

pag. 56 - 17 a 25

17) a) $f(\frac{\pi}{6}) = \sin \frac{\pi}{6} = \sin \frac{2\pi}{6} = 0$
 b) $g(\frac{\pi}{6}) = 1 - \cos \frac{\pi}{6} = 1 - (-1) = 2$
 c) $h(\frac{\pi}{6}) = \sin \frac{\pi}{6} = \sin \frac{2\pi}{6} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$
 d) $D(f) = \mathbb{R}$
 e) $I_m = [0, 2]$
 $g(x) = 1 - \cos x$
 $g(x) = 1 - (-1) = 2$ (MAX)
 $g(x) = 1 - 1 = 0$ (MIN)

18) a)

x	3x	y = cos 3x
0	0	= cos 0 = 1
$\frac{\pi}{6}$	$\frac{\pi}{2}$	= cos $\frac{\pi}{2}$ = 0
$\frac{\pi}{3}$	π	= cos π = -1
$\frac{\pi}{2}$	$\frac{3\pi}{2}$	= cos $\frac{3\pi}{2}$ = 0

 $D = \mathbb{R}$
 $I_m = [-1, 1]$
 $P = \frac{2\pi}{3}$

b)

x	sen x	y = 1 sen x
0	0	= (0) = 0
$\frac{\pi}{2}$	1	= (1) = 1
π	0	= (0) = 0
$\frac{3\pi}{2}$	-1	= (-1) = -1

 $D = \mathbb{R}$
 $I_m = [-1, 1]$
 $P = \pi$

c)

x	sen x	y = 2 sen x	P(x, y)
0	sen 0 = 0	= 2 · 0 = 0	A(0, 0)
$\frac{\pi}{2}$	sen $\frac{\pi}{2}$ = 1	= 2 · 1 = 2	B($\frac{\pi}{2}$, 2)
π	sen π = 0	= 2 · 0 = 0	C(π , 0)
$\frac{3\pi}{2}$	sen $\frac{3\pi}{2}$ = -1	= 2 · (-1) = -2	D($\frac{3\pi}{2}$, -2)
2π	sen 2π = 0	= 2 · 0 = 0	E(2π, 0)

 $D = \mathbb{R}$
 $I_m = [-2, 2]$
 $P = 2\pi$

19) $P = 2\pi \rightarrow \text{sen ou cos}$
 (m) m multiplica a variável
 $P = \frac{\pi}{(m)} \rightarrow \text{TG}$

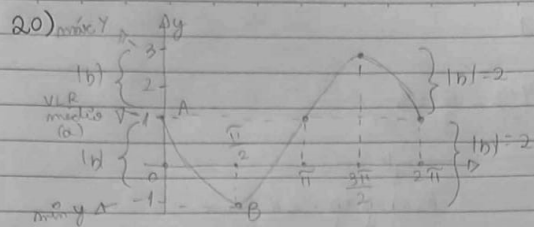
a) $m = 7(\text{sen}) \rightarrow P = \frac{2\pi}{7} = \frac{2\pi}{7}$

b) $m = 2(\text{sen}) \rightarrow P = \frac{2\pi}{2} = \frac{2\pi}{2} = \pi$

c) $m = 2(\text{cos}) \rightarrow P = \frac{2\pi}{2} = \frac{2\pi}{2} = \pi$

d) $m = \pi(\text{TG}) \rightarrow P = \frac{\pi}{\pi} = \frac{\pi}{\pi} = 1$

e) $m = \pi(\text{sen}) \rightarrow P = \frac{2\pi}{\pi} = \frac{2\pi}{\pi} = 2$



$$P(x, y) \rightarrow F(x) = y$$

$$A(0; 1) \quad B\left(\frac{\pi}{2}; -1\right)$$

$$F(0) = 1 \quad F\left(\frac{\pi}{2}\right) = -1$$

$$a + b \cdot \text{sen } 0 = 1 \quad a + b \cdot \text{sen } \frac{\pi}{2} = -1$$

$$a + b \cdot 0 = 1 \quad 1 + b \cdot 1 = -1$$

$$a + 0 = 1 \quad b = -1 - 1$$

$$a = 1 \quad b = -2$$

letra "D"

21) $\max y = 4$ $\min y = 2$ $\rightarrow \text{período } 3$

$$P = \frac{2\pi}{|m|} \rightarrow \frac{2\pi}{|m|} = 3$$

$$|m| = \frac{2\pi}{3}$$

$$y = 3 + \text{sen} \left[\left(\frac{2\pi}{3} \right) \cdot t \right]$$

$$\max \text{sen} = 1 \quad \min \text{sen} = -1$$

a) $y = 2 \pm 2 \rightarrow \text{Im} = [0; 4]$

b) $y = 2 \pm 2 \rightarrow \text{Im} = [0; 4]$

c) $y = 3 \pm 1 \rightarrow \text{Im} = [2; 4]$

d) $y = 3 \pm 1 \rightarrow \text{Im} = [2; 4]$

e) $y = -3 \pm 2 \rightarrow \text{Im} = [-5; -1]$

$$P = 3 \text{ e } P = \frac{2\pi}{|m|}$$

$$\frac{2\pi}{|m|} = 3 \rightarrow \frac{2\pi}{3} = |m| \rightarrow m = \frac{2\pi}{3}$$

$$\max y = 4 \rightarrow \text{Im} = [2; 4]$$

$$\min y = 2$$

b) $y = 2 + 2 \text{sen} \left(\frac{2\pi}{3} \cdot t \right)$

$$y = 2 + 2 \cdot (\pm 1)$$

$$y = 2 \pm 2 \rightarrow \text{Im} = [0; 4]$$

d) $y = 3 + \text{sen} \left(\frac{2\pi}{3} \cdot t \right)$

$$y = 3 \pm 1$$

$$\text{Im} = [2; 4]$$

$$v(t) = y$$

$$\max 4$$

$$|b| = 4 - 2 = \frac{8}{2} = 4$$

$$\min 2$$

$$P = 3 \text{ e } P = \frac{2\pi}{|m|}$$

$$F(x) = a + b \cdot \text{sen} \left(\frac{2\pi}{3} \cdot x \right)$$

$$\frac{2\pi}{|m|} = 3 \rightarrow |m| = \frac{2\pi}{3} \rightarrow F(x) = 3 + 1 \cdot \text{sen} \left(\frac{2\pi}{3} \cdot x \right)$$

22) $C(3) = 2 - \cos \frac{\pi}{2}$

$$C(3) = 2 - \cos \frac{\pi}{2}$$

$$C(3) = 2 - 0 = 2$$

$$V(3) = 3\sqrt{2} \cdot \text{sen} \left(\frac{3\pi}{4} \right)$$

$$V(3) = 3\sqrt{2} \cdot \text{sen} \frac{\pi}{4}$$

$$V(3) = 3\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 3 \cdot \frac{2}{2} = 3$$

$$L(x) = v(x) - C(x)$$

$$L(3) = v(3) - C(3)$$

$$L(3) = 3 - 2 = 1$$

$$\text{lucro} = 1000$$

23) período:

$$\frac{2\pi}{|m|} = 8 \rightarrow \frac{2\pi}{8} = |m|$$

$$|m| = \frac{\pi}{4} \rightarrow m = \frac{\pi}{4}$$

$$\therefore v(x) = 2 \cdot \text{sen} \left(\frac{\pi}{4} \cdot x \right)$$

$$v(x) = a + b \cdot \text{sen}(mx + m)$$

$$a = 0$$

$$b = 2$$

$$\text{Im} = [-2; 2]$$

$$D(6; -2) \rightarrow v(6) = -2$$

$$a + b \cdot \text{sen} \left(\frac{\pi}{4} \cdot 6 + m \right) = -2$$

$$0 + 2 \cdot \text{sen} \left(\frac{\pi}{4} \cdot 6 + m \right) = -2$$

$$\text{sen} \left(\frac{3\pi}{2} + m \right) = \frac{-2}{2}$$

$$\text{sen} \left(\frac{3\pi}{2} + m \right) = -1$$

$$m = 0$$

período: $P = \frac{2\pi}{|m|} = \frac{2\pi}{1} = 2$

$\frac{2\pi}{|m|} = 2 \Rightarrow |m| = 1 \Rightarrow m = 1$

função: $v(x) = a + b \cdot \text{sen}(mx + m)$

$A(0,0) \Rightarrow v(0) = 0$ $B(2,2) \Rightarrow v(2) = 2$

$a + b \cdot \text{sen}(m \cdot 0 + m) = 0$

$a + b \cdot \text{sen}(m \cdot 2 + m) = 2$

$a + b \cdot \text{sen}(m \cdot 0 + 0) = 0$

$a + b \cdot \text{sen}\left(\frac{\pi}{2} \cdot 2 + 0\right) = 2$

$a + b \cdot \text{sen } 0 = 0$

$a + b \cdot \text{sen } \frac{\pi}{2} = 2$

$a + b \cdot 0 = 0$

$a + b \cdot 1 = 2$

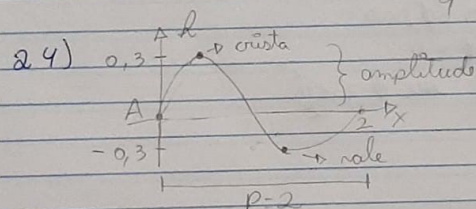
$a + 0 = 0$

$b = 2$

$a = 0$

$v(x) = a + b \cdot \text{sen}(mx + m) = 0 + 2 \cdot \text{sen}\left(\frac{\pi}{2} \cdot x + 0\right)$

$v(x) = 2 \cdot \text{sen } \frac{\pi}{2} x$



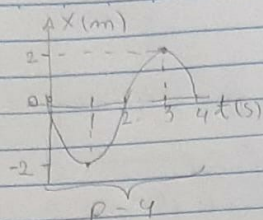
$\text{Im} = [-0.3, 0.3] \Rightarrow a = 0$

Período: $P = \frac{2\pi}{|m|} = 2 = \frac{2\pi}{|m|} \Rightarrow |m| = \frac{2\pi}{2} \Rightarrow m = \pi$

$k(x) = a + b \cdot \text{sen}(mx + m)$

$k(x) = 0.3 \cdot \text{sen}(\pi x)$

25)



$\text{Im}(F) = [-2, 2] \Rightarrow 0$

amplitude = 2 $\Rightarrow |A| = 2 \Rightarrow A = 2$

$x = A \cdot \cos(mt + \phi)$

$F(t) = A \cdot \cos\left(\frac{\pi}{2} t + \phi\right)$

Período: $P = 4$

$\frac{2\pi}{|m|} = 4$

$|m| = \frac{2\pi}{4}$

$|m| = \frac{\pi}{2}$

$m = \frac{\pi}{2}$

$0(0,0) \Rightarrow F(0) = 0$

$A \cdot \cos(mt + \phi) = 0$

$2 \cdot \cos\left(\frac{\pi}{2} \cdot 0 + \phi\right) = 0$

$\cos(0 + \phi) = 0$

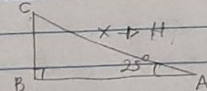
$\cos \phi = 0$

$\phi = \frac{\pi}{2} \text{ ou } \phi = \frac{3\pi}{2}$

pág 57

1) 1º trecho:

$\cos 25^\circ = \frac{220}{x}$



$\text{sen} = \frac{\text{oposto}}{\text{hipotenusa}} \Rightarrow \text{sen } 25^\circ = \frac{220}{x}$

$\Rightarrow 0.423 = \frac{220}{x} \Rightarrow x = \frac{220}{0.423}$

$x \approx 520 \text{ m}$

2º trecho:

$$\begin{aligned} \text{SEN } 25^\circ &= \frac{0,13,5}{11} \\ 396 &\approx 220 \\ 176, & \\ \cos 13,5^\circ & \\ 0,233 &= \frac{176}{Y} \rightarrow Y = \frac{176}{0,233} \\ Y &\approx 755 \text{ m} \end{aligned}$$

2) Lei dos Senos:

$$\cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$$

$$x^2 = 89^2 + 20^2 - 2 \cdot 89 \cdot 20 \cdot \cos 120^\circ$$

$$x^2 = 7921 + 400 - 2 \cdot 89 \cdot 20 \cdot \left(-\frac{1}{2}\right)$$

$$x^2 = 7921 + 400 + 1780 \rightarrow x = \sqrt{10101} \approx 100 \text{ km}$$

pág 58 e 59

1) $\text{SEN } 10^\circ \text{ DE:}$

$$\tilde{\pi} \approx 3,14 \rightarrow F(3,14) = 5$$

$$F(3\tilde{\pi}) \approx F(3 \cdot 3,14) = F(9,42)$$

$$F(3\tilde{\pi}) = F(\tilde{\pi}) = 5$$

letra b

2) Imagem: $+1$ ou -1 (máx e mín)

$$F(t) = \frac{\tilde{\pi}}{9} \cdot \text{sen} \left[\frac{2\tilde{\pi}}{3} \left(\frac{t-3}{4} \right) \right]$$

$$\text{máx } \text{SEN} = 1 \quad \text{mín } \text{SEN} = -1$$

$$F(t) = \frac{\tilde{\pi}}{9} \cdot 1 \quad F(t) = \frac{\tilde{\pi}}{9} \cdot (-1)$$

$$F(t) = \frac{\tilde{\pi}}{9} \text{ máx} \quad F(t) = -\frac{\tilde{\pi}}{9} \text{ mín}$$

$$\text{máx} \rightarrow F(t) = \frac{\tilde{\pi}}{9} \cdot \frac{180}{\tilde{\pi} \text{ Rad}} = \frac{180}{9} = 20^\circ \text{ letra c}$$

3) $D = 60 \rightarrow R = 90$

Lei dos Senos:

$$NE^2 = NO^2 + EO^2 - 2 \cdot NO \cdot EO \cdot \cos \theta$$

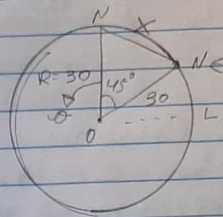
$$x^2 = 30^2 + 30^2 - 2 \cdot 30 \cdot 30 \cdot \cos 45^\circ$$

$$x^2 = 900 + 900 - 2 \cdot 900 \cdot \frac{\sqrt{2}}{2}$$

$$x^2 = 1800 - 900 \cdot 1,4$$

$$x^2 = 1800 - 1260$$

$$x = \sqrt{540}$$



4) $T(h) > 28$

$$8 \cos \left[\left(\frac{h+9}{12} \right) \cdot \tilde{\pi} \right] > 22 - 18$$

$$\cos \left[\left(\frac{h+9}{12} \right) \cdot \tilde{\pi} \right] > \frac{4}{8} > \frac{1}{2}$$

$$\cos \left[\left(\frac{h+9}{12} \right) \cdot \tilde{\pi} \right] > \frac{1}{2}$$

$11 < h < 19$
intervalo de 8 hrs

$$\frac{5\tilde{\pi}}{3} < x < \frac{7\tilde{\pi}}{3}$$

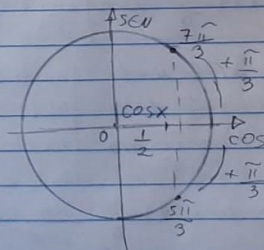
$$\frac{5\tilde{\pi}}{3} < \left(\frac{h+9}{12} \right) \cdot \tilde{\pi} < \frac{7\tilde{\pi}}{3}$$

$$\frac{5\tilde{\pi}}{3} \cdot \frac{12}{\tilde{\pi}} < h+9 < \frac{7\tilde{\pi}}{3} \cdot \frac{12}{\tilde{\pi}}$$

$$20 < h+9 < 28$$

$$20-9 < h < 28-9$$

$$11 < h < 19$$



$$\cos x = \frac{1}{2} \rightarrow x = 60^\circ = \frac{\tilde{\pi}}{3}$$

5) mór radiocár $\rightarrow 1 = 300 + 250.1$

$$\text{SEN} \left[\frac{2\pi(d-77)}{365} \right] = 1$$

JAN $\rightarrow 31$

FEB $\rightarrow 28$

MAR $\rightarrow 31$

ABR $\rightarrow 30$

MAI $\rightarrow 31 \rightarrow 151$

JUN $\rightarrow 30 \rightarrow 181$

$\rightarrow 168 \rightarrow \text{SUN} 40$

$$\frac{2\pi(d-77)}{365} = \frac{\pi}{2} \Rightarrow d-77 = \frac{365}{2}$$

$$\frac{2(d-77)}{365} = \frac{1}{2} \Rightarrow d-77 = \frac{365}{2}$$

$$d-77 = \frac{1.365}{2.2}$$

6) $F(x) = A + B \cdot \text{SEN}(\frac{\pi}{12} \cdot x)$

mór = 70
A = 50 } $|B| = 20$

mín = 30

P = 24

$\frac{2\pi}{12} = 24$

$\frac{2\pi}{12} = 12$

$\therefore m = \pi/12$

$F(x) = 50 + 20 \cdot \text{SEN}(\frac{\pi}{12} \cdot x)$

7) lei dos cossenos:

$PB^2 = AB^2 + AP^2 - 2 \cdot AB \cdot AP \cdot \cos \theta$

$d^2 = 2,5^2 + 4,5^2 - 2 \cdot 2,5 \cdot 4,5 \cdot \cos 60$

$d^2 = 6,25 + 20,25 - 2 \cdot 11,25 \cdot \frac{1}{2}$

$d^2 = 86,5 - 11,25$

$d = \sqrt{75,25} \approx 8,67$

MAX m

PAB $\rightarrow 16$ 7

BP $\rightarrow x$ 4

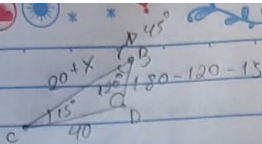
$7x = 16.4$

$x = 64 = 9,1$

$x = 9 \text{ min} \cdot 0,1 \cdot 605$

$\rightarrow 55$

8)



$BC = CD$
 $\text{SEN} D \text{ SEN} B$

$x = \frac{40\sqrt{3}}{\sqrt{2}} - \frac{20\sqrt{2}}{\sqrt{2}}$

$\frac{20+x}{\text{SEN} 120} = \frac{40}{\text{SEN} 45}$

$x = \frac{40\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} - 20$

$\frac{20+x}{\frac{\sqrt{3}}{2}} = \frac{40}{\frac{\sqrt{2}}{2}}$

$x = \frac{20\sqrt{6}}{\sqrt{2}} - 20$

$x = 20\sqrt{6} - 20$

$(20+x)\sqrt{2} = 40\sqrt{3}$

$x = 48 - 20 = 28 \text{ m}$

$20\sqrt{2} + \sqrt{2}x = 40\sqrt{3}$

$\sqrt{2}x = 40\sqrt{3} - 20\sqrt{2}$

9) lei cossenos

AP = 16

$AB^2 = AP^2 + BP^2 - 2 \cdot AP \cdot BP \cdot \cos \hat{P}$

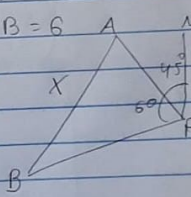
PB = 6

$x^2 = 16^2 + 6^2 - 2 \cdot 16 \cdot 6 \cdot \cos 60$

$x^2 = 256 + 36 - 2 \cdot 96 \cdot \frac{1}{2}$

$x^2 = 292 - 96$

$x = \sqrt{196} \rightarrow x = 14 \text{ km}$



10) $P = \frac{2\pi}{6} = \frac{2\pi}{3} = \frac{2\pi}{6} = 2\pi \cdot \frac{6}{\pi} = 12$

$T(k) = 24 + 3 \cdot 1 = 24 + 3 = 27$

$\cos(\frac{\pi}{6}t + \frac{\pi}{3}) = 1 \rightarrow \frac{\pi}{6}t + \frac{\pi}{3} = \frac{2\pi}{3} \rightarrow \frac{\pi}{6}t = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3} \rightarrow t = 2$

ou 2π

$t = 12 - 2 = 10$

momento = 5 + 10 = 15 h