

# HOPFIELD NETWORK HYPERPARAMETER STUDY (PLAIN-LANGUAGE NOTES)

## WHAT WE WANT TO KNOW

- How the code boosts or shrinks the strength of the outside input ('w\_external').  
\*Outside input = the  $\hat{a}$  push $\hat{a}$  coming from stimuli.\*
- How the code boosts or shrinks the internal feedback ('w\_s').  
\*Internal feedback = neurons reinforcing each other to stay in a memory.\*
- Where random noise enters the maths.  
\*Noise = random wiggles that can shake the system into a new state.\*
- Where we could plug in controls for these three knobs and later add scoring.

Line numbers below refer to the files in this repository snapshot.

## WHERE THE IMPORTANT PIECES LIVE

### MEMORY GENERATOR ('HNN\_GEN')

'HNN\_Gen.HNN' ('HNN\_Gen.py:12-37') builds the stored patterns ( $\hat{a}$  memories $\hat{a}$ ) and sets the initial neuron activity. The network size is 'self.N = 2\*\*6' (64 neurons). The number of memories is 'self.P = 10'. 'self.eps' sets the size of the random kick around each stored memory. Building the Hebbian matrix ('mems @ mems $\epsilon\mu$ ') sets the baseline internal feedback.

### STOCHASTIC INTEGRATOR ('EUL\_MAY.EM')

'EM.Eu\_Ma\_Test' ('Eul\_May.py:37-81') steps the activity forward in time using the Euler $\hat{a}$  Maruyama method.

\*Euler $\hat{a}$  Maruyama = a recipe for simulating noisy differential equations.\*

Each step combines:

- Recurrent drive 'np.dot(W, z)' where 'z = tanh(delta \* y)' ('Eul\_May.py:87-107').  
\*tanh' squashes values between  $\hat{a}$  1 and 1 so the neurons saturate gently.\*
- Leak '-y', which pulls neurons back toward zero.
- External input 'u', included when we pick additive mode ('C == 'A').
- Noise term 'sigma \* random', where 'sigma' controls the noise strength.

Noise enters through the Wiener increment ('Eul\_May.py:79').

\*Wiener increment = the small random jump that models Brownian motion.\*

### EXPERIMENT DRIVER ('MAINHNN\_SHB')

'MainHNN\_SHB' ('MainHNN\_SHB.py:40-125') sets time span, generates memories, and builds the external input.

'u\_proto' ('MainHNN\_SHB.py:62-84') contains amplitudes for each memory and each stimulus slot. The diagonal entries (2.0 $\hat{a}$  3.5) push the intended memory; small off-diagonal terms (0.05 $\hat{a}$  0.2) create gentle cross-talk. Combining 'u\_proto' with the memory patterns yields

'u\_tran', the actual input sent to the integrator. The list 'sigma = [0.5]' sets noise levels for repeats.

## AUTAPSE TOY MODEL ('PARETO/FIGURES/FIG\_MODEL.PY')

This self-feedback toy system introduces clear knobs 'w\_in' (external weight) and 'w\_self' (internal weight) ('pareto/figures/fig\_model.py:33-68').

\*Autapse = a neuron that talks to itself, used here as a simple test bed.\*

The update rule:

[CODE BLOCK]

```
x = x + dt * (-x - leak + w_in * h + w_self * activation(x)) + dt * sigma_n * random
```

[CODE BLOCK]

It then sweeps these knobs to show a trade-off: stronger input speeds up responses but can shorten memory ('pareto/figures/fig\_model.py:183-369').

## HOW THE THREE KNOBS APPEAR

### EXTERNAL INPUT STRENGTH ('W\_EXTERNAL')

- Right now the strength lives inside 'u\_proto' ('MainHNN\_SHB.py:62-84'). There is no single scalar knob; each stimulus draw sets its own value.
- 'EM.hop\_field\_test' adds the vector 'u' directly ('Eul\_May.py:107'). To mirror Dan's 'w\_external', we can wrap it as 'w\_external \* u'.
- The autapse model already exposes 'w\_in' ('pareto/figures/fig\_model.py:33-46'), and later plots assume the total input weight is 'w\_self + w\_in'.

**\*\*Takeaway:\*\*** add a global multiplier, either when computing 'u\_tran' or inside 'hop\_field\_test', so experiments can scale outside input with one parameter.

### INTERNAL COUPLING STRENGTH ('W\_S')

- The recurrent drive comes from the Hebbian matrix  $W = (1/N) * M @ M^T$  ('Eul\_May.py:104-105').
- \*Hebbian = neurons that fire together wire together; this matrix encodes that rule.\*
- A new knob 'w\_s' could multiply 'np.dot(W, z)' to deepen or flatten the attractor wells.
- \*Attractor well = a stable state the system tends to fall into.\*
- In the autapse toy, 'w\_self' already plays this role.

### NOISE LEVEL ('SIGMA')

- 'sigma' passes through every call to 'Eu\_Ma\_Test' and multiplies the random part of the step ('Eul\_May.py:79').
- Adjusting the 'sigma' list in 'MainHNN\_SHB' lets you run several noise settings back to back.

## HOW THESE KNOBS INTERACT

1. **External input vs. stability:** Larger diagonal entries in 'u\_proto' make the network snap faster to the target memory, but can overpower the internal matrix if cross-talk is too big. The autapse sweeps show the same story: higher 'w\_in' raises responsiveness but shrinks memory duration ('pareto/figures/fig\_model.py:183-260').
2. **Internal gain vs. flexibility:** A bigger 'w\_s' would make attractor wells deeper, giving longer recall but slower switching. Autapse plots confirm that higher 'w\_self' stretches memory but slows response ('pareto/figures/fig\_model.py:307-365').
3. **Noise vs. control:** 'sigma' helps the system leave a memory when the input changes, but too much noise destroys stored patterns. Sweeping 'sigma' will highlight where the model shifts from *stuck* to *chaotic*.

## PRACTICAL SUGGESTIONS

1. Add explicit knobs 'w\_external' and 'w\_s' in 'EM.hop\_field\_test' so every experiment can tune them without changing random seeds or inputs.
2. When building 'u\_proto', multiply by the global 'w\_external' so different runs stay comparable.
3. Borrow the simple metrics from the autapse scripts (response rate, memory length, reaction time) and apply them to the overlap traces produced in 'HNPlot'.

## LINK TO THE SCIENCE ADVANCES PAPER

The repository follows *Stimulus-Driven Dynamics for Robust Memory Retrieval in Hopfield Networks* (Science Advances, doi:10.1126/sciadv.adu6991). The paper discusses balancing quick stimulus-driven retrieval against stable memory storage. Our code reflects that balance: stimuli ('MainHNN\_SHB.py:62-125') drive switching, recurrent connections and noise ('Eul\_May.py:79-107') control how long memories stick. Putting the 'w\_external', 'w\_s', and 'sigma' knobs on the surface will let us explore the same trade-off in a controlled way.

## NEXT STEPS

- Wire in the new scaling knobs and expose them through configuration so sweeps are simple.
- Reuse the autapse scoring logic to measure responsiveness and memory retention directly on the Hopfield outputs.

- Run structured sweeps (for example, a grid over 'w\_external' and 'w\_s') and log results into the time-stamped folders in 'outputs/'.