S4 GROUPE n

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Exercice 1:

1. $y' = \sin(y)$

2. $x^2 y' = e^y$

$$\begin{aligned}
x^2 y' &= e^y \\
x^2 \frac{dy}{dx} &= e^y \\
\frac{dy}{e^y} &= \frac{dx}{x^2} \\
\int \frac{1}{e^y}\, dy &= \int \frac{1}{x^2}\, dx \\
\int e^{-y}\, dy &= \int x^{-2}\, dx \\
-e^{-y} &= -\frac{1}{x} + C, \quad C \in \mathbb{R} \\
\ln(e^{-y}) &= \ln|\frac{1}{x} + C| \\
-y &= \ln|\frac{1}{x} + C| \\
y &= -\ln|\frac{1}{x} + C|
\end{aligned}$$

$$\boxed{y = -\ln|\frac{1}{x} + C|}$$

3. $(x^2+1)y' + 3xy = x^2$
   Etape 1: ESSM

$$
\begin{aligned}
(x^2+1)y'+3xy &= 0\\
(x^2+1)y' &= -3xy\\
y'y^{-1} &= \frac{-3x}{x^2+1}\\
\frac{dy}{y} &= \frac{-3x}{x^2+1}dx\\
\int \frac{1}{y}\,dy &= \int -\frac{3x}{x^2+1}\,dx\\
ln|y| &= -\frac{3}{2}ln|x^2+1|+C, \quad C\in\mathbb{R}\\
y &= \lambda(x^2+1)^{-3/2}, \quad \lambda=e^C \quad (1)
\end{aligned}
$$

Etape 2: EASM MVC

$$
\begin{aligned}
y' &= \lambda'(x^2+1)^{-\frac{3}{2}}+(-\frac{3}{2})\lambda(2x)(x^2+1)^{-\frac{3}{2}-1}\\
y' &= \lambda'(x^2+1)^{-\frac{3}{2}}-\frac{3}{2}\lambda(2x)(x^2+1)^{-\frac{5}{2}} \quad (2)
\end{aligned}
$$

(1) et (2) dans (E) ($(x^2+1)y'+3xy=x^2$):

$$
\begin{aligned}
(x^2+1)(\lambda'(x^2+1)^{-\frac{3}{2}}-\frac{3}{2}\lambda(2x)(x^2+1)^{-\frac{5}{2}})+3x(\lambda(x^2+1)^{-3/2} &= x^2\\
\lambda'(x^2+1)^{\frac{1}{2}} &= x^2\\
\lambda' &= x^2(x^2+1)^{\frac{1}{2}}\\
d\lambda &= x^2(x^2+1)^{\frac{1}{2}}dx\\
\int d\lambda &= \int x^2(x^2+1)^{\frac{1}{2}}dx
\end{aligned}
$$

$$
\begin{aligned}
x^2 &= \sinh^2(t) \\
x &= \sinh(t) \\
dx &= \cosh(t)dt \\
x^2 + 1 &= \sinh^2(t) + 1 = \cosh^2(t) \\
\sqrt{x^2+1} &= \sqrt{\cosh^2(t)} = \cosh(t), \quad \cosh(t) > 0 \\
\int x^2\sqrt{x^2+1}\,dx &= \int \sinh^2(t)\cosh(t)\cosh(t)\,dt \\
\int x^2\sqrt{x^2+1}\,dx &= \int \sinh^2(t)\cosh^2(t)\,dt
\end{aligned}
$$

$$\sinh^2(t) = \frac{\cosh(2t)-1}{2} \quad , \ \cosh^2(t) = \frac{\cosh(2t)+1}{2}$$

$$\sinh^2(t)\cosh^2(t) = (\frac{\cosh(2t)-1}{2})(\frac{\cosh(2t)+1}{2})$$

$$\sinh^2(t)\cosh^2(t) = \frac{1}{4}(\cosh(2t)-1)(\cosh(2t)+1)$$

$$\sinh^2(t)\cosh^2(t) = \frac{1}{4}(\cosh^2(2t)-1)$$

$$\sinh^2(t)\cosh^2(t) = \frac{1}{4}\sinh^2(2t)$$

Donc

$$\int \sinh^2(t)\cosh^2(t)\,dt = \int \frac{1}{4}\sinh^2(2t)\,dt$$

$$\int \sinh^2(t)\cosh^2(t)\,dt = \frac{1}{4}\int \sinh^2(2t)\,dt$$

$$\text{or } \ \sinh^2(2t)\,dt = \int \frac{\cosh(4t)-1}{2}\,dt$$

$$\text{car } \ \sinh^2(t) = \frac{\cosh(2t)-1}{2}$$

$$\frac{1}{2}\int \cosh(4t) - \frac{1}{2}\int 1dt$$

$$\cosh(4t)dt = \frac{1}{4}\sinh(4t)$$

$$\int \sinh^2(t)dt = \frac{1}{2}.\frac{1}{4}\sinh(4t) - \frac{1}{2}t$$

$$\int \sinh^2(t)dt = \frac{1}{8}\sinh(4t) - \frac{1}{2}t + C, \quad C \int \mathbb{R}$$

$$\int \sinh^2(t)dt = \frac{1}{4}(\frac{1}{8}\sinh(4t) - \frac{1}{2}t + C)$$

$$\int \sinh^2(t)dt = \frac{1}{32}\sinh(4t) - \frac{1}{8}t + C$$

$x = \sinh(t), t = arcsinh(x) = \ln(x + \sqrt{x^2+1})$

$$\sinh(4t) = 2\sin(2t)\cosh(2t)$$

$$\sinh(2t) = 2\sinh(t)\cosh(t) = 2x\sqrt{1+x^2}$$

$$\begin{aligned}
\cosh(2t) &= \sqrt{1+\sinh^2(2t)} = \sqrt{1+(2x\sqrt{x^2+1})^2} \\
\cosh(2t) &= \sqrt{1+4x^2(x^2+1)} = \sqrt{1+4x^4+4x^2} \\
\cosh(2t) &= \sqrt{(2x^2+1)^2} = 2x^2+1, \quad x^2 >= 0
\end{aligned}$$

$$\begin{aligned}
\sinh(4t) &= 2(2x\sqrt{1+x^2})(2x^2+1) \\
\sinh(4t) &= 4x(2x^2+1)\sqrt{1+x^2}
\end{aligned}$$

$$\begin{aligned}
\frac{1}{32}\sinh(4t) - \frac{1}{8}t &= \frac{1}{32}4x(2x^2+1)\sqrt{1+x^2} - \frac{1}{8}t + C \\
\int x^2\sqrt{x^2+1}dx &= \frac{1}{8}x(2x^2+1)\sqrt{x^2+1} - \frac{1}{8}ln|x+\sqrt{x^2+1}|+C
\end{aligned}$$

$$\begin{aligned}
\lambda &= \frac{1}{8}x(2x^2+1)\sqrt{x^2-1} - \frac{1}{8}\ln|x+\sqrt{x^2+1}|+C \\
y &= (\frac{1}{8}x(2x^2+1)\sqrt{x^2-1} - \frac{1}{8}\ln|x+\sqrt{x^2+1}|+C)(x^2+1)^{-\frac{3}{2}}
\end{aligned}$$

$$\boxed{y = (\frac{1}{8}x(2x^2+1)\sqrt{x^2-1} - \frac{1}{8}\ln|x+\sqrt{x^2+1}|+C)(x^2+1)^{-\frac{3}{2}}}$$

4. $y' + y = 2e^x + 4\sin(x)$
   <u>Etape 1: ESSM</u>

$$\begin{aligned}
y' + y &= 0 \\
y' &= -y \\
\frac{dy}{y} &= -1 \\
\int \frac{1}{y}dy &= -\int 1dx \\
ln|y| &= -x + C, \quad C \in \mathbb{R} \\
y &= -e^{-x}.e^C \\
y &= \lambda e^{-x} \quad (1)
\end{aligned}$$

Etape 2: EASM MVC

$$y' = -\lambda e^{-x} + \lambda' e^{-x}$$
$$y' = e^{-x}(\lambda + \lambda')(2)$$

(1) et (2) dans (E):

$$\begin{aligned}
(-\lambda + \lambda)e^{-x} + \lambda e^{-x} &= 2e^{x} + 4\sin(x) \\
\lambda' &= 2e^{x} + 4sin(x) \\
\frac{d\lambda}{dx}e^{-x} &= 2e^{x} + 4sin(x) \\
\int d\lambda &= \int (2e^{2x} + 4e^{x}\sin(x))\,dx \\
\int d\lambda &= \int 2e^{2x}\,dx + 4\int e^{x}\sin(x))\,dx \\
\int d\lambda &= e^{2x} + C + 4[-\cos(x)e^{x} + \int e^{x}\cos(x)\,dx]
\end{aligned}$$

Soit $I = \int e^{x}\cos(x)$

$$\begin{cases} u = e^{x} \implies u' = e^{x} \\ v = -\cos(x) \implies v' = \sin(x) \end{cases}$$

$-\cos(x)e^{x} + \int e * x\cos(x)\,dx$

$$\begin{cases} u = e^{x} \implies v' = \cos(x) \\ u' = e^{x} \implies v' = \sin(x) \end{cases}$$

$$-e^{x}cos(x) + -e^{x}sin(x) - \int e^{x}\sin(x))\,dx$$
$$-e^{x}cos(x) + -e^{x}sin(x) - I$$
$$\begin{aligned}
I &= e^{x}(sin(x) - cos(x)) - I \\
2I &= e^{x}(sin(x) - cos(x)) \\
I &= \frac{e^{x}}{2}(sin(x) - cos(x))
\end{aligned}$$

$$\int d\lambda = e^{2x} + c + 2e^{x}(sin(x) - cos(x)) + k$$
$$\lambda x = e^{2x} + e^{2x}(\sin(x) - \cos(x)) + C$$
$$\lambda x = e^{x}(e^{x} + 2(\sin(x) - \cos(x)) + C$$

$$y = e^{x}.e^{-x}(e^{x} + 2(\sin(x) - \cos(x)) + Ce^{-x}$$
$$y = e^{x} + 2\sin(x) - 2\cos(x) + Ce^{-x}$$

$$\boxed{y = e^{x} + 2\sin(x) - 2\cos(x) + Ce^{-x}}$$

5. $y' - 2y = 2x^3 + x$ ou $y(3) = 1$

6. $y' + 2xy = e^{x-x^2}$

7. $y'\cos(2y) - \sin(y) = 0$

8. $y' - 2y = 2x^3 + x \quad \text{ou} \quad y(3) = 1$

Exercice 2:

1. $xy' = y + 3xy^2$

2. $y' + \frac{y}{x+1} = \frac{1}{2}(x+1)^3 y^3$

ETAPE 1: ESSM

$$\begin{aligned} y' + \frac{y}{x+1} &= 0 \\ \frac{dy}{dx} &= -\frac{y}{x+1} \\ \frac{dy}{y} &= -\frac{dx}{x+1} \\ \int \frac{dy}{y} &= -\int \frac{dx}{x+1} \\ \ln|y| &= -\ln|x+1| + C, \quad C \int \mathbb{R} \\ y &= e^{-\ln|x+1|}.e^{C} \\ y &= \lambda e^{-\ln|x+1|}, \quad \lambda = e^{C} \\ y &= \frac{\lambda}{e^{\ln|x+1|}} \\ y &= \frac{\lambda}{x+1} \end{aligned}$$

$y = y_0 = \frac{\lambda}{x+1}(1)$ solution homogene de (E)

ETAPE 2: EASM (MVC)

$$\begin{aligned} y_0' &= (\frac{\lambda}{x+1})' \\ y_0' &= \frac{\lambda'(x+1) - 1(\lambda)}{(x+1)^2} \quad (2) \end{aligned}$$

(1) et (2) dans (E):

$$\begin{aligned}
\frac{\lambda'(x+1)-\lambda}{(x+1)^2} + \frac{\frac{\lambda}{x+1}}{x+1} &= \frac{1}{2}(x+1)^3(\frac{\lambda}{x+1})^3 \\
\frac{\lambda'(x+1)-\lambda}{(x+1)^2} + \frac{\lambda}{(x+1)^2} &= \frac{1}{2}(x+1)^3\frac{\lambda^3}{(x+1)^3} \\
\frac{\lambda'(x+1)-\lambda+\lambda}{(x+1)^2} &= \frac{1}{2}\lambda^3 \\
\lambda'(x+1) &= \frac{1}{2}\lambda^3(x+1)^2 \\
\lambda' &= \frac{1}{2}\lambda^3(x+1) \\
\frac{d\lambda}{dx} &= \frac{1}{2}\lambda^3(x+1) \\
\frac{d\lambda}{\lambda^3} &= \frac{1}{2}(x+1)dx \\
\int \frac{d\lambda}{\lambda^3} &= \frac{1}{2}\int (x+1)dx \\
\int \lambda^{-3}d\lambda &= \frac{1}{2}(\int xdx + \int dx) \\
\frac{\lambda^{-3+1}}{-3+1} &= \frac{1}{2}(\frac{x^2}{2}+x)+K, \quad K \in \mathbb{R} \\
-\lambda^{-2} &= \frac{x^2+2x+4K}{2} \\
\lambda^2 &= -\frac{2}{x^2+2x+L}, \quad L = 4K \\
\lambda &= \pm\sqrt{-\frac{2}{x^2+2x+L}}
\end{aligned}$$

Alors $\boxed{y = \pm\sqrt{-\frac{2}{x^2+2x+L}} \times \frac{1}{x+1}}$

3. $y'(1-\sin(s)\cos(x)) + y^2\cos(x) - y' + \sin(x) = 0$
   si $y = \cos(x)$ est une solution particuliere

4. $y = xy' + (y')^3$