 1$

---

2. (E): $y'' - 4y' + y = xe^{2x}$

---

3. (E): $y'' + 2y' + 5y = cosx$ ou $y(0) = y'(0) = 0$
Etape 1 : ESSM:

$$y'' + 2y^t + 5y = 0$$

Equation caracteristique:

$$r^2 + 2r + 5 = 0$$
$$\Delta = 2^2 - 4 \times 1 \times 5 = 4 - 20 = -16$$
$$r = \frac{-2 \pm \sqrt{-16}}{2 \times 1} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

D'ou on a:

$$y = e^{-x}(E\cos(2x) + F\sin(2x))$$

Etape 2 : EASM:
$y'' + 2y' + 5y = \cos(x) \quad (E)$

$$\begin{aligned} y_p &= A\cos(x) + B\sin(x) \quad (1) \\ y_p' &= -A\sin(x) + B\cos(x) \quad (2) \\ y_p'' &= -A\cos(x) - B\sin(x) \quad (3) \end{aligned}$$

(1), (2), (3) dans (E):

$$\begin{aligned} (-A\cos(x) - B\sin(x)) + 2(-A\sin(x) + B\cos(x)) + 5(A\cos(x) + B\sin(x)) &= \cos(x) \\ (-A + 2B + 5A)\cos(x) + (-B - 2A + 5B)\sin(x) &= \cos(x) \\ (4A + 2B)\cos(x) + (4B - 2A)\sin(x) &= \cos(x) \end{aligned}$$

Systeme:

$$\begin{aligned} 4A + 2B &= 1 \\ -2A + 4B &= 0 \end{aligned}$$

Resolution:

$$
\begin{aligned}
-2A + 4B = 0 \Rightarrow A = 2B \\
4(2B) + 2B = 1 \Rightarrow 8B + 2B = 1 \\
10B = 1 \Rightarrow B = \frac{1}{10} \\
A = 2B = \frac{2}{10} = \frac{1}{5}
\end{aligned}
$$

D'ou:

$$
\begin{aligned}
y &= e^{-x}(E\cos(2x) + F\sin(2x)) + \frac{1}{5}\cos(x) + \frac{1}{10}\sin(x) \\
y(0) &= e^{0}(E\cos(0) + F\sin(0)) + \frac{1}{5}\cos(0) + \frac{1}{10}\sin(0) = 0 \\
E + \frac{1}{5} &= 0 \Rightarrow E = -\frac{1}{5}
\end{aligned}
$$

Derivee:

$$
\begin{aligned}
y'(0) &= 0 \\
y' &= e^{-x}(E\cos(2x) + F\sin(2x)) - e^{-x}(-2E\sin(2x) + 2F\cos(2x)) - \frac{1}{5}\sin(\\
y'(0) &= e^{0}(E\cos(0) + F\sin(0)) - e^{0}(-2E\sin(0) + 2F\cos(0)) - \frac{1}{5}\sin(0) + \frac{1}{10} \\
-E + 2F + \frac{1}{10} &= 10 \\
-(-\frac{1}{5}) + 2F + \frac{1}{10} &= 0 \implies \frac{1}{5} + 2F + \frac{1}{10} = 0 \\
2F &= -\frac{1}{5} - \frac{1}{10} \\
2F &= -\frac{3}{10} \\
F &= -\frac{3}{20}
\end{aligned}
$$

$$
\boxed{y = e^{-x}(-\frac{1}{5}\cos(2x) - \frac{3}{20}\sin(2x)) + \frac{1}{5}\cos(x) + \frac{1}{10}\sin(x)}
$$

---

4. (E): $y'' - 4y + 13y = cos(3x)$

Etape 1 : ESSM
$y'' - 4y + 13y = 0$

Equation caracteristique :

$$\begin{aligned} r^2 - 4r + 13 = 0 \\ \Delta = (-4)^2 - 4 \times 1 \times 13 = 16 - 52 \\ r = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i \end{aligned}$$

$\Delta < 0$, donc $y = e^{2x}(A\cos 3x + B\sin 3x)$

Etape 2 : EASM

$$\begin{aligned} y'' - 4y' + 13y &= \cos(3x) \\ f(x) &= \cos(3x) \\ y_p &= m\cos(3x) + n\sin(3x) \\ y_p' &= -3m\sin(3x) + 3n\cos(3x) \\ y_p'' &= -9m\cos(3x) - 9n\sin(3x) \end{aligned}$$

Substituons dans l'equation :

$$\begin{aligned} (-9m\cos(3x) - 9n\sin(3x)) - 4(-3m\sin(3x) + 3n\cos(3x)) + 13(m\cos(3x) + n\sin(3x)) &= \mathrm{c} \\ (-9m + 12n + 13m)\cos(3x) + (-9n - 12m + 13n)\sin(3x) &= \mathrm{c} \\ (4m + 12n)\cos(3x) + (4n - 12m)\sin(3x) &= \mathrm{c} \end{aligned}$$

On obtient le systeme :

$$\begin{aligned} 4m + 12n = 1 \\ -12m + 4n = 0 \end{aligned}$$

Resolution :

$$\begin{aligned}
-12m + 4n &= 0 \\
4n &= 12m \\
n &= 3m
\end{aligned}$$

Substitution dans la premiere equation :

$$\begin{aligned}
4m + 12(3m) &= 1 \\
4m + 36m &= 1 \\
40m &= 1 \\
m &= \frac{1}{40}
\end{aligned}$$

Donc, on a solution :

$$\boxed{y = e^{2x}(A\cos(3x) + B\sin(3x)) + \frac{1}{40}\cos(3x) + \frac{3}{40}\sin(3x)}$$