Nomenclature

Joint space

q Joint positions
M Inertia matrix

Posture task space

β Desired acceleration of the first joint q_1
k Posture stiffness

Impedance task space (all variables are expressed in world frame)

R Rotation matrix of the end effector
ω Angular velocity vector of the end effector
α Angular acceleration vector of the end effector
p Position vector of the end effector
v Linear velocity vector of the end effector
a Linear acceleration vector of the end effector
f Virtual wrench vector acting on the end effector
K Impedance stiffness matrix
Λ Impedance inertia matrix

Reference Spreading

φ Time of impact
$\Delta\varphi$ Offset between measured impact and reference extension
γ Normalized time since impact
T Duration of interim impact mode
m RS mode selector (-1: no RS, 0: no interim, 1: Jari, 2: Sven, 3: mixing)
j Number of jumps that have occured

Sub- and superscripts

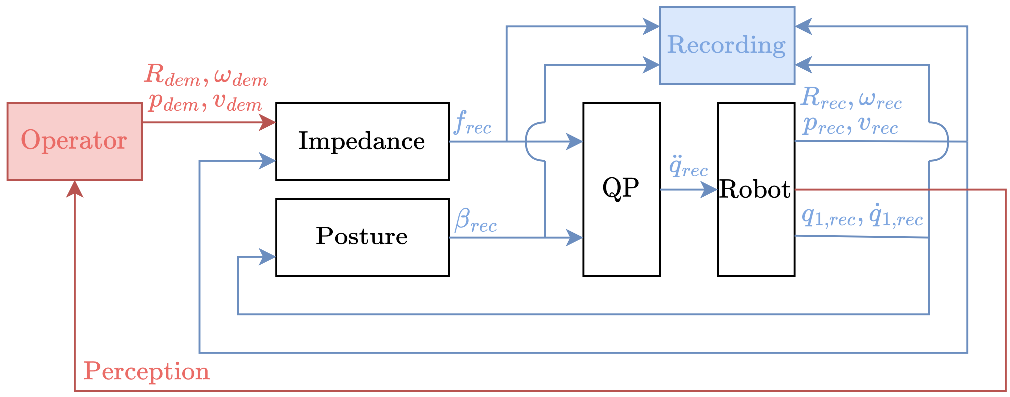
$(\cdot)_{dem}$ Signals from VR device during recording
$(\cdot)_{rec}$ Signals measured from robot or calculated during recording
$(\cdot)_{ext}$ Signals resulting from the splitting and extending of the recorded signal
$(\cdot)_{ref}$ Signals determined using RS scheme
$(\cdot)_{rep}$ Signals measured from robot or calculated during replay

Recording

$$\ddot{q}_{rec} = \min_{\ddot{q}} \left\| \begin{bmatrix} \alpha(\ddot{q}) \\ a(\ddot{q}) \end{bmatrix} - \Lambda^{-1} f_{rec} \right\| + \|\ddot{q}_1 - \beta_{rec}\|$$

$$f_{rec} = 2(\Lambda K_{rec})^{\frac{1}{2}} \begin{bmatrix} \omega_{dem} - \omega_{rec} \\ v_{dem} - v_{rec} \end{bmatrix} + K_{rec} \begin{bmatrix} R_{rec}(\log({R_{rec}}^T R_{dem}))^\vee \\ p_{dem} - p_{rec} \end{bmatrix}$$

$$\beta_{rec} = 2\sqrt{k_{rec}}\,[-\dot{q}_{1,rec}] + k_{rec}\,[-q_{1,rec}]$$

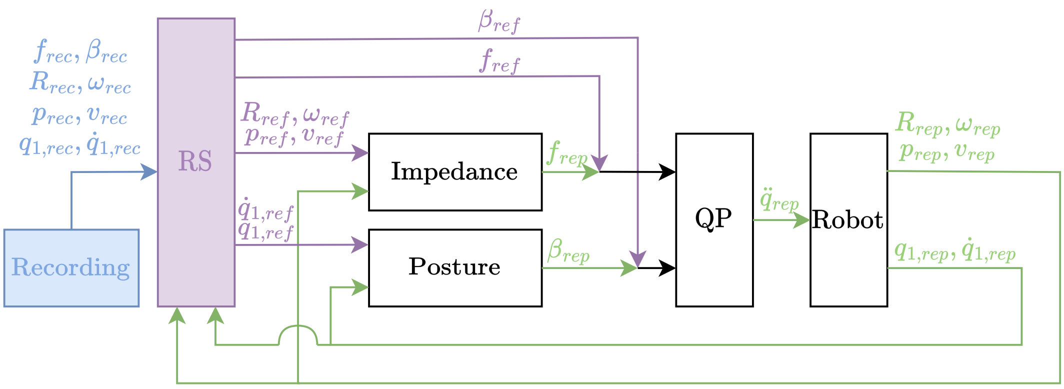


Replaying

$$\ddot{q}_{rep} = \min_{\ddot{q}} \left\| \begin{bmatrix} \alpha(\ddot{q}) \\ a(\ddot{q}) \end{bmatrix} - \Lambda^{-1}(f_{rep} + f_{ref}) \right\| + \|\ddot{q}_1 - (\beta_{rep} + \beta_{ref})\|$$

$$f_{rep} = 2(\Lambda K_{rep})^{\frac{1}{2}} \begin{bmatrix} \omega_{ref} - \omega_{rep} \\ v_{ref} - v_{rep} \end{bmatrix} + K_{rep} \begin{bmatrix} R_{rep}(\log({R_{rep}}^T R_{ref}))^\vee \\ p_{ref} - p_{rep} \end{bmatrix}$$

$$\beta_{rep} = 2\sqrt{k_{rep}}\,[\dot{q}_{1,ref} - \dot{q}_{1,rep}] + k_{rep}\,[q_{1,ref} - q_{1,rep}]$$



Reference Extending

$$\varphi_{rec}^{-} = \varphi_{rec}(j) + \Delta\varphi$$
$$\varphi_{rec}^{+} = \varphi_{rec}(j+1) - \Delta\varphi$$

$$\Omega = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

$$S_{hold}(x) = \begin{cases} x(\varphi_{rec}^{-}), & \text{if } t < \varphi_{rec}^{-} \\ x(\varphi_{rec}^{+}), & \text{if } t > \varphi_{rec}^{+} \\ x(t), & \text{otherwise} \end{cases}$$

$$S_{integ}(x,y) = S_{hold}(x) + \begin{cases} y(\varphi_{rec}^{-})(t - \varphi_{rec}^{+}), & \text{if } t < \varphi_{rec}^{-} \\ y(\varphi_{rec}^{+})(t - \varphi_{rec,j}^{-}), & \text{if } t > \varphi_{rec}^{+} \\ 0, & \text{otherwise} \end{cases}$$

$$S_{rot}(R,\omega) = \begin{cases} R(\varphi_{rec}^{-})e^{\Omega(\varphi_{rec}^{-})(t-\varphi_{rec}^{-})}, & \text{if } t < \varphi_{rec}^{-} \\ R(\varphi_{rec}^{+})e^{\Omega(\varphi_{rec}^{+})(t-\varphi_{rec}^{+})}, & \text{if } t > \varphi_{rec}^{+} \\ R(t), & \text{otherwise} \end{cases}$$

$$R_{ext,j}(t,j) = S_{rot}(R_{rec}(t), \omega_{rec}(t))$$

$$\begin{bmatrix} p_{ext,j}(t,j) \\ q_{1,ext,j}(t,j) \end{bmatrix} = S_{integ}\left(\begin{bmatrix} p_{rec}(t) \\ q_{1,rec}(t) \end{bmatrix}, \begin{bmatrix} v_{rec}(t) \\ \dot{q}_{1,rec}(t) \end{bmatrix} \right)$$

$$\begin{bmatrix} \omega_{rec}(t,j) \\ v_{ext}(t,j) \\ \dot{q}_{1,ext}(t,j) \\ f_{ext}(t,j) \\ \beta_{ext}(t,j) \end{bmatrix} = S_{hold}\left(\begin{bmatrix} \omega_{rec}(t) \\ v_{rec}(t) \\ \dot{q}_{1,rec}(t) \\ f_{rec}(t) \\ \beta_{rec}(t) \end{bmatrix} \right)$$

Note that the functions S_{hold}, S_{integ}, and S_{rot} also require φ_{rec} to be evaluated. This dependancy is excluded from notation for brevity.

Reference Switching

$\gamma = \frac{t-\varphi_{rep,j}}{T}$

$$U_{jump}(x_{rep}, x_{ext}) = \begin{cases} x_{ext}(t, j_{rec}), & \text{if } m = -1 \\ x_{ext}(t, j_{rep}), & \text{if } m = 0 \\ x_{ext}(t, j_{rep}), & \text{if } (\gamma \geq 1) \wedge (m \notin \{-1, 0\}) \\ x_{ext}(t, j_{rep}), & \text{if } (\gamma < 1) \wedge (m = 0) \\ x, & \text{if } (\gamma < 1) \wedge (m = 1) \\ 0, & \text{if } (\gamma < 1) \wedge (m = 2) \\ \gamma x_{ext}(t, j_{rep}), & \text{if } (\gamma < 1) \wedge (m = 3) \end{cases}$$

$$U_{cont}(x_{ext}) = \begin{cases} x_{ext}(t, j_{rec}), & \text{if } m = -1 \\ x_{ext}(t, j_{rep}), & \text{if } m = 0 \\ x_{ext}(t, j_{rep}), & \text{if } (\gamma \geq 1) \wedge (m \notin \{-1, 0\}) \\ x_{ext}(t, j_{rep}), & \text{if } (\gamma < 1) \wedge (m = 0) \\ x_{ext}(t, j_{rep} - 1), & \text{if } (\gamma < 1) \wedge (m = 1) \\ x_{ext}(t, j_{rep} - 1), & \text{if } (\gamma < 1) \wedge (m = 2) \\ \gamma x_{ext}(t, j_{rep}) + (1-\gamma) x_{ext}(t, j_{rep} - 1), & \text{if } (\gamma < 1) \wedge (m = 3) \end{cases}$$

$$U_{cont}(R_{ext}) = \begin{cases} R_{ext}(t, j_{rec}), & \text{if } m = -1 \\ R_{ext}(t, j_{rep}), & \text{if } m = 0 \\ R_{ext}(t, j_{rep}), & \text{if } (\gamma \geq 1) \wedge (m \notin \{-1, 0\}) \\ R_{ext}(t, j_{rep}), & \text{if } (\gamma < 1) \wedge (m = 0) \\ R_{ext}(t, j_{rep} - 1), & \text{if } (\gamma < 1) \wedge (m = 1) \\ R_{ext}(t, j_{rep} - 1), & \text{if } (\gamma < 1) \wedge (m = 2) \\ R_{ext}(t, j_{rep} - 1)(R_{ext}^T(t, j_{rep} - 1) R_{ext}(t, j_{rep}))^{\gamma}, & \text{if } (\gamma < 1) \wedge (m = 3) \end{cases}$$

$$R_{ref}(t) = U_{rot}(R_{ext}(t, j))$$

$$\begin{bmatrix} p_{ref}(t) \\ q_{1,ref}(t) \\ f_{ref}(t) \\ \beta_{ref}(t) \end{bmatrix} = U_{cont}\left(\begin{bmatrix} p_{ext}(t, j) \\ f_{ext}(t, j) \\ \beta_{ext}(t, j) \end{bmatrix} \right)$$

$$\begin{bmatrix} \omega_{ref}(t) \\ v_{ref}(t) \\ \dot{q}_{1,ref}(t, j) \end{bmatrix} = U_{jump}\left(\begin{bmatrix} \omega_{rep}(t) \\ v_{rep}(t) \\ \dot{q}_{1,rep}(t) \end{bmatrix}, \begin{bmatrix} \omega_{ext}(t, j) \\ v_{ext}(t, j) \\ \dot{q}_{1,ext}(t, j) \end{bmatrix} \right)$$

Note that the functions U_{jump}, U_{cont}, and U_{rot} also require j_{rep}, j_{rec}, γ and m to be evaluated. These dependencies are excluded from notation for brevity.

Miscelaneous

$$\begin{bmatrix} \omega \\ v \end{bmatrix} = J\dot{q}$$

$$\begin{bmatrix} \alpha(\ddot{q}) \\ a(\ddot{q}) \end{bmatrix} = J\ddot{q} + \dot{J}\dot{q}$$

$$\Lambda^{-1} = J^T M^{-1} J$$

Need a different symbol for $\dot{q}_1$ since it is not always the derivative of q_1.