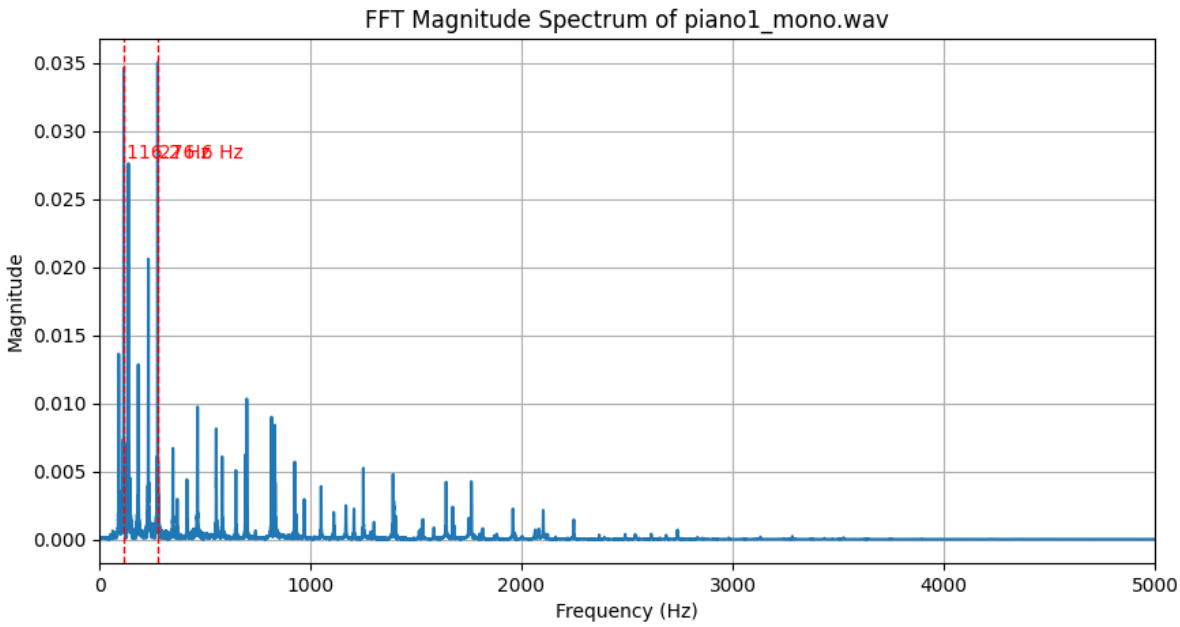
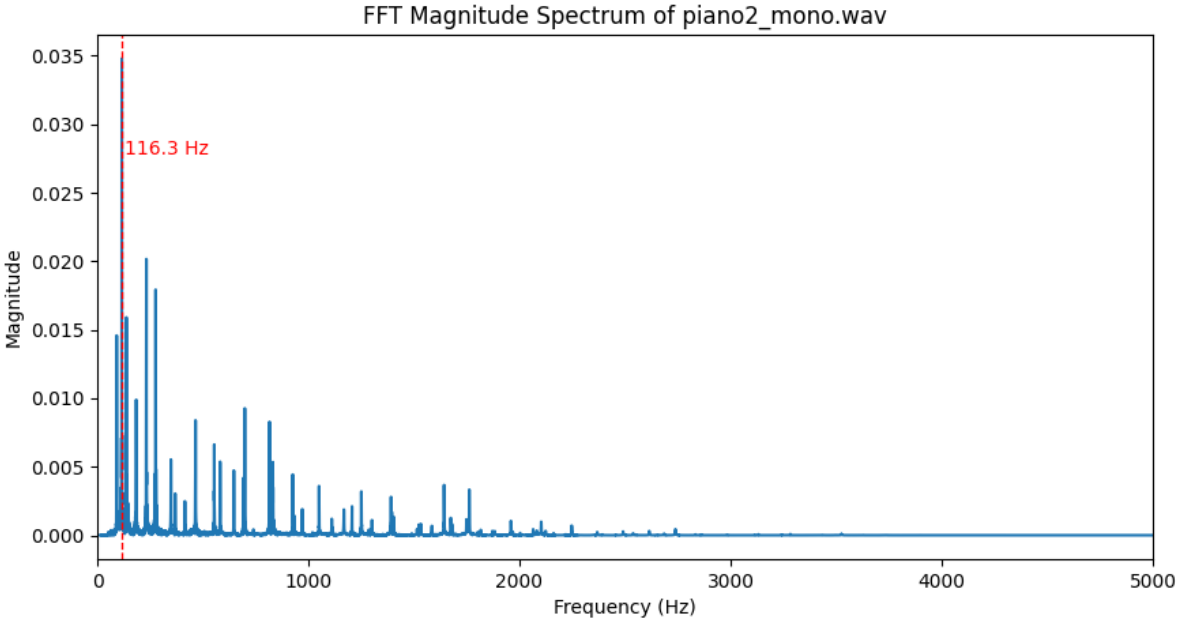
Pitch Detection - Signals and Systems

File Name	Fundamental Frequency (Hz)	Simultaneous / Sequential	Notes
piano1_mono.wav	116.23, 276.598	Sequential	Piano notes occur at different times, causing multiple sequential fundamental frequencies.
piano2_mono.wav	116.379	Neither?	Sequential piano notes are occuring at different times, but the FFT only shows one fundamental frequency for some reason.
trumpet.csv	981.107	Neither	A single fundamental frequency occurs due to a single trumpet emitting the sound signal.
twotrumpetsAB.csv	872.094, 981.107	Simultaneous	Simultaneous multiple fundamental frequencies occur due to two different trumpets emitting a sound signal

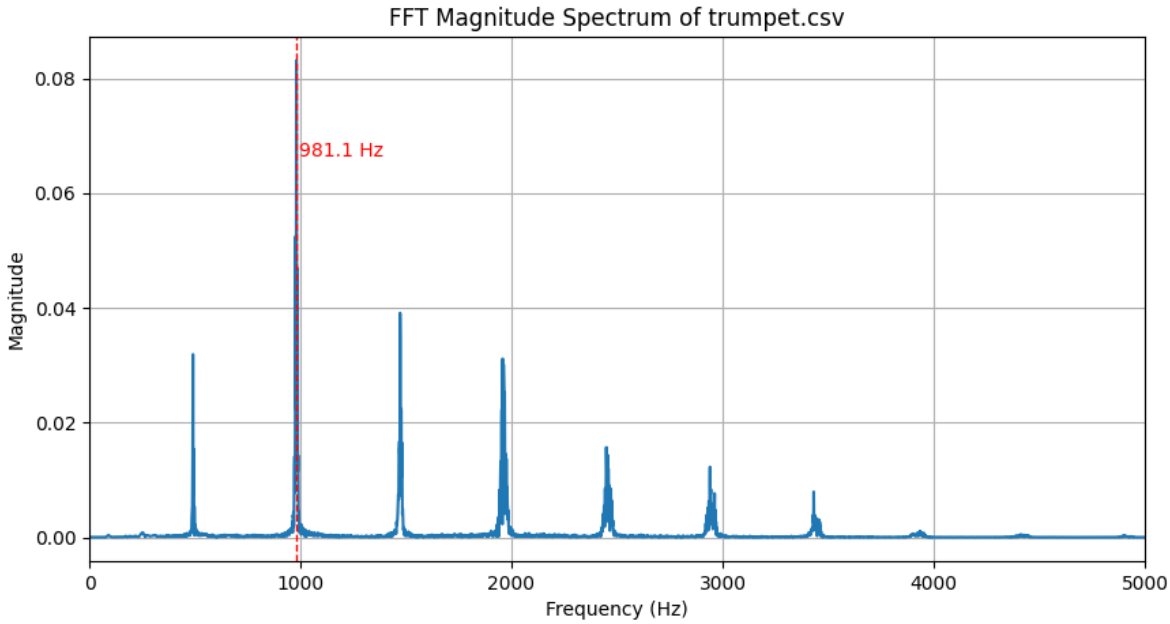
Piano1 Harmonics



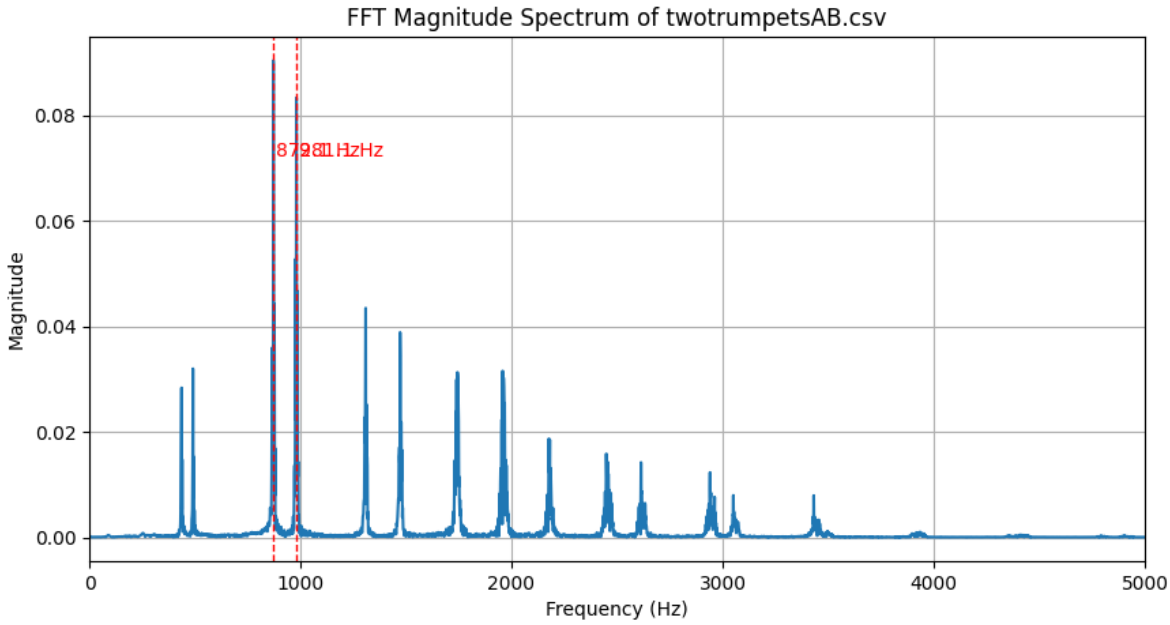
Piano2 Harmonics



Trumpet Harmonics



TwoTrumpets Harmonics



Techniques Used to Analyze the Data

This section describes the techniques I used to analyze the sound signals and how I used the numbered equations described below.

(1): The first step was to extract the data from the four sound signal files, two .csv files and two .wav files, into a **discrete-time signal**, formatted for Python signal analysis. I used numpy's `loadtxt()` function for the .csv files and SciPy's `wavfile.read()` function for the .wav files.

(2): The **discrete Fourier transform** was then performed on all four signals using SciPy's `fft()` algorithm.

(5 & 6): To normalize the signal data that was extracted from the .wav files, I used numpy's `abs()` function to get the **magnitude of the complex signals**, then divided the magnitude of the signals by `N` to **normalize the magnitudes**.

(3): The **FFT bins are then mapped to physical frequencies** for all four FFT signals using SciPy's `fftfreq()` function. The function uses **Equation (3)** to internally compute the results.

(7): Because these are real signals, only the real-side of the data matters and the second half of the data is redundant. Using the **Niquist limit**, we get only the first half of each FFT signal. This is shown in our code between lines 49 and 57.

Equations Used During Analysis

$$ x[n], \quad n = 0, 1, 2, \dots, N-1 $$

Equation (1): Discrete-time signal obtained by sampling the continuous-time audio waveform.

$$ X[k] = \sum_{n=0}^{N-1} x[n] e^{-j 2\pi kn / N} $$

Equation (2): Definition of the discrete Fourier transform (DFT).

$$ f_k = \frac{k F_s}{N} $$

Equation (3): Mapping FFT bin index to physical frequency.

$$ T_s = \frac{1}{F_s} $$

Equation (4): Sampling interval in seconds per sample.

$$ |X[k]| = \sqrt{\Re{X[k]}^2 + \Im{X[k]}^2} $$

Equation (5): Magnitude of the complex FFT output.

$$ |X_{\text{norm}}[k]| = \frac{|X[k]|}{N} $$

Equation (6): Normalized FFT magnitude.

$$ 0 \le f \le \frac{F_s}{2} $$

Equation (7): Frequency range of the single-sided spectrum (Nyquist limit).