공학박사학위논문

Mathematical Optimization Approaches for Solving Production Planning Problems

생산 계획 문제 해결을 위한 최적화 기법

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산업공학과

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Abstract

Mathematical Optimization Approaches for Solving Production Planning Problems

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In this thesis, we consider a production planning problem arising in real-world industry. We propose mathematical optimization techniques for solving the problem. We verify that the proposed solution approaches are more effective than existing methods.

Keywords: Production planning, Optimization, Industrial engineering

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Chapter 1

Introduction

Production planning problem has been an important issue for the last decades. For the high-technology industry, in particular, importance of the elaborate production plan greatly increases since each product has high value.

In this thesis, we consider the production planning problem occurs in real-world high-technology industry. We start by describing the problem in Section 1.1.

1.1 Problem Description

This problem is very hard to solve as shown in Copil et al. [2]. An illustration of the problem is shown in Figure 1.1.

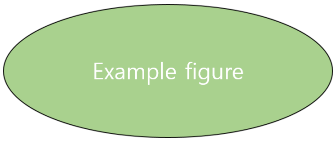

Figure 1.1: An illustration of the problem

1.2 Research Motivation and Contribution

Our research motivations and main contributions of the thesis are as follow:

(a) We introduce a production planning problem which is not studied yet.

(b) We propose various solution approaches based on mathematical optimization techniques.

(c) We conduct computational experiments on various datasets to verify the effectiveness of proposed solution approaches

1.3 Organization of the Thesis

The thesis is composed of 5 chapters. In Chapter 2, we review literatures related to the problem. In Chapter 3, we propose various solution approaches. In Chapter 4, results of computational experiments are presented. Finally, in Chapter 5, we give concluding remarks and possible future research directions of this thesis.

Chapter 2

Literature Review

2.1 Review on Production Planning Problem

Lots of studies to solve production planning problem are done (see e.g. [1, 4, 5, 6]). We extend the solution approach proposed by Duarte et al. [3].

Chapter 3

Solution Approaches

3.1 Exact Approaches

Total cost $f_0(x)$ is the sum of fixed cost $fc(x)$ and variable cost $vc(x)$. In other words, following equation is true.

$$f_0(x) = fc(x) + vc(x) \tag{3.1}$$

Now we can formulate the problem as follow.

$$
\begin{aligned}
\text{minimize} \quad & f_0(x) && && (3.2)\\
\text{subject to} \quad & f_i(x) \le 0 && \forall i \in I && (3.3)\\
& g_j(x) = 0 && \forall j \in J && (3.4)\\
& x \in \mathbb{X} && && (3.5)
\end{aligned}
$$

(3.2) is an objective function for the problem. (3.3) and (3.4) are constraints of the problem. Domain of the decision variables is expressed as (3.5).

Proposition 3.1. *We can solve (3.2)-(3.5) in polynomial time.*

Proof. Since f_0, f_i, g_j are linear functions and $\mathbb{X}$ is a polyhedron, we can use LP. □

3.2 Heuristic Approaches

Pseudo code for the heuristic algorithm is as follows.

Algorithm 1 Heuristic Algorithm $(\mathcal{P}, r, l)$

```
RP ← LP relaxation of P,
n ← 0;
while nl + r ≤ |T| do
    foreach i ∈ I, t ∈ [nl, nl + r] do
        declare x_it ∈ {0, 1} in RP;
    end for
    solve RP and get solution (x̄);
    foreach i ∈ I, t ∈ [nl, (n + 1)l] do
        if x̄_it = 1 then
            fix x_it = x̄_it
        end if
    end for
    n ← n + 1;
end while
return feasible solution for P;
```

Chapter 4

Computational Experiments

4.1 Test Instances

Description of test instance sets are shown in Table 4.1.

Table 4.1: Description of test instances

Set	Fixed Cost	Variable Cost	Demand level
I	0	0	low
II			high
III	$DU(fc^{min}, fc^{max})$	$DU(vc^{min}, vc^{max})$	low
IV			high

4.2 Test Results

We use a commercial MIP-solver [7] to conduct tests. Test results are shown in Table 4.2. This results verify that the proposed method (*New*) is more effective than existing method (*Old*).

Table 4.2: Test results

Dim	*Set*	*Gap*		*Time*		*#Opt/#Feas*	
		Old	*New*	*Old*	*New*	*Old*	*New*
30×30	I	61.11	3.41	0.94	0.07	10/10	10/10
	II	17.21	0.23	0.90	0.05	10/10	10/10
	III	48.62	8.17	0.94	0.18	10/10	10/10
	IV	15.46	0.28	0.79	0.09	10/10	10/10
100×100	I	85.75	5.24	559.19	9.71	2/10	10/10
	II	9.87	0.53	394.92	2.82	8/10	10/10
	III	77.11	61.47	600	600	0/3	6/10
	IV	20.71	10.47	496.36	409.06	0/5	5/10

Chapter 5

Conclusion

5.1 Conclusion

We proposed solution approaches for a production planning problem.

5.2 Future Direction

We can extend our approaches to apply to other problems.

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국문초록

본 논문에서는 실제 산업에서 발생하는 생산계획문제를 소개하고 최적화 기법을 이용하여 이 문제를 해결한다. 실험을 통해 제안된 해법의 효과성을 입증한다.

주요어: 생산계획, 최적화, 산업공학

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감사의 글

서울대학교 산업공학과의 모든 식구들께 감사드립니다.