Nomenclature

Joint space
q Joint positions
M Inertia matrix

Posture task space
β Desired acceleration of the first joint q_1
k Posture stiffness

Impedance task space (all variables are expressed in world frame)
R Rotation matrix of the end effector
ω Angular velocity vector of the end effector
α Angular acceleration vector of the end effector
p Position vector of the end effector
v Linear velocity vector of the end effector
a Linear acceleration vector of the end effector
f Virtual wrench vector acting on the end effector
K Impedance stiffness matrix
Λ Impedance inertia matrix

Reference Spreading
φ Time of impact
γ Normalized time since impact
T Duration of interim impact mode
m Intermediate impact mode selector (0: no RS, 1: Jari, 2: Sven, 3: mixing)
j Number of jumps that have occured

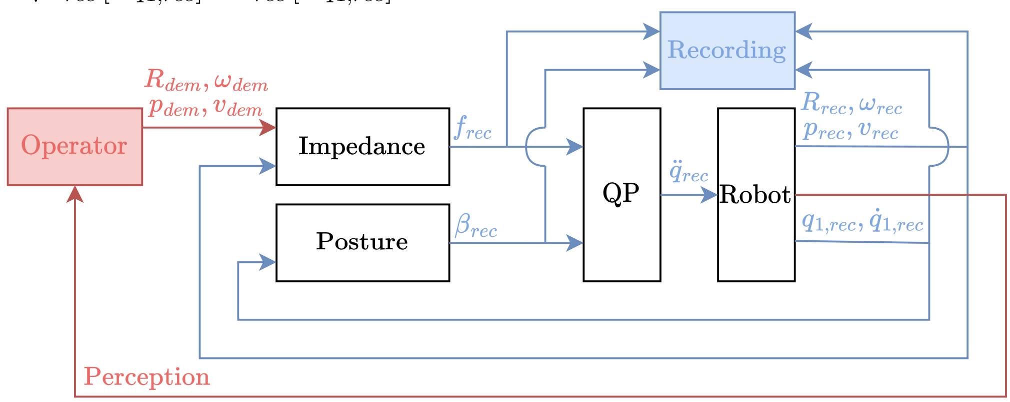
Sub- and superscripts
$(\cdot)_{dem}$ Signals from VR device during recording
$(\cdot)_{rec}$ Signals measured from robot or calculated during recording
$(\cdot)_{ext}$ Signals resulting from the splitting and extending of the recorded signal
$(\cdot)_{ref}$ Signals determined using RS scheme
$(\cdot)_{rep}$ Signals measured from robot or calculated during replay

Recording

$$\ddot{q}_{rec} = \min_{\ddot{q}} \left\| \begin{bmatrix} \alpha(\ddot{q}) \\ a(\ddot{q}) \end{bmatrix} - \Lambda^{-1} f_{rec} \right\| + \|\ddot{q}_1 - \beta_{rec}\|$$

$$f_{rec} = 2(\Lambda K_{rec})^{\frac{1}{2}} \begin{bmatrix} \omega_{dem} - \omega_{rec} \\ v_{dem} - v_{rec} \end{bmatrix} + K_{rec} \begin{bmatrix} R_{rec}(\log({R_{rec}}^T R_{dem}))^{\vee} \\ p_{dem} - p_{rec} \end{bmatrix}$$

$$\beta_{rec} = 2\sqrt{k_{rec}}\,[-\dot{q}_{1,rec}] + k_{rec}\,[-q_{1,rec}]$$

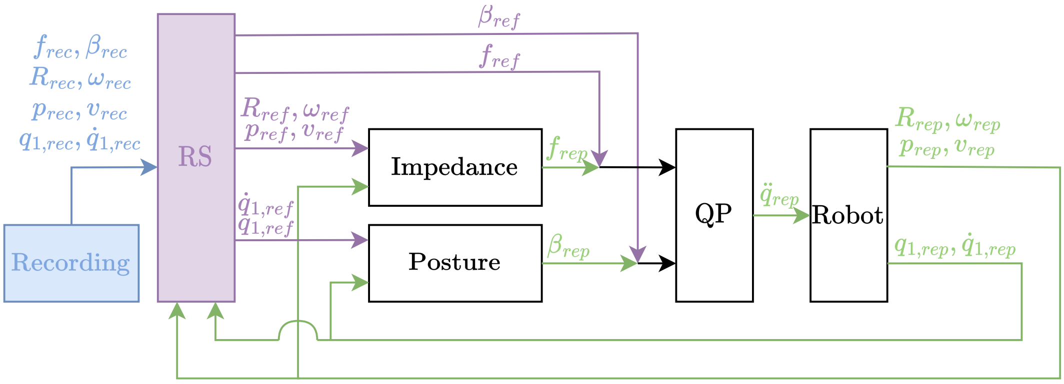


Replaying

$$\ddot{q}_{rep} = \min_{\ddot{q}} \left\| \begin{bmatrix} \alpha(\ddot{q}) \\ a(\ddot{q}) \end{bmatrix} - \Lambda^{-1}(f_{rep} + f_{ref}) \right\| + \|\ddot{q}_1 - (\beta_{rep} + \beta_{ref})\|$$

$$f_{rep} = 2(\Lambda K_{rep})^{\frac{1}{2}} \begin{bmatrix} \omega_{ref} - \omega_{rep} \\ v_{ref} - v_{rep} \end{bmatrix} + K_{rep} \begin{bmatrix} R_{rep}(\log({R_{rep}}^T R_{ref}))^{\vee} \\ p_{ref} - p_{rep} \end{bmatrix}$$

$$\beta_{rep} = 2\sqrt{k_{rep}}\,[\dot{q}_{1,ref} - \dot{q}_{1,rep}] + k_{rep}\,[q_{1,ref} - q_{1,rep}]$$



Reference Extending

$$S_{hold}(x,\varphi,t,j) = \begin{cases} x(\varphi_j), & \text{if } t < \varphi_j \\ x(\varphi_j), & \text{if } t > \varphi_{j+1} \\ x(t), & \text{otherwise} \end{cases}$$

$$S_{integ}(x,y,\varphi,t,j) = \begin{cases} x(\varphi_j) + y(\varphi_j)(t-\varphi_j), & \text{if } t < \varphi_j \\ x(\varphi_{j+1}) + y(\varphi_{j+1})(t-\varphi_{j+1}), & \text{if } t > \varphi_{j+1} \\ x(t), & \text{otherwise} \end{cases}$$

$$S_{rot}(R,\omega,\varphi,t,j) = \begin{cases} R(\varphi_j)e^{\omega(\varphi_j)(t-\varphi_j)}, & \text{if } t < \varphi_j \\ R(\varphi_{j+1})e^{\omega(\varphi_{j+1})(t-\varphi_{j+1})}, & \text{if } t > \varphi_{j+1} \\ R(t), & \text{otherwise} \end{cases}$$

$$R_{ext,j}(t) = S_{rot}(R_{rec}, \omega_{rec}, \varphi_{rec}, t, j)$$

$$\begin{bmatrix} p_{ext,j}(t) \\ q_{1,ext,j}(t) \end{bmatrix} = S_{integ}\left(\begin{bmatrix} p_{rec} \\ q_{1,rec} \end{bmatrix}, \begin{bmatrix} v_{rec} \\ \dot{q}_{1,rec} \end{bmatrix}, \varphi_{rec}, t, j \right)$$

$$\begin{bmatrix} \omega_{rec}(t) \\ v_{ext,j}(t) \\ \dot{q}_{1,ext,j}(t) \\ f_{ext,j}(t) \\ \beta_{ext,j}(t) \end{bmatrix} = S_{hold}\left(\begin{bmatrix} \omega_{rec} \\ v_{rec} \\ \dot{q}_{1,rec} \\ f_{rec} \\ \beta_{rec} \end{bmatrix}, \varphi_{rec}, t, j \right)$$

Reference Switching

$\gamma = \frac{t-\varphi_{rep,j}}{T}$

$$U_{jump}(x, x_{ext}, \gamma, m, j) = \begin{cases} x_{ext,j}, & \text{if } \gamma \geq 1 \\ x_{ext,j}, & \text{if } (\gamma < 1) \wedge (m = 0) \\ x, & \text{if } (\gamma < 1) \wedge (m = 1) \\ 0, & \text{if } (\gamma < 1) \wedge (m = 2) \\ \gamma x_{ext,j}, & \text{if } (\gamma < 1) \wedge (m = 3) \end{cases}$$

$$U_{cont}(x_{ext}, \gamma, m, j) = \begin{cases} x_{ext,j}, & \text{if } \gamma \geq 1 \\ x_{ext,j}, & \text{if } (\gamma < 1) \wedge (m = 0) \\ x_{ext,j-1}, & \text{if } (\gamma < 1) \wedge (m = 1) \\ x_{ext,j-1}, & \text{if } (\gamma < 1) \wedge (m = 2) \\ \gamma x_{ext,j} + (1-\gamma) x_{ext,j-1}, & \text{if } (\gamma < 1) \wedge (m = 3) \end{cases}$$

$$U_{rot}(R_{ext}, \gamma, m, j) = \begin{cases} R_{ext,j}, & \text{if } \gamma \geq 1 \\ R_{ext,j}, & \text{if } (\gamma < 1) \wedge (m = 0) \\ R_{ext,j-1}, & \text{if } (\gamma < 1) \wedge (m = 1) \\ R_{ext,j-1}, & \text{if } (\gamma < 1) \wedge (m = 2) \\ R_{ext,j-1}(R_{ext,j-1}^T R_{ext,j})^{\gamma}, & \text{if } (\gamma < 1) \wedge (m = 3) \end{cases}$$

$$R_{ref} = U_{rot}(R_{ext}, \gamma, m, j)$$

$$\begin{bmatrix} p_{ref} \\ q_{1,ref} \\ \dot{q}_{1,ref} \\ f_{ref} \\ \beta_{ref} \end{bmatrix} = U_{cont}\left(\begin{bmatrix} p_{ext} \\ q_{1,ext} \\ \dot{q}_{1,ext} \\ f_{ext} \\ \beta_{ext} \end{bmatrix}, \gamma, m, j \right)$$

$$\begin{bmatrix} \omega_{ref} \\ v_{ref} \end{bmatrix} = U_{jump}\left(\begin{bmatrix} \omega_{ext,j}(t) \\ v_{ext,j}(t) \end{bmatrix}, \varphi_{rec}, \gamma, m, j \right)$$

Miscelaneous

$$\begin{bmatrix} \omega \\ v \end{bmatrix} = J\dot{q}$$

$$\begin{bmatrix} \alpha(\ddot{q}) \\ a(\ddot{q}) \end{bmatrix} = J\ddot{q} + \dot{J}\dot{q}$$

$$\Lambda^{-1} = J^T M^{-1} J$$