IST 772 Midterm Exam - Summer 2021

Instructions: Compose brief answers to each of the following questions, typing your response in *italics* below each question. Please also include a .R file with IF RELEVANT.

1. Describe the conceptual connection between μ ("mu," the population mean) , {$\bar{?}$}("x-bar," a sample mean) and the sampling distribution. How are they connected to each other?

   When a sample of values is taken from a population, the mean of these values represents a sample mean. A sample mean may not be the same as a population mean because a sample mean contains sampling error. As this process is repeated, and many sample means are collected, this forms a sampling distribution. The sampling distribution is interpreted to make reasonable inferences about the population mean.

2. Your boss at the New York Times asks you to conduct an A/B test on two different headlines about the same story. Each headline is displayed on n=140 high traffic social media pages:

   Headline 1 gets an average of 2400 clicks per hour.
   Headline 2 gets an average of 2200 clicks per hour.

The 95% confidence interval is as follows:

100 < (mean difference between Headline 1 and 2) < 300.

Answer the following questions about that confidence interval:

a. On the basis of this confidence interval conduct a hypothesis test at the 0.05 level under the alternative hypothesis that average clicks per hour are not equal. State the null and alternative hypotheses.

H_0: the mean difference in clicks per hour between headline 1 and headline 2 is equal to zero.

H_A: the mean difference in clicks per hour between headline 1 and headline 2 is not equal to zero.

b. Would you reject or not reject the null hypothesis. Why or why not?

It is reasonable to reject the null hypothesis because zero is not in the 95% confidence interval.

c. Based on your answer in b. What is your conclusion about the difference between the headlines? Is headline 1 or 2 better and why?

I am reasonably confident that headline 1 is better than headline 2 because the confidence interval does not contain zero. Headline 1 has between 100 and 300 more clicks per hour on average and that difference is statistically significant.

d. Your friend calculates the p-value for the hypothesis mentioned in questions a-c and finds that p = 0.25. Does this sound plausible to you? Why or why not?

I am surprised by a p value of 0.25 because a sample size of n = 140 is not a small sample size. I would have expected that to be sufficient to produce statistically significant results.

e. Your boss tells you to run the same experiment 999 more times, calculating a new confidence interval each time. Now you have a collection of 1000 confidence intervals, each of which was constructed in the same way, but from new data samples: What can you say about this collection of confidence intervals?

950 out of the 1000 confidence intervals contain the population mean difference in clicks per hour.

f. Which command in R would you use to produce the confidence interval for each of the 1000 that you constructed?

t.test()

3. Tests for detecting diseases such as HIV are not 100% accurate and one can use Bayes' theorem to assess the probability that someone is actually infected HIV given a positive test. Please use the following facts to calculate the probability that someone has HIV after having received a positive test:

- For someone with HIV, the probability of a positive test is 99%
- The probability that someone has HIV is 3%.
- The probability of getting a positive test is 4%.

*P(Has HIV | Positive Test) = (P(Positive Test | Has HIV) * P (Has HIV)) / P(Positive Test)*
*P(Has HIV | Positive Test) = (0.99 * 0.03) / 0.04*
P(Has HIV | Positive Test) = 0.74

4. : The Null Hypothesis Significance test (NHST) is the classic inferential test used throughout the 20th century. The NHST comprises a set of logical steps that lead to a consideration of the viability of a stated null hypothesis. Following the material presented on page 77 of *Reasoning with Data*, here is an unordered list of the steps:

Calculate the test statistic
Assert a null hypothesis
Collect data
Find the p-value associate with the test statistic
Choose an alpha level
Reject the null hypothesis
Fail to reject the null hypothesis
Evaluate the p-value with respect to alpha

Place these steps in the correct order and add a brief one or two sentence explanation that describes the purpose and importance of each step.

1. Assert a null hypothesis

The null hypothesis is always stated first. It states that the true difference in means is equal to zero. Sufficient evidence against the null hypothesis must be collected in order to reject it.

2. Choose an alpha level

At this point the alpha is selected which will be used to determine whether or not the results are statistically significant. An alpha of 0.05 is commonly chosen but sometimes a more stringent or less stringent alpha may be used.

3. Collect data

Now that the hypothesis has been stated and the alpha level has been selected, the data collection process begins. This is also important because the method of sampling and the sample size are decided on.

4. Calculate the test statistic

Now that the data has been collected a test statistic can be calculated. The test statistic will be interpreted to make reasonable inferences about the population.

5. Find the p-value associate with the test statistic

A corresponding p-value is calculated which will determine whether or not the results are statistically significant based on the alpha that was selected.

6. Evaluate the p-value with respect to alpha

The p-value will be evaluated with respect to the selected alpha. This will determine whether or not the results are statistically significant.

7. Reject the null hypothesis

If the p-value is less than the selected alpha, then the null hypothesis is rejected.

7. Fail to reject the null hypothesis

If the p-value is greater than the selected alpha, then the null hypothesis is not rejected.