LENS COMBINATION FORMULAS

Many optical tasks require several lenses in order to achieve an acceptable level of performance. One possible approach to lens combinations is to consider each image formed by each lens as the object for the next lens and so on. This is a valid approach, but it is time consuming and unnecessary.

It is much simpler to calculate the effective (combined) focal length and principal-point locations and then use these results in any subsequent paraxial calculations (see figure 4.8). They can even be used in the optical invariant calculations described in the preceding section.

EFFECTIVE FOCAL LENGTH

The following formulas show how to calculate the effective focal length and principal-point locations for a combination of any two arbitrary components. The approach for more than two lenses is very simple: Calculate the values for the first two elements, then perform the same calculation for this combination with the next lens. This is continued until all lenses in the system are accounted for.

The expression for the combination focal length is the same whether lens separation distances are large or small and whether f_1 and f_2 are positive or negative:

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} .$$ (4.16)

This may be more familiar in the form

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} .$$ (4.17)

Notice that the formula is symmetric with respect to the interchange of the lenses (end-for-end rotation of the combination) at constant d. The next two formulas are not.

COMBINATION FOCAL-POINT LOCATION

For all values of f_1, f_2, and d, the location of the focal point of the combined system (s_2''), measured from the secondary principal point of the second lens (H_2''), is given by

$$s_2'' = \frac{f_2(f_1 - d)}{f_1 + f_2 - d} .$$ (4.18)

This can be shown by setting $s_1 = d - f_1$ (see figure 4.8a), and solving

$$\frac{1}{f_2} = \frac{1}{s_1} + \frac{1}{s_2''}$$

for s_2''.

COMBINATION SECONDARY PRINCIPAL-POINT LOCATION

Because the thin-lens approximation is obviously highly invalid for most combinations, the ability to determine the location of the secondary principal point is vital for accurate determination of d when another element is added. The simplest formula for this calculates the distance from the secondary principal point of the final (second) element to the secondary principal point of the combination (see figure 4.8b):

$$z = s_2'' - f .$$ (4.19)

COMBINATION EXAMPLES

It is possible for a lens combination or system to exhibit principal planes that are far removed from the system. When such systems are themselves combined, negative values of d may occur. Probably the simplest example of a negative d-value situation is shown in figure 4.9. Meniscus lenses with steep surfaces have external principal planes. When two of these lenses are brought

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into contact, a negative value of d can occur. Other combined-lens examples are shown in figures 4.10 through 4.13.

Symbols

f_c = combination focal length (EFL), positive if combination final focal point falls to the right of the combination secondary principal point, negative otherwise (see figure 4.8c).

f_1 = focal length of the first element (see figure 4.8a).

f_2 = focal length of the second element.

d = distance from the secondary principal point of the first element to the primary principal point of the second element, positive if the primary principal point is to the right of the secondary principal point, negative otherwise (see figure 4.8b).

$s_1"$ = distance from the primary principal point of the first element to the final combination focal point (location of the final image for an object at infinity to the right of both lenses), positive if the focal point is to left of the first element's primary principal point (see figure 4.8d).

$s_2"$ = distance from the secondary principal point of the second element to the final combination focal point (location of the final image for an object at infinity to the left of both lenses), positive if the focal point is to the right of the second element's secondary principal point (see figure 4.8b).

z_H = distance to the combination primary principal point measured from the primary principal point of the first element, positive if the combination secondary principal point is to the right of secondary principal point of second element (see figure 4.8d).

$z_H"$ = distance to the combination secondary principal point measured from the secondary principal point of the second element, positive if the combination secondary principal point is to the right of the secondary principal point of the second element (see figure 4.8c).

Note: These paraxial formulas apply to coaxial combinations of both thick and thin lenses immersed in air or any other fluid with refractive index independent of position. They assume that light propagates from left to right through an optical system.
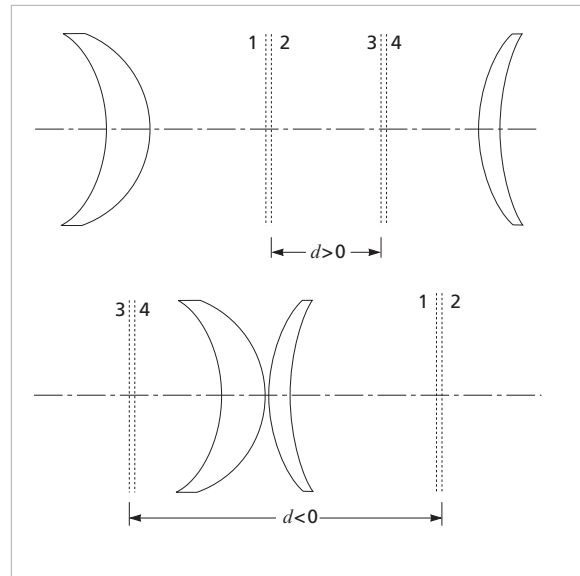

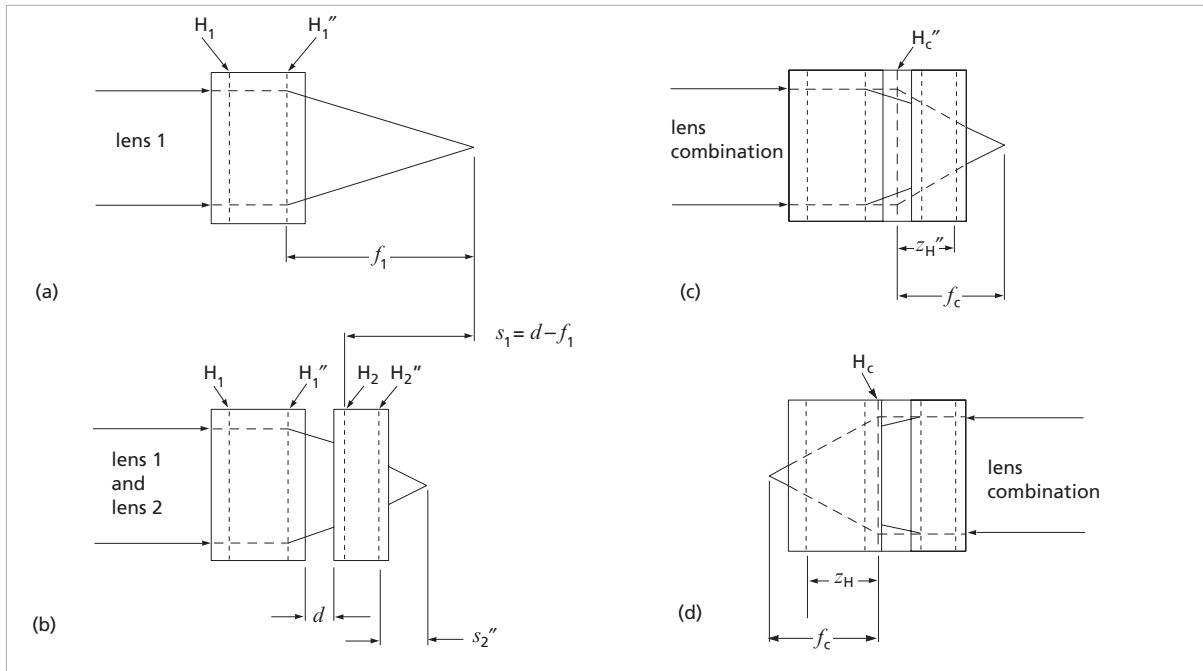Figure 4.9   "Extreme" meniscus-form lenses with external principal planes (drawing not to scale)

Optical Coatings & Materials

Material Properties

Optical Specifications

Fundamental Optics

Gaussian Beam Optics

Machine Vision Guide

Laser Guide

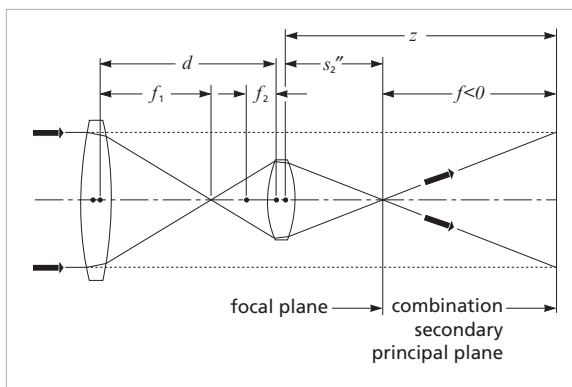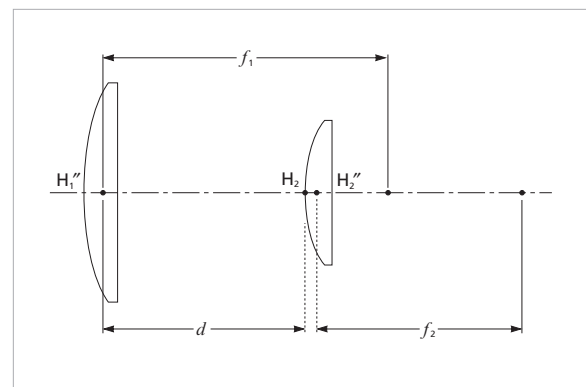Figure 4.8   Lens combination focal length and principal planes

Figure 4.10   Positive lenses separated by distance greater than $f_1 = f_2$: f is negative and both s_2'' and z are positive. Lens symmetry is not required.

Figure 4.11   Achromatic combinations: Air-spaced lens combinations can be made nearly achromatic, even though both elements are made from the same material. Achieving achromatism requires that, in the thin-lens approximation,

$$d = \frac{(f_1 + f_2)}{2}.$$

This is the basis for Huygens and Ramsden eyepieces.

This approximation is adequate for most thick-lens situations. The signs of f_1, f_2, and d are unrestricted, but d must have a value that guarantees the existence of an air space. Element shapes are unrestricted and can be chosen to compensate for other aberrations.

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