Introduction to Graph Theory

Competitive programming workshop @ UniTn

Day 3 - 23/07/2020

Tutors: G. Fabris, Q. F. Lotito

Last contest

Problem A - Bijele

Element-wise subtract the input array of pieces from the expected one.

Problem B - Dicecup

Just count the frequency of the possible outcomes and find the maximum.

Problem C - I Can guess the Data Structure!

Simulate! You can use a stack, a queue and a priority queue to simulate the input and see the output you get.

Problem D - Union Find

Just a theoretical problem. You need to implement a union find disjoint set. Giacomo explained it last week.

Problem E - Disastrous Downtime

Use a priority queue to store requests: push “arrivals” in the queue, but also “departures” (arrivals + 1000). Let *X* be the maximum number of requests in the system at the same time, and *K* be the number of servers, the answer is *ceil(X / K)*.

Problem F -Trending Topic

Combination of maps to store the frequency of the words and sorting to print. Not that easy to write.. 😈

Problem G - Distributing Ballot Boxes

Assign one box to every city, then greedily assign the remaining boxes to the city with the worst people to boxes ratio. You should use a priority queue instead of a linear search to check the ratios.

Problem H - Greeting Card

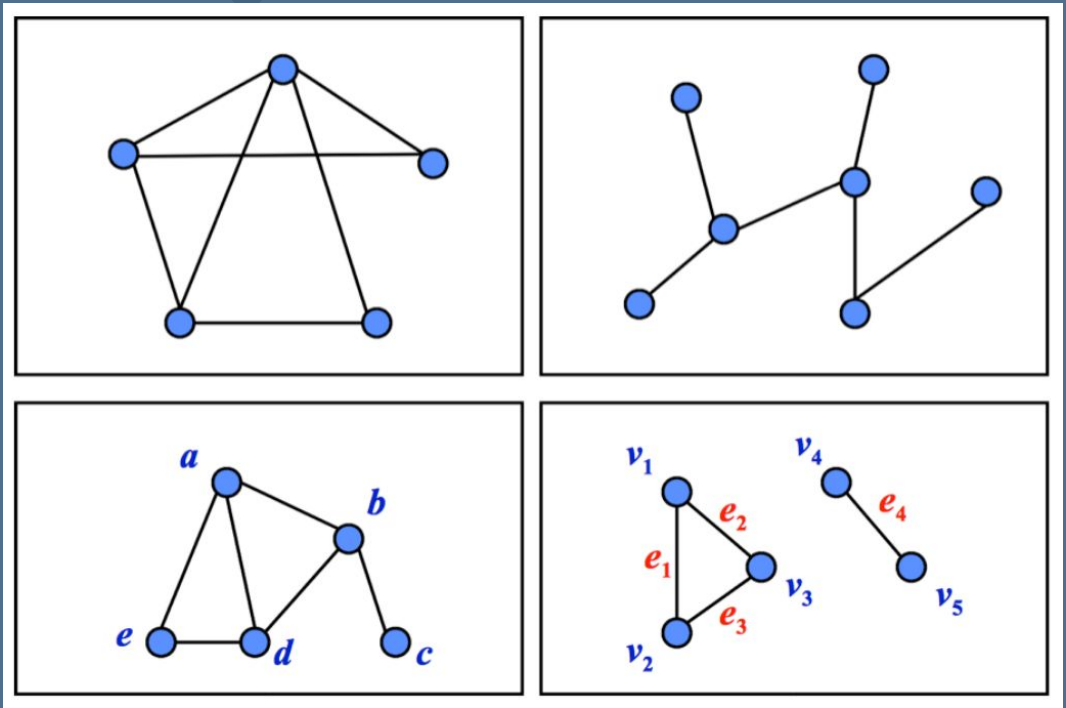
We need to find points such that $\Delta x^2 + \Delta y^2 = 2018^2$

You can precompute the possible 12 pairs (Δx, Δy) satisfying the above equation (by checking all the possible combinations): (0, 2018), (2018, 0), (1118, 1680), (1680, 1118), and negatives..

Then you can use a set / hashmap to check if a given pair of deltas can be applied to an input (i.e., also the resulting point is in the input).

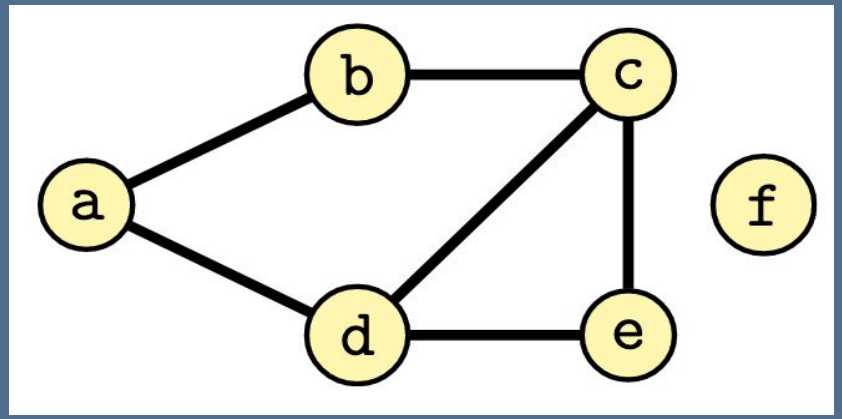
Introduction to Graph Theory

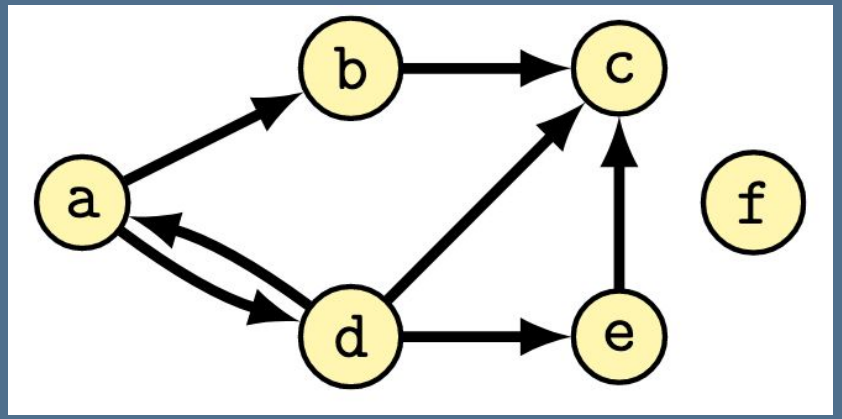
What is a graph?



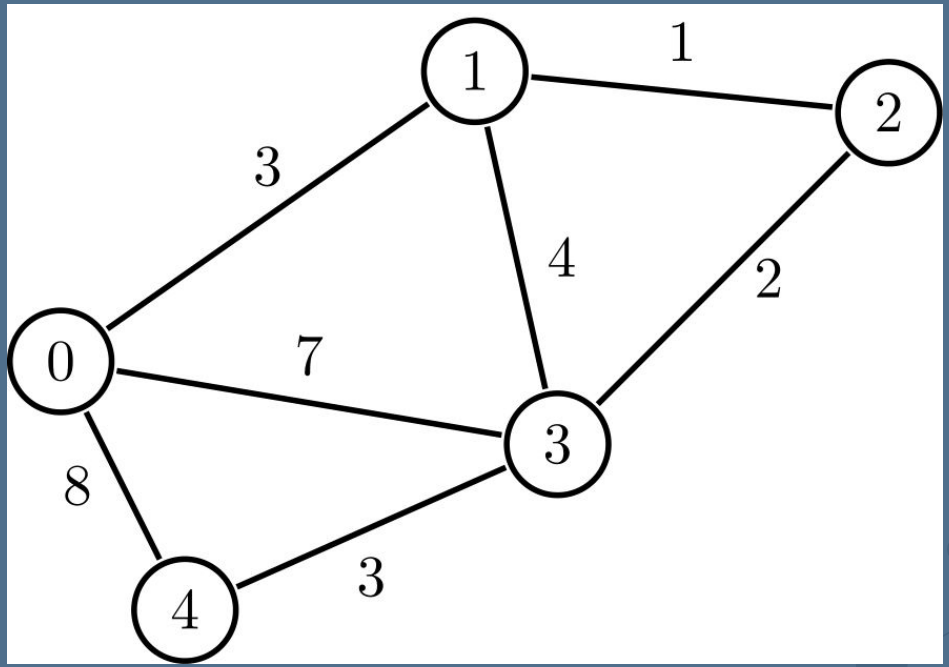
- Vertices
- Edges
- Paths
- Subgraphs
- Connected graphs
- Degree of a vertex
- Tree

Orientation

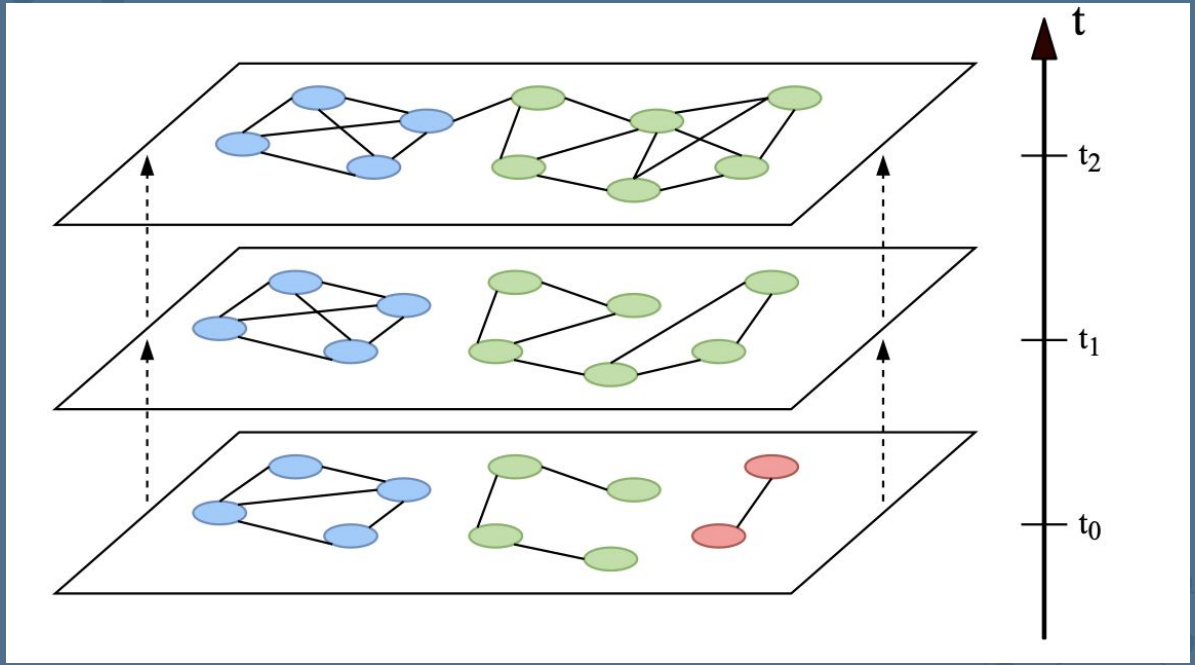




Weight



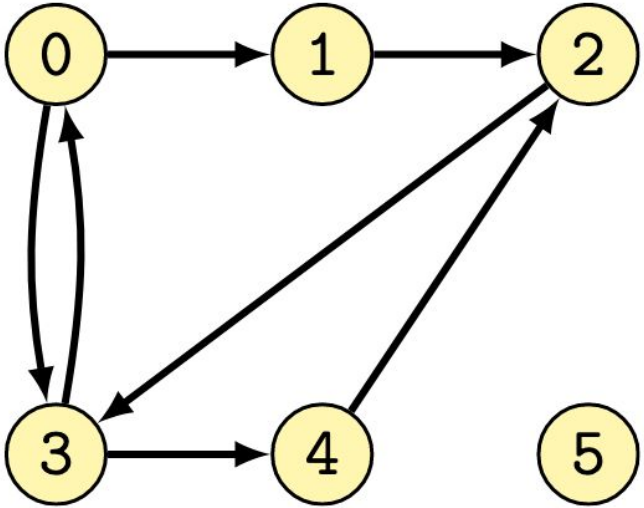
Time



Applications

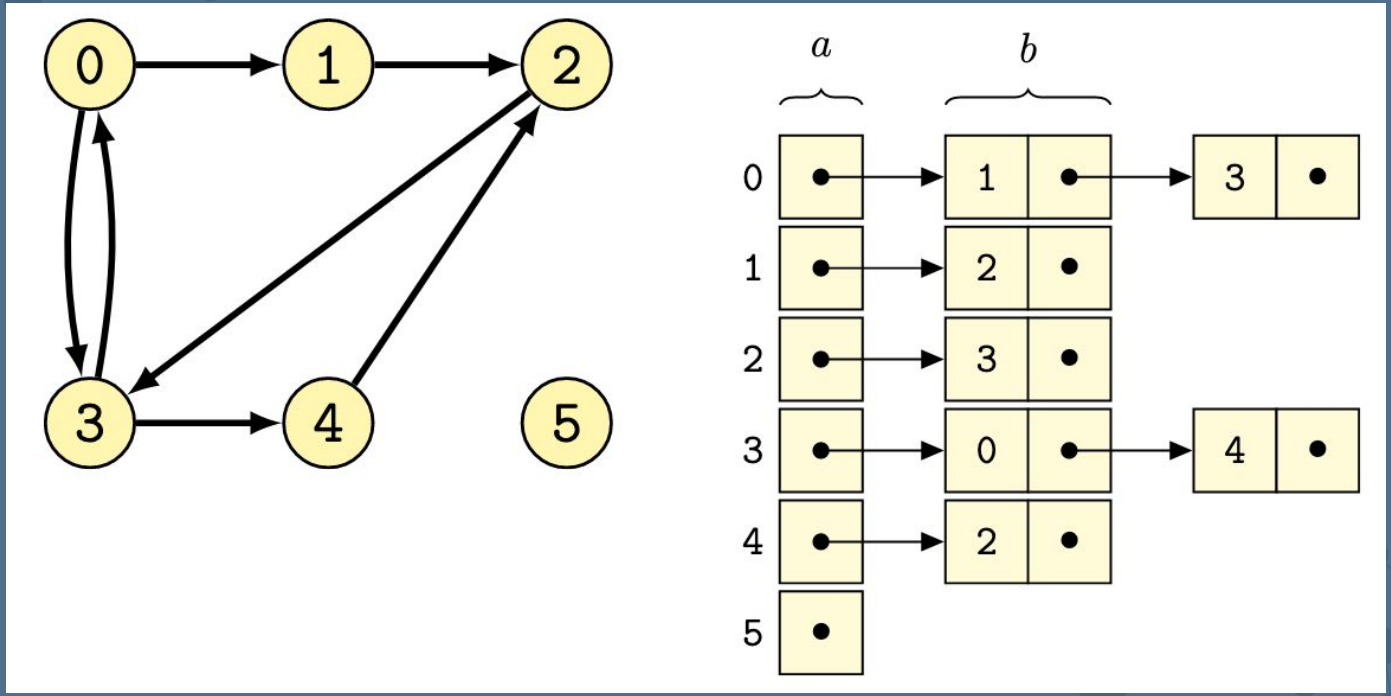
- Find the shortest path between two cities
- Find communities on social networks
- Analyze interactions among proteins
- Find how many links on average you have to click to go from a Wikipedia page to another
- ...

How to represent a graph



	0	1	2	3	4	5
0	0	1	0	1	0	0
1	0	0	1	0	0	0
2	0	0	0	1	0	0
3	1	0	0	0	1	0
4	0	0	1	0	0	0
5	0	0	0	0	0	0

How to represent a graph

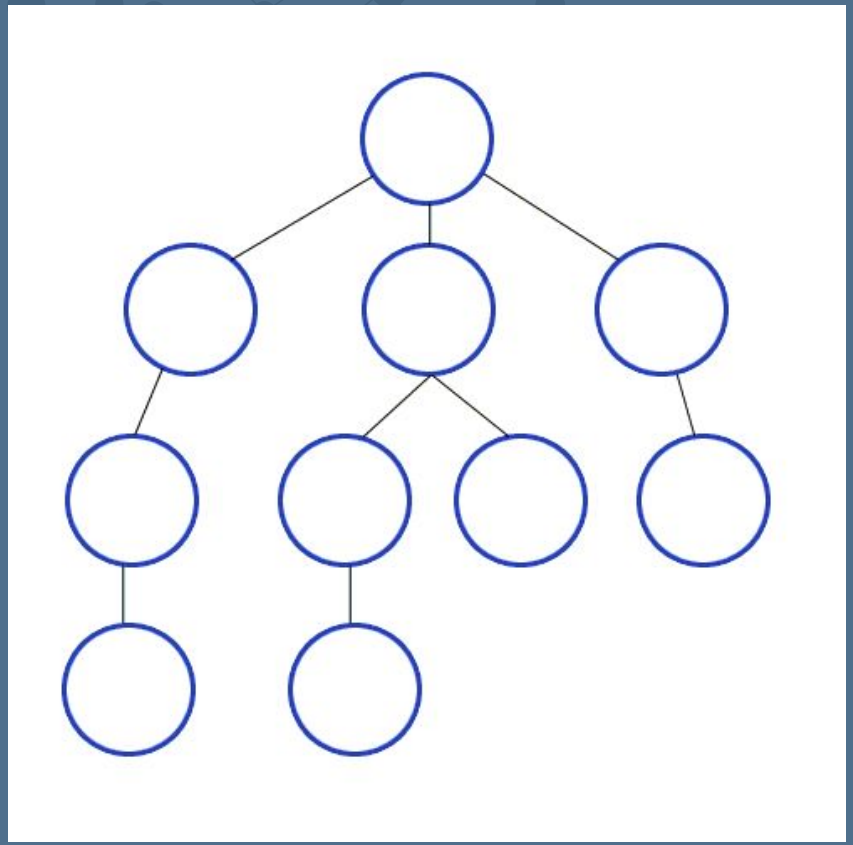


How to represent a graph in C++

- A NxN matrix (easy, maybe too expensive)
- A vector of vectors: *v [i]* contains the ID of the neighbors of vertex *i* (vector<int> grafo[N])

```
for(int i = 0; i < n; i++)
{
    int num;
    cin >> peso[i] >> num;
    for(int j = 0; j < num; j++)
    {
        int a;
        cin >> a;
        grafo[a-1].push_back(i);
        in_deg[i]++;
    }
}
```

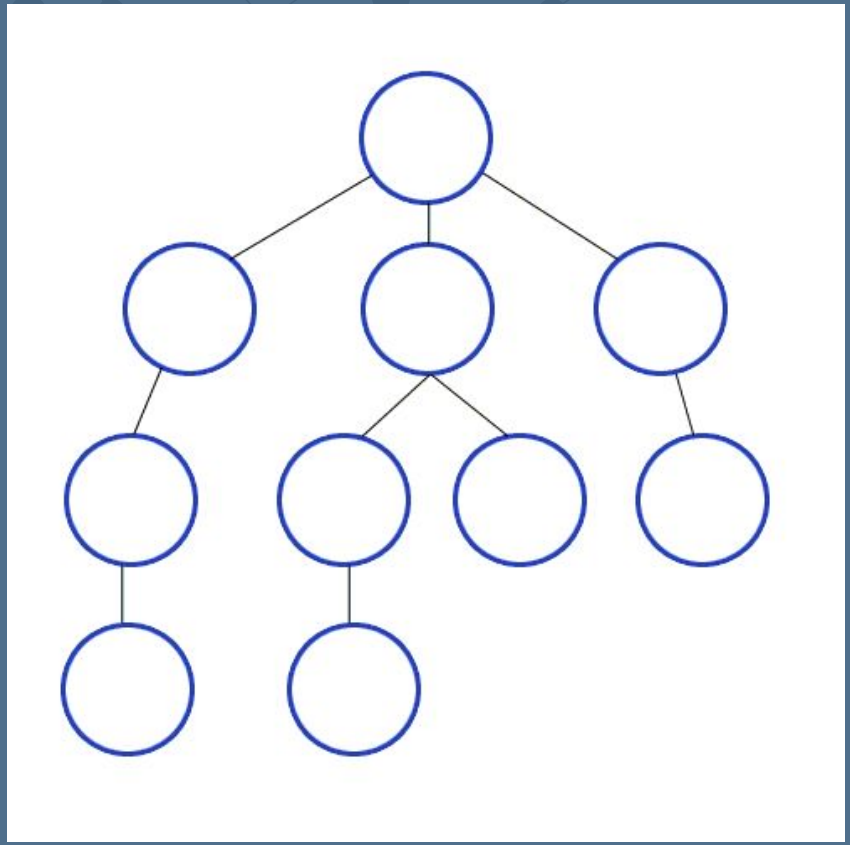
Visiting a graph - DFS



- Implemented with an explicit stack (LIFO)
- Implemented with an implicit stack (recursion)

```
dfs(GRAPH G, NODE u, boolean[] visited)
visited[u] = true
{ visita il nodo u (pre-order) }
foreach v ∈ G.adj(u) do
    if not visited[v] then
        { visita l'arco (u, v) }
        dfs(G, v, visited)
{ visita il nodo u (post-order) }
```

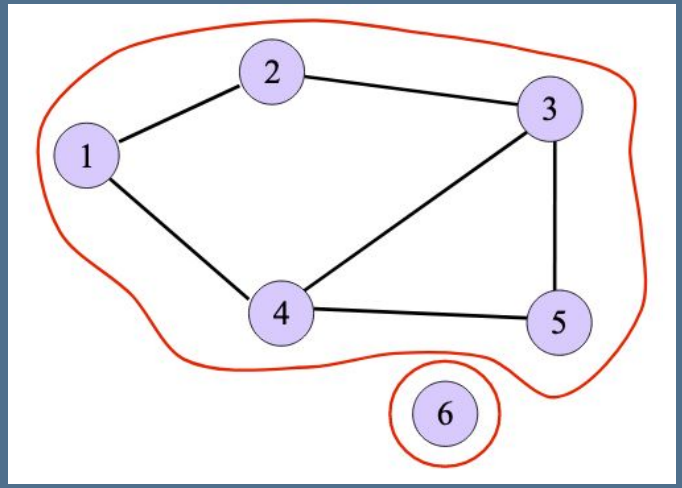
Visiting a graph - BFS



- Implemented with an explicit queue (FIFO)

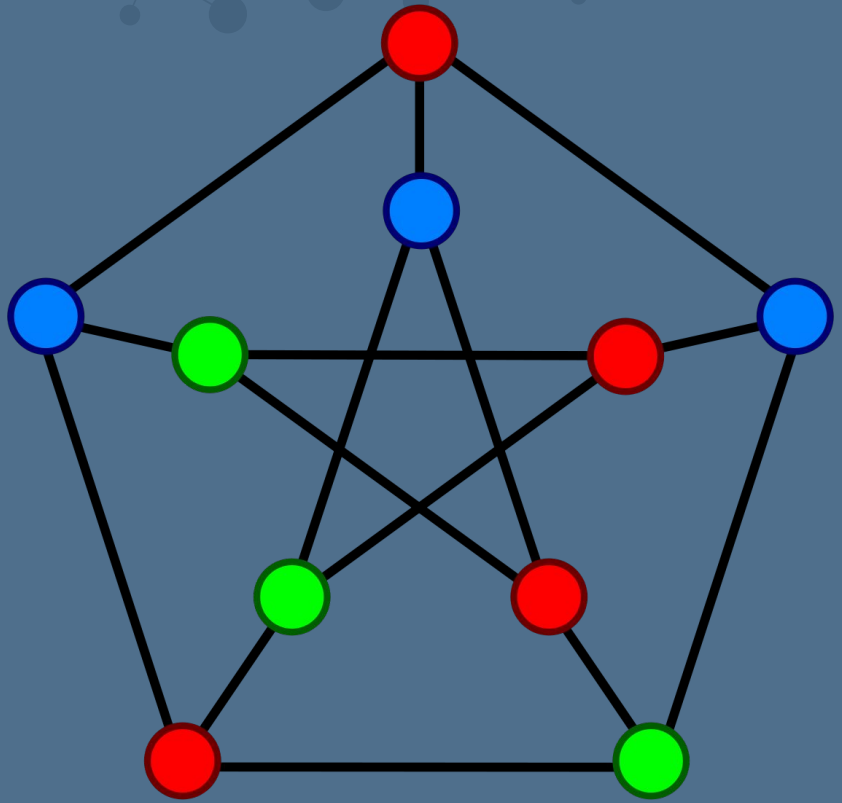
```
bfs(GRAPH G, NODE r)
QUEUE Q = Queue()
S.enqueue(r)
boolean[] visited = new boolean[G.size()]
foreach u ∈ G.V() − {r} do
    visited[u] = false
visited[r] = true
while not Q.isEmpty() do
    NODE u = Q.dequeue()
    { visita il nodo u}
    foreach v ∈ G.adj(u) do
        { visita l'arco (u, v) }
        if not visited[v] then
            visited[v] = true
            Q.enqueue(v)
```

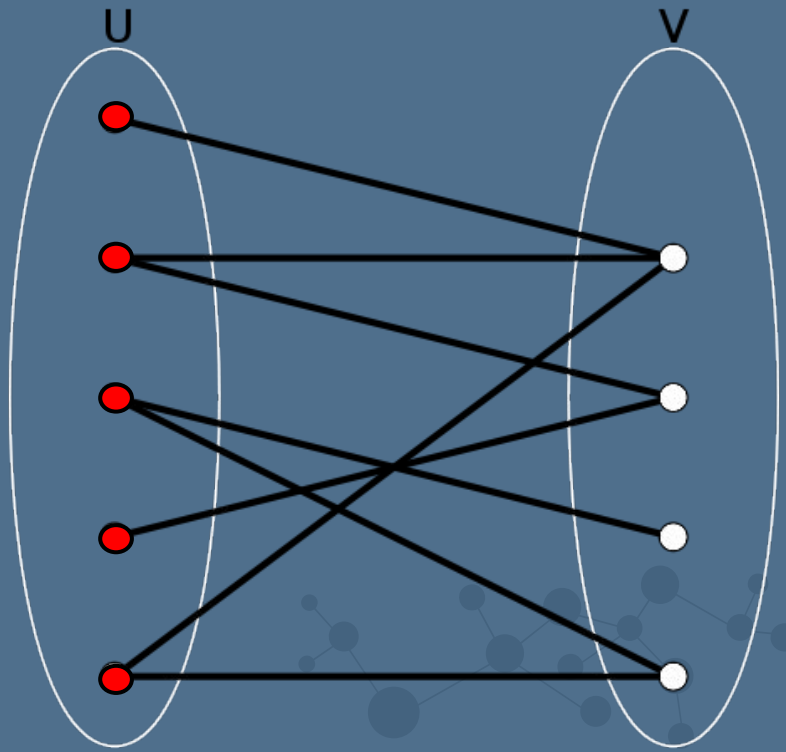
Connected Components



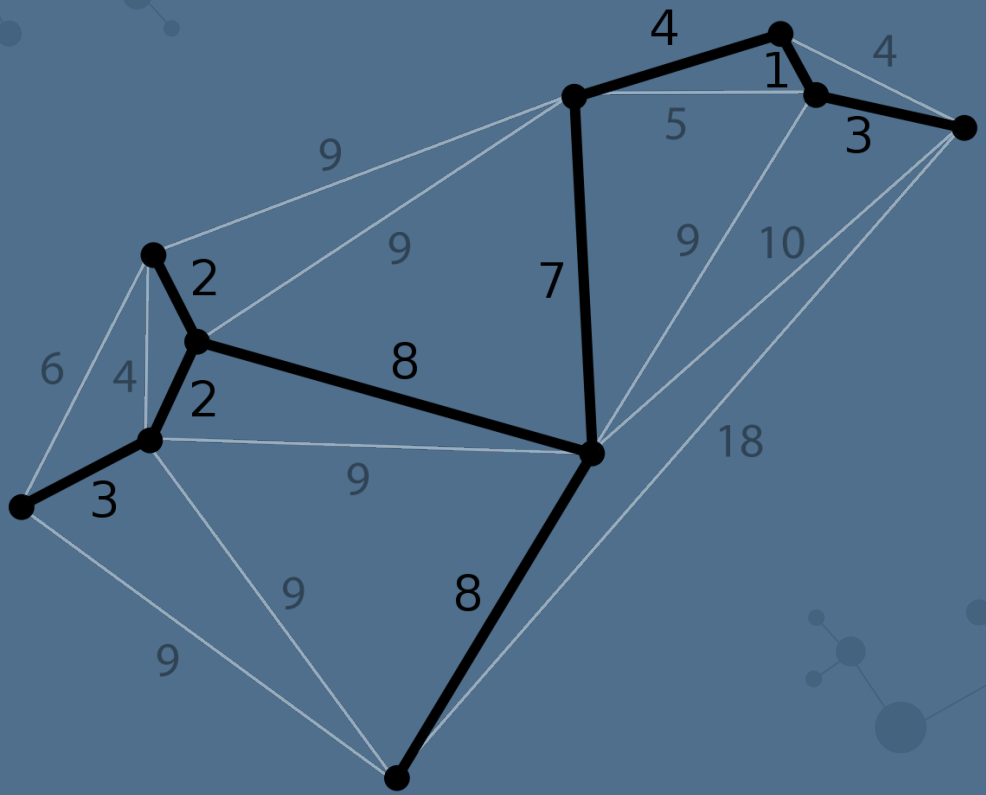

```
int[] cc(GRAPH G)
int[] id = new int[G.size()]
foreach u ∈ G.V() do
    id[u] = 0
int counter = 0
foreach u ∈ G.V() do
    if id[u] == 0 then
        counter = counter + 1
        ccdfs(G, counter, u, id)
return id
```

Coloring and Bipartite Checking





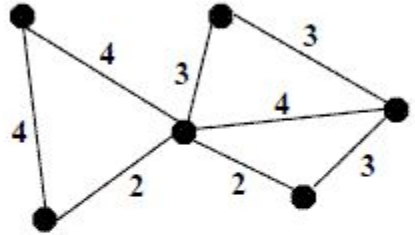
Minimum Spanning Tree



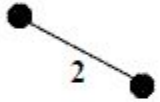
Kruskal's Algorithm

Kruskal's Algorithm

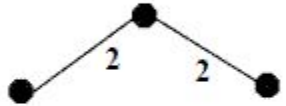
1 *Given a network……*



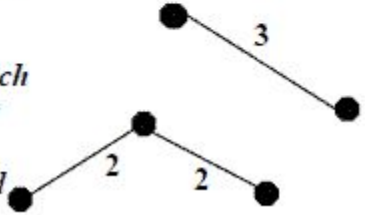
2 *Choose the shortest edge (if there is more than one, choose any of the shortest)……..*



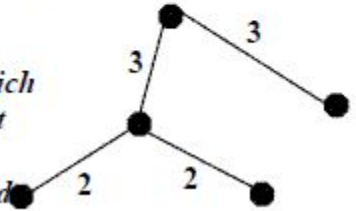
3 *Choose the next shortest edge and add it……..*



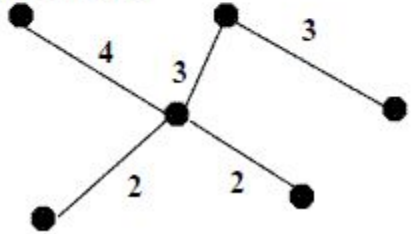
4 *Choose the next shortest edge which wouldn't create a cycle and add it.*



5 *Choose the next shortest edge which wouldn't create a cycle and add it.*



6 *Repeat until you have a minimal spanning tree.*



Kruskal's Algorithm

- MAKE-SET(v) puts v in a set by itself
- FIND-SET(v) returns the name of v's set
- UNION(u, v) combines the sets that u and v are in

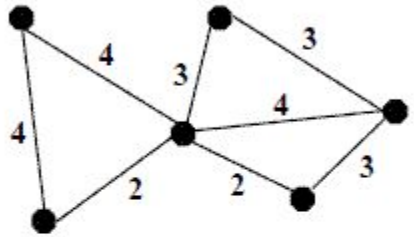
MST-Kruskal(G, w)

```
1  A ← ∅
2  for each vertex v ∈ V[G]
3        do MAKE-SET(v)
4  sort the edges of E into nondecreasing order by weight w
5  for each edge (u, v) ∈ E, taken in nondecreasing order by weight
6        do if FIND-SET(u) ≠ FIND-SET(v)
7              then A ← A ∪ {(u, v)}
8                   UNION(u, v)
9  return A
```

Prim's Algorithm

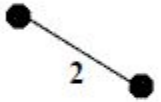
Prim's Algorithm

1 *Given a network……*

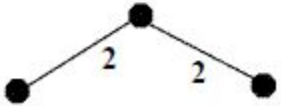


2 *Choose a vertex*

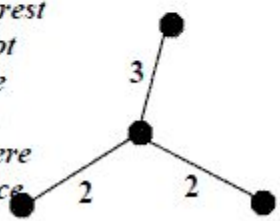
3 *Choose the shortest edge from this vertex.*



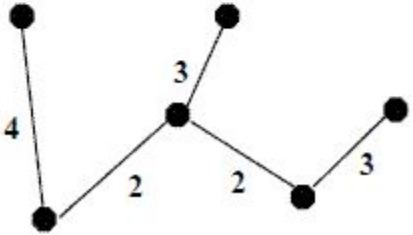
4 *Choose the nearest vertex not yet in the solution.*



5 *Choose the next nearest vertex not yet in the solution, when there is a choice choose either.*



6 *Repeat until you have a minimal spanning tree.*



Prim's Algorithm

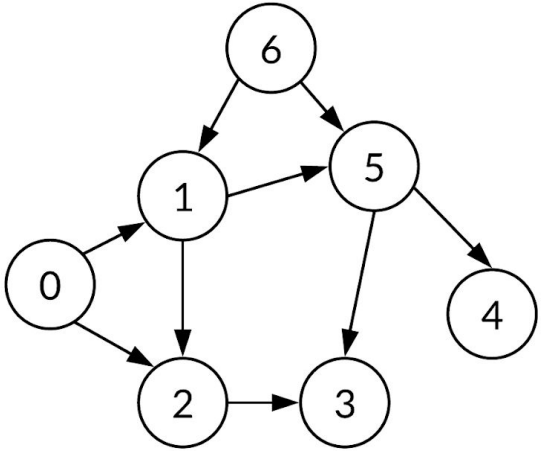
- INSERT(v) **puts** v **in the structure**
- EXTRACT-MIN() **finds and returns the node with minimum key value**
- DECREASE-KEY(v, w) **updates (decreases) the key of** v

```
MST-Prim(G, w, r)
 1  for each u ∈ V[G]
 2        do key[u] ← ∞
 3           π[u] ← NIL
 4  key[r] ← 0
 5  Q ← V[G]
 6  while Q ≠ ∅
 7        do u ← EXTRACT-MIN(Q)
 8           for each v ∈ Adj[u]
 9                 do if v ∈ Q and w(u, v) < key[v]
10                       then π[v] ← u
11                            key[v] ← w(u, v)
```

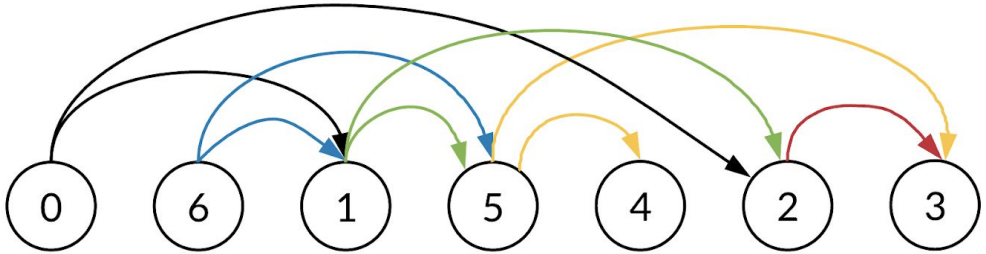
Topological Sort

A topological sort of a directed graph is a linear ordering of its vertices such that for every directed edge uv from vertex *u* to vertex *v*, *u* comes before *v* in the ordering. Only possible in there are no cycles.
Applications: precedence of events, ...

Unsorted graph



Topologically sorted graph



Topological Sort based on DFS

- Consider nodes with no incoming edges.
- Start a DFS and store in a stack the nodes at exit time.
- Iterate over the stack to get a possible topological ordering.

Thanks!

Identifying Information

MARS

Despite being red, Mars is a cold place full of iron oxide dust

JUPITER

It’s a gas giant and the biggest planet in the Solar System

VENUS

Venus has a beautiful name and a poisonous atmosphere

Patient Medical History

Age: 75

Gender: Male

Allergies: None

Location:
Madrid, Spain

2005	2008	2014
Despite being red, Mars is a cold place	Mercury is the smallest planet	Saturn is a gas giant and has rings

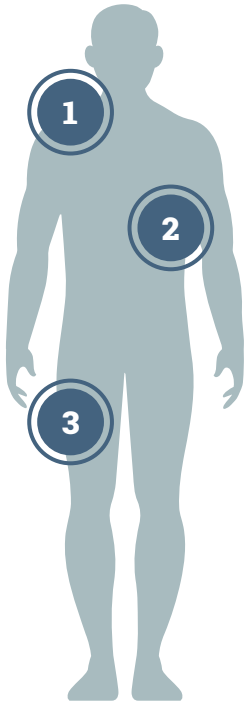
Physical Examination

1. SATURN

It’s a gas giant and has rings around itself

3. JUPITER

Jupiter is the biggest planet in the Solar System



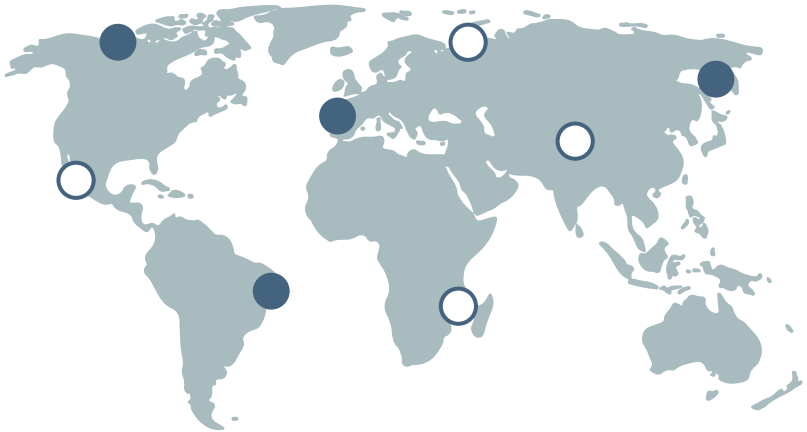
2. MARS

Mars is actually a cold place full of iron oxide dust

A Picture Is Worth a Thousand Words

Findings

EVOLUTION



Mars Venus

PREVALENCE

To modify this graph, click on it, follow the link, change the data and paste the new graph here

Discussion

DR. JENNA DOE

“Saturn is a gas giant and has several rings”

DR. JOHN JAMES

“Jupiter is the biggest planet in the Solar System”

Discussion Summary

Mercury is the closest planet to the Sun and the smallest one in the Solar System. It’s only a bit larger than the Moon.

The planet’s name has nothing to do with the liquid metal, since Mercury was named after the Roman messenger god

Comparison

	JUPITER	MERCURY	VENUS	MARS
	✗	✗	✓	✓
	✓	✓	✓	✓
	✓	✗	✓	✓
	✗	✓	✓	✗
	✗	✓	✓	✗

Diagnosis

1

MERCURY

Mercury is the closest planet to the Sun

2

VENUS

Venus is the second planet from the Sun

3

MARS

Despite being red, Mars is actually a cold place

4

JUPITER

Jupiter is a gas giant and the biggest planet

5

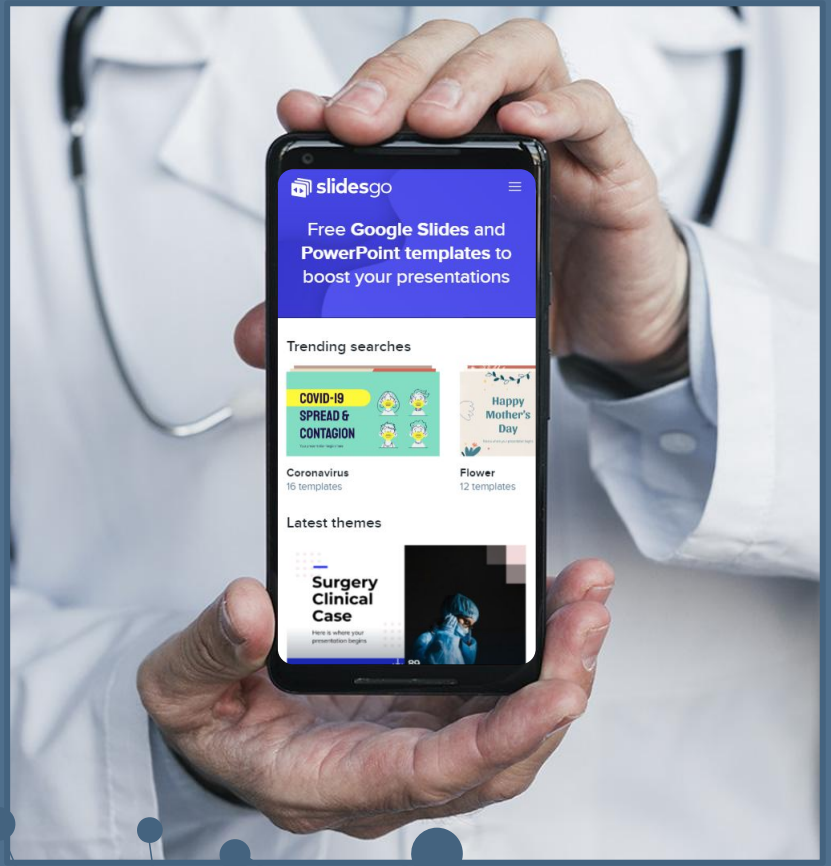
SATURN

It’s composed of hydrogen and helium

6

NEPTUNE

Neptune is the farthest planet from the Sun



Factors to Consider

Mercury is the closest planet to the Sun and the smallest one in the Solar System—it's only a bit larger than the Moon

Treatment

Neptune is the farthest planet from the Sun

01

Despite being red, Mars is actually a cold place

02

Jupiter is the biggest planet in the Solar System

Patient Monitoring

WEEK 1

Despite being red, Mars is actually a cold place

WEEK 2

Venus is the second planet from the Sun

WEEK 3

Mercury is the closest planet to the Sun

WEEK 4

Earth is the third planet from the Sun

8,300,000

Big numbers catch your audience’s attention

Contraindications & Indications

- Here you can describe the reason to start the treatment
- Here you can describe the reason to start the treatment

- Here you can describe the reason to stop the treatment
- Here you can describe the reason to stop the treatment

Post-Prevention

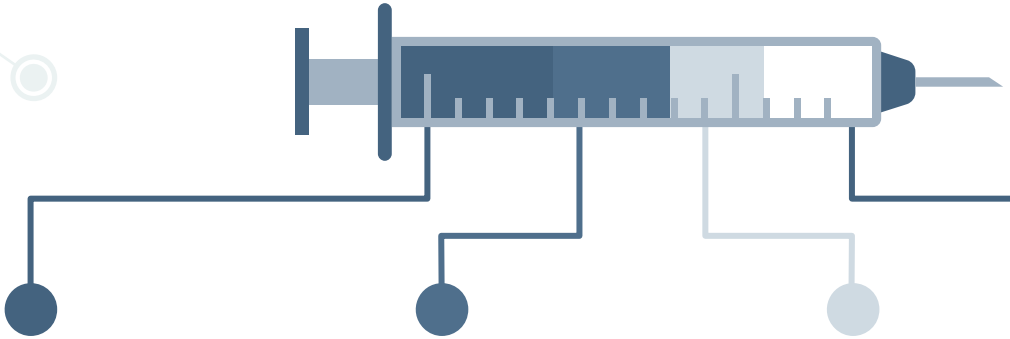
Jupiter is a gas giant and the biggest planet

Mercury is the closest planet to the Sun

Neptune is the farthest planet from the Sun

Venus has a beautiful name, but it’s very hot

Case Timeline



Mercury is the closest planet to the Sun

Despite being red, Mars is actually a cold place

Neptune is the farthest planet from the Sun

Earth is the third planet from the Sun

Conclusions

Mercury is the closest planet to the Sun and the smallest one in the Solar System—it’s only a bit larger than the Moon

References

- AUTHOR (YEAR). *Title of the publication*. Publisher
- AUTHOR (YEAR). *Title of the publication*. Publisher
- AUTHOR (YEAR). *Title of the publication*. Publisher
- AUTHOR (YEAR). *Title of the publication*. Publisher
- AUTHOR (YEAR). *Title of the publication*. Publisher
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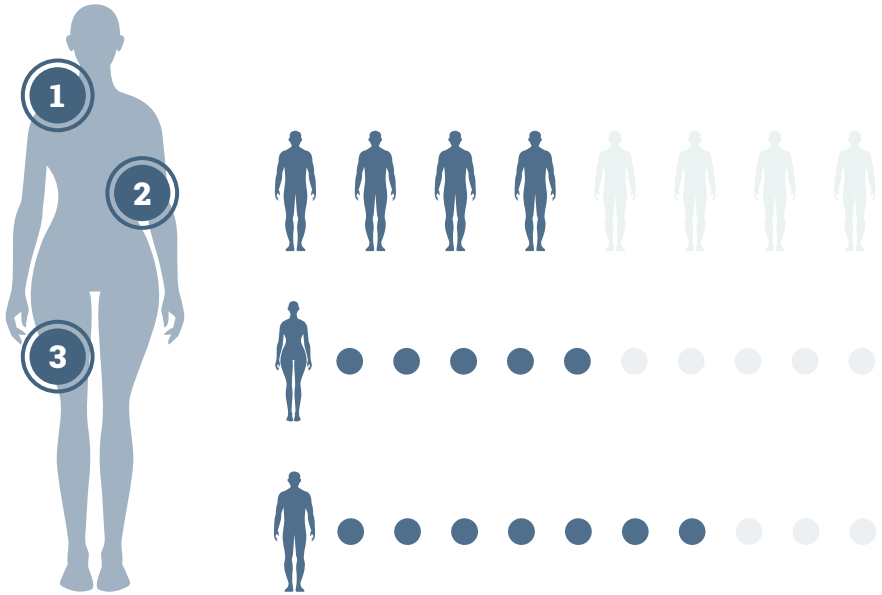
Thanks!

DO YOU HAVE ANY QUESTIONS?

youremail@freepik.com
+91 620 421 838
yourcompany.com

Alternative Resources



Resources

PHOTOS

- Doctor holding stethoscope
- Doctor sitting desk
- Medical team doctor’s office
- Doctor analyzing with microscope
- Medical team doctor’s office
- Nurse pointing window while talking old man
- Doctor’s hand tending patient
- Nurse writing her clipboard
- Young doctor writing report
- Doctor showing mobile phone

VECTORS

- Geometric background molecules line
- Medical infographic collection template
- Colorful medical infographic pack
- Medical infographic template with human body

Instructions for use

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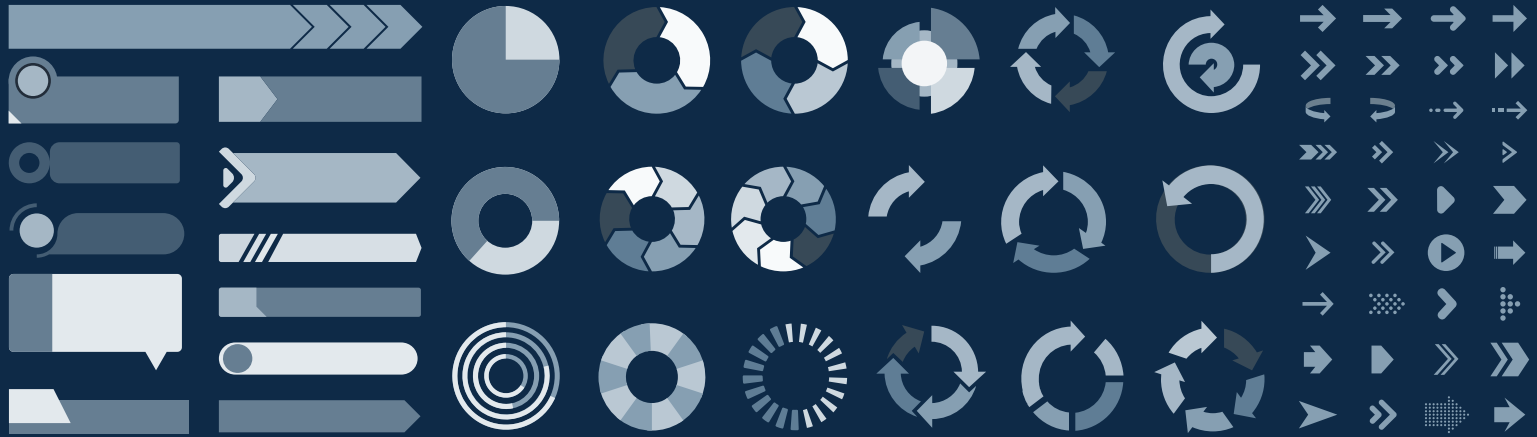






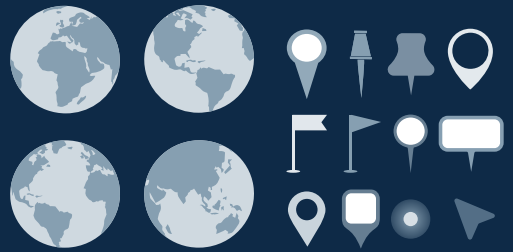

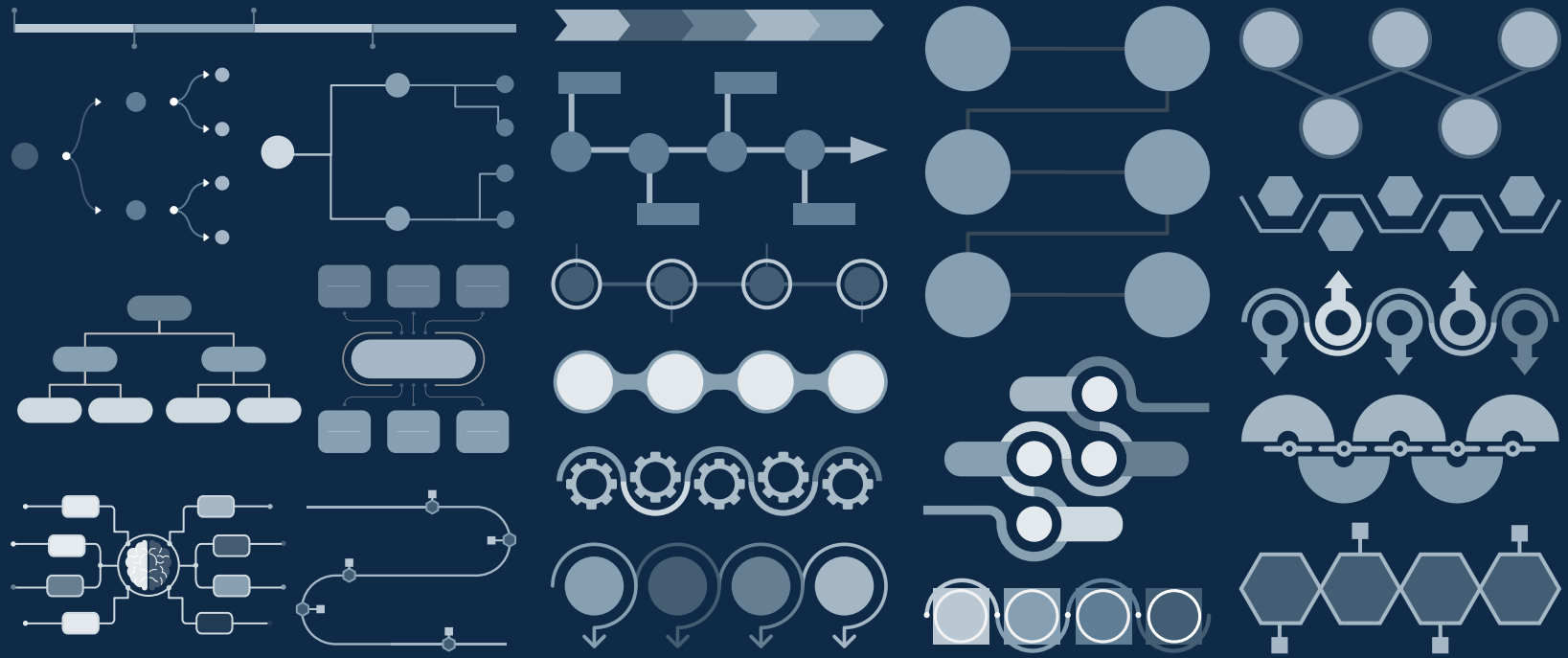

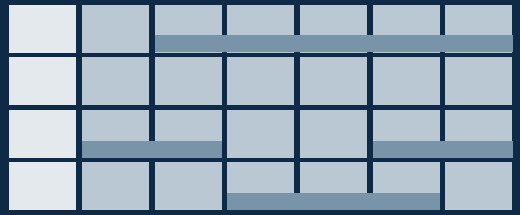

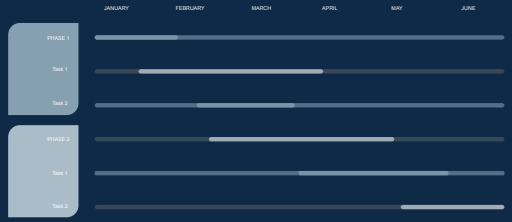
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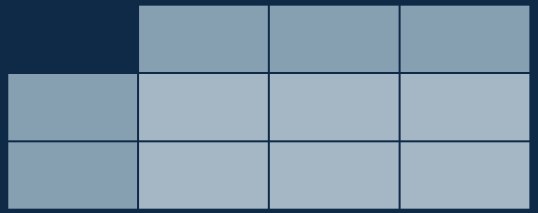

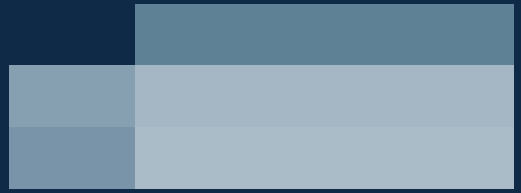

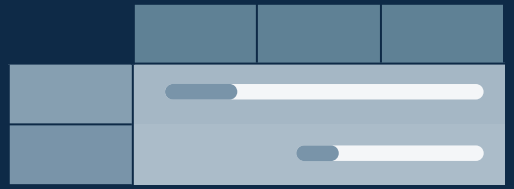

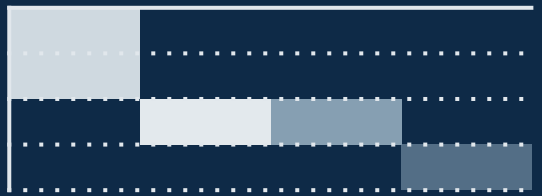

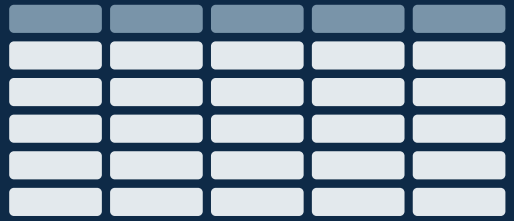

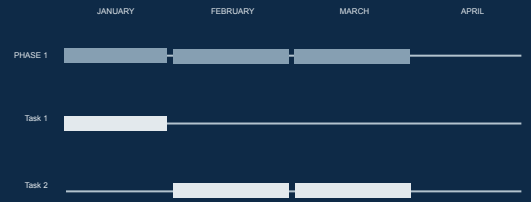
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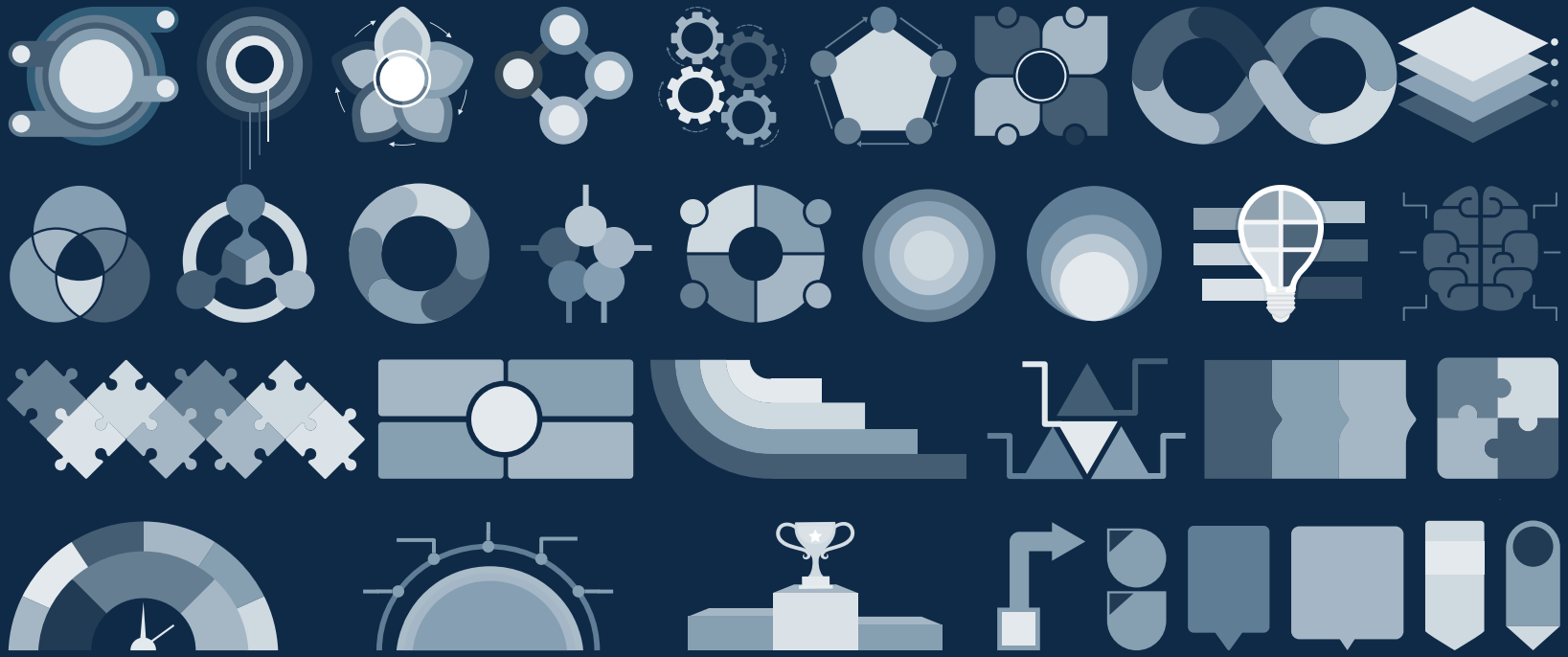
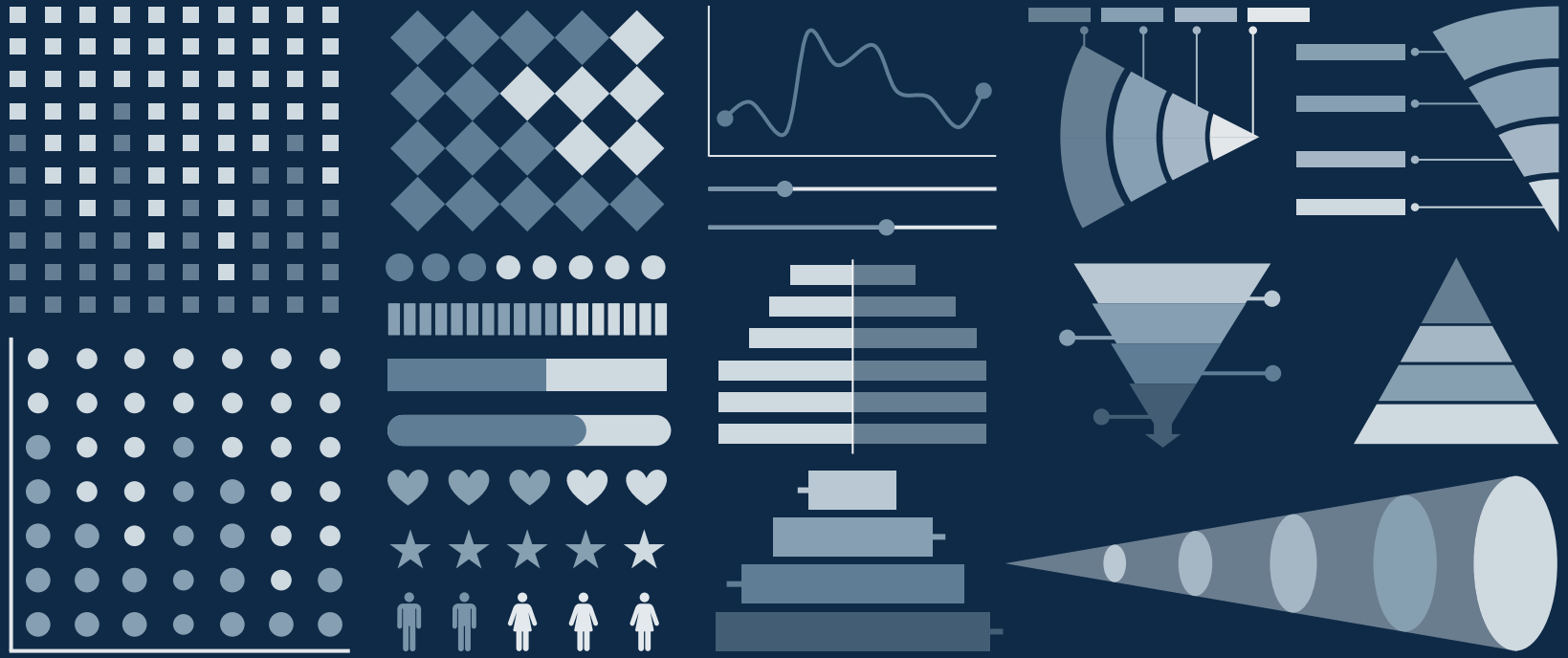
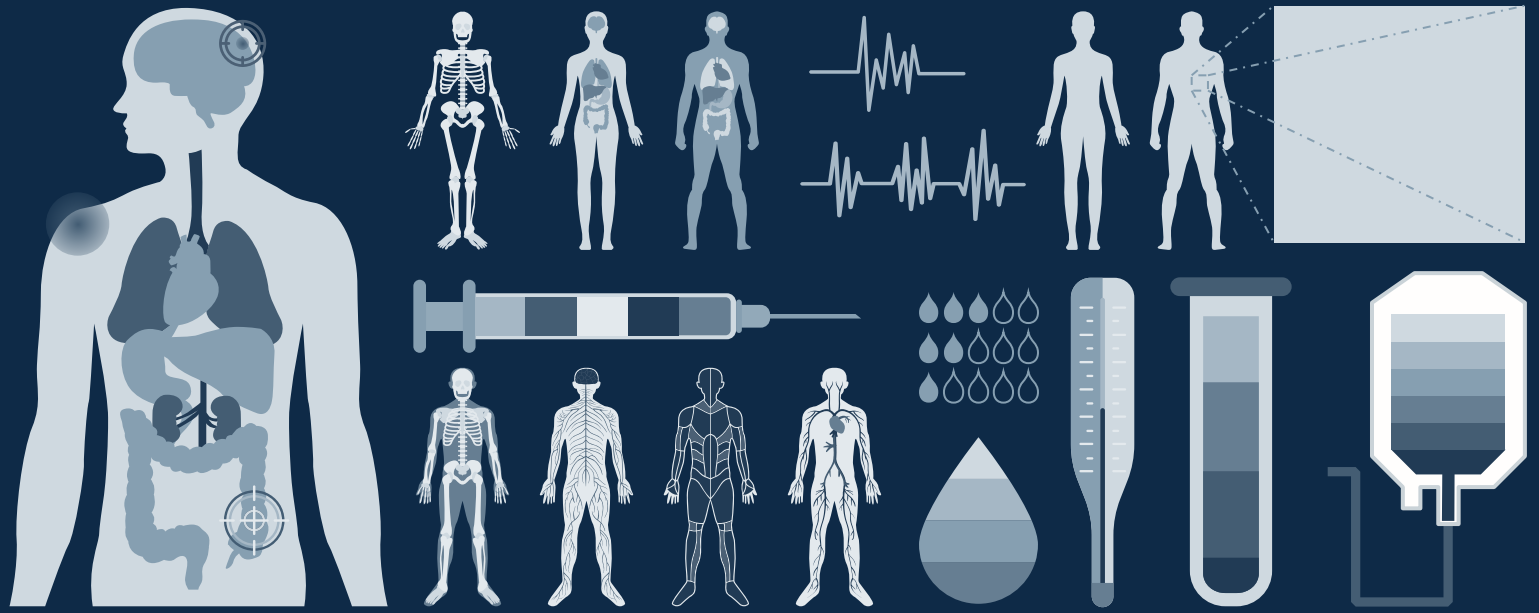
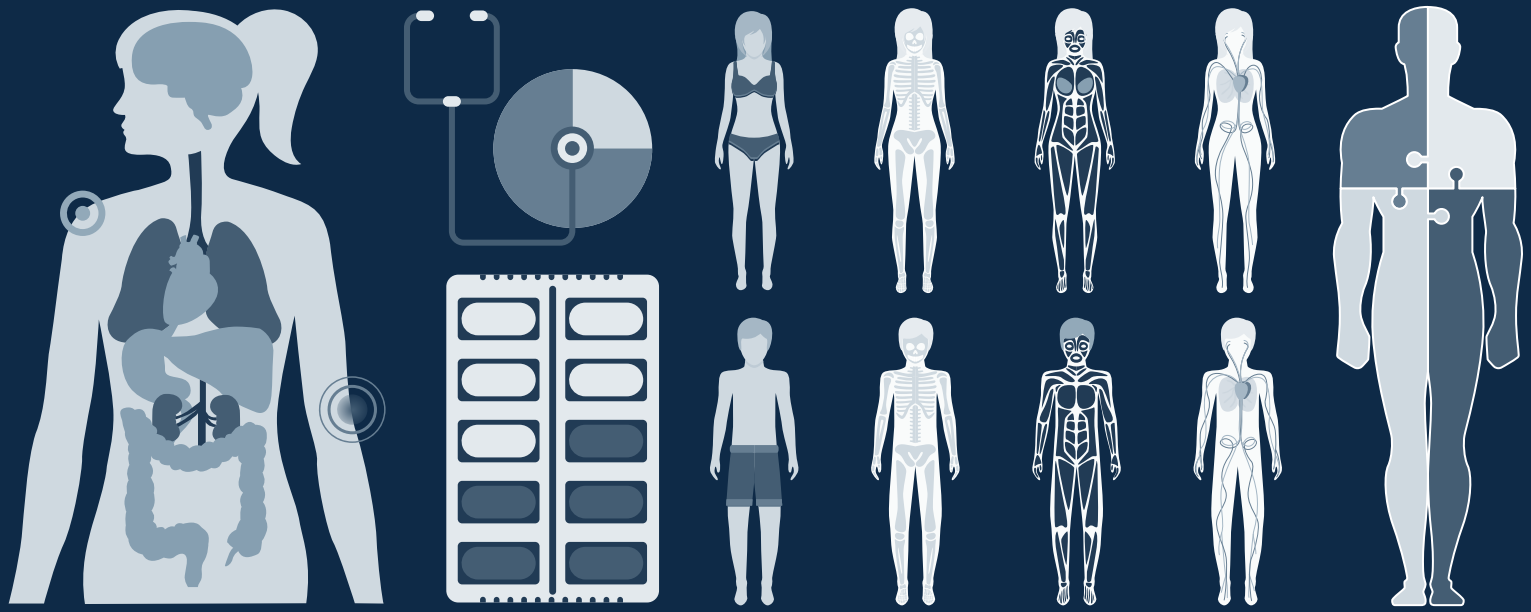
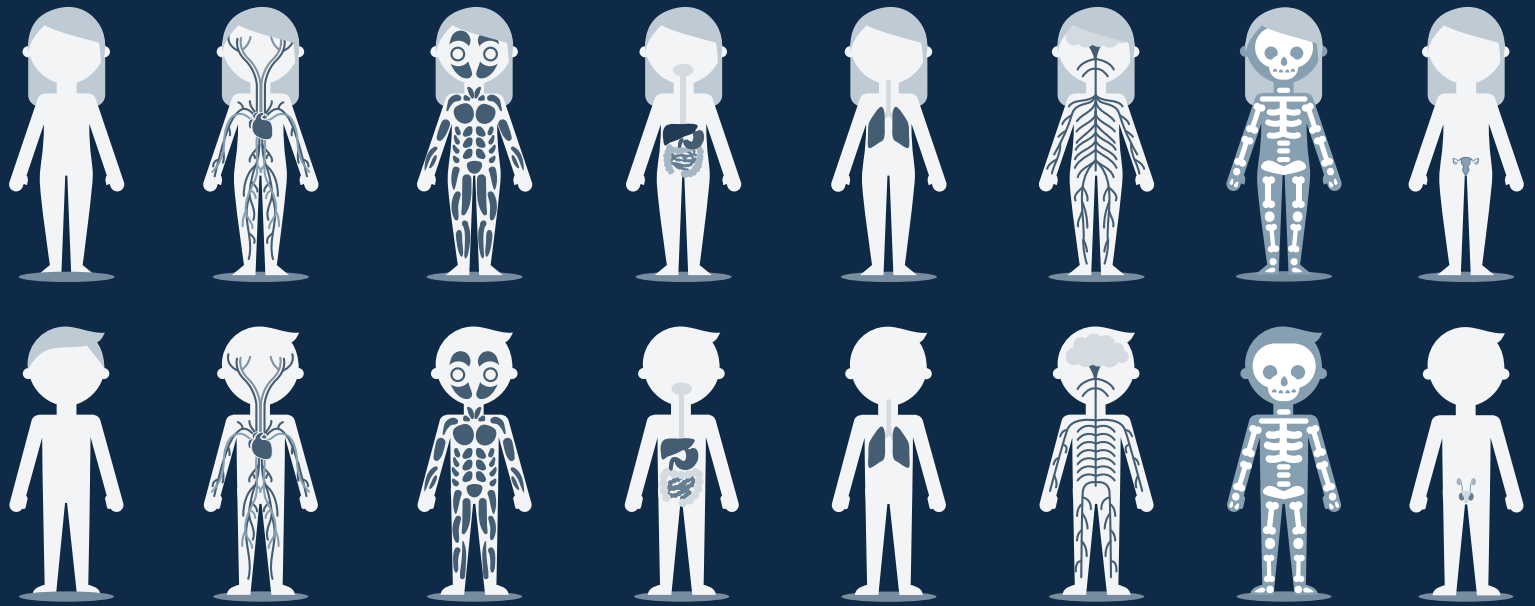




Medical Infographics







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