Nomenclature

Joint space
q Joint positions
M Inertia matrix

Posture task space
β Desired acceleration of the first joint q_1
k Posture stiffness

Impedance task space (all variables are expressed in world frame)
R Rotation matrix of the end effector
ω Angular velocity vector of the end effector
α Angular acceleration vector of the end effector
p Position vector of the end effector
v Linear velocity vector of the end effector
a Linear acceleration vector of the end effector
f Virtual wrench vector acting on the end effector
K Impedance stiffness matrix
Λ Impedance inertia matrix

Reference Spreading
φ Time of impact
γ Impact time-based interpolation variable
T Duration of interim impact mode
m Intermediate impact mode(0: no RS, 1: Jari, 2: Sven, 3: mixing)

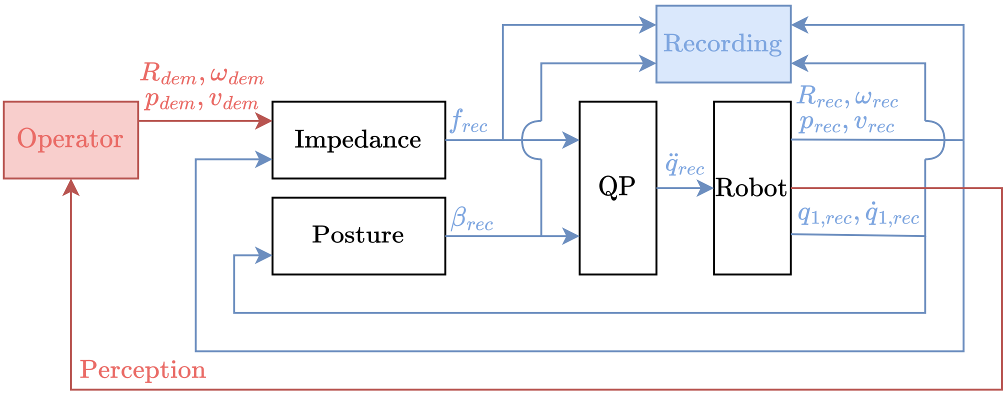
Sub- and superscripts
$(\cdot)^a$ Ante-impact signal
$(\cdot)^p$ Post-impact signal
$(\cdot)_{dem}$ Signals from VR device during recording
$(\cdot)_{rec}$ Signals measured from robot or calculated during recording
$(\cdot)_{ref}$ Signals determined using RS scheme
$(\cdot)_{rep}$ Signals measured from robot or calculated during replay

Recording

$$\ddot{q}_{rec} = \min_{\ddot{q}} \left\| \begin{bmatrix} \alpha(\ddot{q}) \\ a(\ddot{q}) \end{bmatrix} - \Lambda^{-1} f_{rec} \right\| + \|\ddot{q}_1 - \beta_{rec}\|$$

$$f_{rec} = 2(\Lambda K_{rec})^{\frac{1}{2}} \begin{bmatrix} \omega_{dem} - \omega_{rec} \\ v_{dem} - v_{rec} \end{bmatrix} + K_{rec} \begin{bmatrix} R_{rec}(\log({R_{rec}}^T R_{dem}))^{\vee} \\ p_{dem} - p_{rec} \end{bmatrix}$$

$$\beta_{rec} = 2\sqrt{k_{rec}}\,[-\dot{q}_{1,rec}] + k_{rec}\,[-q_{1,rec}]$$



Reference Extending

$$REhold(x, t, \varphi) = \begin{cases} \begin{bmatrix} x(t) & x(\varphi) \end{bmatrix}, & \text{if } t \leq \varphi \\ \begin{bmatrix} x(\varphi) & x(t) \end{bmatrix}, & \text{if } t > \varphi \end{cases}$$

$$REintegrate(x, y, t, \varphi) = \begin{cases} \begin{bmatrix} x(t) & x(\varphi) \end{bmatrix} + \begin{bmatrix} 0 & y(\varphi)(t-\varphi) \end{bmatrix}, & \text{if } t \leq \varphi \\ \begin{bmatrix} x(\varphi) & x(t) \end{bmatrix} + \begin{bmatrix} y(\varphi)(t-\varphi) & 0 \end{bmatrix}, & \text{if } t > \varphi \end{cases}$$

$$\begin{bmatrix} R^a_{rec}(t) & R^p_{rec}(t) \end{bmatrix} = \begin{cases} \begin{bmatrix} R_{rec}(t) & R_{rec}(\varphi_{rec}) e^{\omega_{rec}(\varphi_{rec})(t-\varphi_{rec})} \end{bmatrix}, & \text{if } t \leq \varphi_{rec} \\ \begin{bmatrix} R_{rec}(\varphi_{rec}) e^{\omega_{rec}(\varphi_{rec})(t-\varphi_{rec})} & R_{rec}(t) \end{bmatrix}, & \text{if } t > \varphi_{rec} \end{cases}$$

$$\begin{bmatrix} p^a_{rec}(t) & p^p_{rec}(t) \end{bmatrix} = REintegrate(p_{rec}, \dot{q}_{1,rec}, t, \varphi_{rec})$$
$$\begin{bmatrix} q^a_{1,rec}(t) & q^p_{1,rec}(t) \end{bmatrix} = REintegrate(q_{1,rec}, \dot{q}_{1,rec}, t, \varphi_{rec})$$
$$\begin{bmatrix} f^a_{rec}(t) & f^p_{rec}(t) \end{bmatrix} = REhold(f_{rec}, t, \varphi_{rec})$$
$$\begin{bmatrix} \beta^a_{rec}(t) & \beta^p_{rec}(t) \end{bmatrix} = REhold(\beta_{rec}, t, \varphi_{rec})$$
$$\begin{bmatrix} v^a_{rec}(t) & v^p_{rec}(t) \end{bmatrix} = REhold(v_{rec}, t, \varphi_{rec})$$
$$\begin{bmatrix} \omega^a_{rec}(t) & \omega^p_{rec}(t) \end{bmatrix} = REhold(\omega_{rec}, t, \varphi_{rec})$$
$$\begin{bmatrix} \dot{q}^a_{1,rec}(t) & \dot{q}^p_{1,rec}(t) \end{bmatrix} = REhold(\dot{q}_{1,rec}, t, \varphi_{rec})$$

Reference Switching

$\gamma_{rep} = \frac{t-\varphi_{rep}}{T}$

$$RS(x^a(t), x^p(t), t, \gamma) = \begin{cases} x^a, & \text{if } \gamma \leq 0 \\ x^p, & \text{if } \gamma \geq 1 \\ x^p, & \text{if } (0 < \gamma < 1) \wedge (m = 0) \\ x^a, & \text{if } (0 < \gamma < 1) \wedge (m = 1) \\ x^a, & \text{if } (0 < \gamma < 1) \wedge (m = 2) \\ (1-\gamma)x^a + \gamma x^p, & \text{if } (0 < \gamma < 1) \wedge (m = 3) \end{cases}$$

$$RSvel(x, x^a(t), x^p(t), t, \gamma) = \begin{cases} x^a, & \text{if } \gamma \leq 0 \\ x^p, & \text{if } \gamma \geq 1 \\ x^p, & \text{if } (0 < \gamma < 1) \wedge (m = 0) \\ x, & \text{if } (0 < \gamma < 1) \wedge (m = 1) \\ 0, & \text{if } (0 < \gamma < 1) \wedge (m = 2) \\ \gamma x^p, & \text{if } (0 < \gamma < 1) \wedge (m = 3) \end{cases}$$

$$RScont(x^a(t), x^p(t), t, \gamma) = RSvel(0, x^a(t), x^p(t), t, \gamma) + \begin{cases} x^a, & \text{if } (0 < \gamma < 1) \wedge (m = 1) \\ x^a, & \text{if } (0 < \gamma < 1) \wedge (m = 2) \\ (1-\gamma)x^a, & \text{if } (0 < \gamma < 1) \wedge (m = 3) \\ 0, & \text{otherwise} \end{cases}$$

$f_{ref} = RS(f^a_{rec}, f^p_{rec}, t, \gamma_{rep})$
$\beta_{ref} = RS(\beta^a_{rec}, \beta^p_{rec}, t, \gamma_{rep})$
$p_{ref} = RS(p^a_{rec}, p^p_{rec}, t, \gamma_{rep})$
$q_{1,ref} = RS(q^a_{1,rec}, q^p_{1,rec}, t, \gamma_{rep})$
$\dot{q}_{1,ref} = RSvel(\dot{q}_{1,rep}, \dot{q}^a_{1,rec}, \dot{q}^p_{1,rec}, t, \gamma_{rep})$
$v_{ref} = RSvel(v_{rep}, v^a_{rec}, v^p_{rec}, t, \gamma_{rep})$
$\alpha_{ref} = RSvel(\alpha_{rep}, \alpha^a_{rec}, \alpha^p_{rec}, t, \gamma_{rep})$

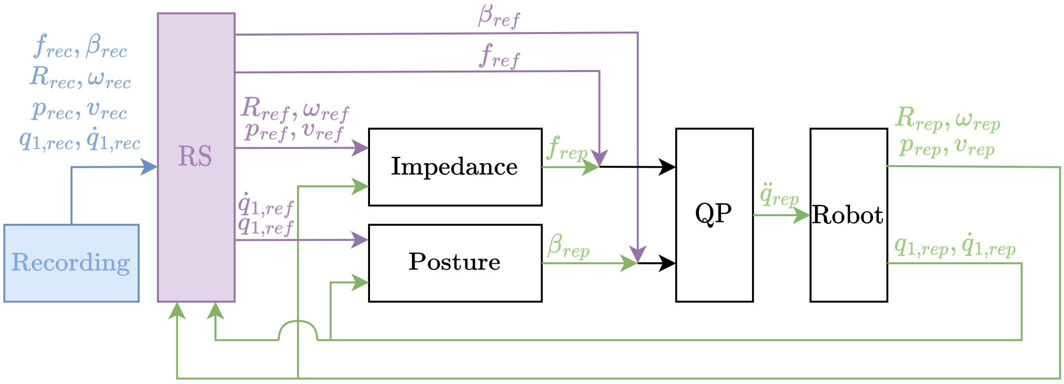
$$R_{ref} = \begin{cases} R^a_{rec}, & \text{if } \gamma \leq 0 \\ R^p_{rec}, & \text{if } \gamma \geq 1 \\ R^p_{rec}, & \text{if } (0 < \gamma < 1) \wedge (m = 0) \\ R_{rep}, & \text{if } (0 < \gamma < 1) \wedge (m = 1) \\ I, & \text{if } (0 < \gamma < 1) \wedge (m = 2) \\ R_{rep}(R^T_{rep}R_{rec})^{\gamma}, & \text{if } (0 < \gamma < 1) \wedge (m = 3) \text{Klopt deze interpolatie???} \end{cases}$$

Replaying

$$\ddot{q}_{rep} = \min_{\ddot{q}} \left\| \begin{bmatrix}\alpha(\ddot{q})\\ a(\ddot{q})\end{bmatrix} - \Lambda^{-1}(f_{rep}+f_{rec}) \right\| + \|\ddot{q}_1 - (\beta_{rep}+\beta_{rec})\|$$

$$f_{rep} = 2(\Lambda K_{rep})^{\frac{1}{2}} \begin{bmatrix}\omega_{rec}-\omega_{rep}\\ v_{rec}-v_{rep}\end{bmatrix} + K_{rep} \begin{bmatrix}R_{rep}(\log({R_{rep}}^T R_{rec}))^{\vee}\\ p_{rec}-p_{rep}\end{bmatrix}$$

$$\beta_{rep} = 2\sqrt{k_{rep}}\,[\dot{q}_{1,rec}-\dot{q}_{1,rep}] + k_{rep}\,[q_{1,rec}-q_{1,rep}]$$



Miscelaneous

$$\begin{bmatrix}\omega\\ v\end{bmatrix} = J\dot{q}$$

$$\begin{bmatrix}\alpha(\ddot{q})\\ a(\ddot{q})\end{bmatrix} = J\ddot{q} + \dot{J}\dot{q}$$

$$\Lambda^{-1} = J^T M^{-1} J$$