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ממ"ן 11

1

Prove:

$A\Delta B\subseteq D \wedge B\Delta C\subseteq D \to A\Delta C\subseteq D$

Since $(X\to Z) \wedge (Y\to Z) \equiv (X\vee Y)\to Z$, also:

$(A\Delta B\subseteq D \wedge B\Delta C\subseteq D) \equiv (A\Delta B \cup B\Delta C) \subseteq D$. Proof:

$(X\to Z) \wedge (Y\to Z)$

$Z\vee\neg X \wedge Z\vee\neg Y$

$Z\vee\neg(X\vee Y)$

$(X\vee Y) \to Z$

$A\Delta B\subseteq D \wedge B\Delta C\subseteq D$

$(x\in(A\Delta B) \to x\in D) \wedge (x\in(B\Delta C) \to x\in D)$

$x\in(A\Delta B \cup B\Delta C) \to x\in D$

$(A\Delta B \cup B\Delta C) \subseteq D$

Therefore:

$(A\Delta B\subseteq D \wedge B\Delta C\subseteq D) \equiv (A\Delta B \cup B\Delta C) \subseteq D$ // (1)

I'll prove that $A\Delta C \subseteq (A\Delta B \cup B\Delta C)$, then it would follow by transience that $A\Delta C\subseteq C$

Expanding $(A\Delta B \cup B\Delta C)$:

$(\overline{B} \cap A) \cup (\overline{B} \cap C) \cup (B\cap \overline{A}) \cup (B\cap \overline{C})$

$[\overline{B} \cap(A \cup C)] \cup [B\cap(\overline{A} \cup \overline{C})]$

$\boldsymbol{(A\Delta B \cup B\Delta C) \equiv [\overline{B} \cap(A \cup C)] \cup [B\cap(\overline{A} \cup \overline{C})]}$ // (2)

Expanding $A\Delta C$:

$(A\cap \overline{C}) \cup (\overline{A} \cap C)$

$(A\cup C) \cap (\overline{A} \cup \overline{C})$

$\boldsymbol{A\Delta C \equiv (A\cup C) \cap (\overline{A} \cup \overline{C})}$ // (3)

Define:

$P = B$; $Q = (A \cup C)$; $R = (\overline{A} \cup \overline{C})$

So proving $Q\wedge R \to (\neg P\vee Q \vee P\wedge R)$,

will prove that $A\Delta C \subseteq (A\Delta B \cup B\Delta C)$

Proving $Q \wedge R \rightarrow (\neg P \vee Q \vee P \wedge R)$ is always true:
Premise: $(Q \wedge R)$; Conclusion: $(\neg P \vee Q \vee P \wedge R)$.
Premise → Conclusion is always true if the following holds:
Whenever the Premise is true, also the Conclusion is true.
Assuming that Premise is true:
$Q \wedge R \equiv \mathbf{T} \Longrightarrow$
$Q \equiv \mathbf{T} \wedge R \equiv \mathbf{T}$
Using that in the Conclusion:
$(\neg P \vee Q \vee P \wedge R) \equiv (\neg P \vee \mathbf{T} \vee P \wedge \mathbf{T}) \equiv \neg P \vee P \equiv \mathbf{T}$
Therefore the conclusion is dependent upon the premise, therefore
$\boldsymbol{Q \wedge R \rightarrow (\neg P \vee Q \vee P \wedge R) \equiv T}$ // (4)

P, Q and R are placeholders (defined above), so I'll use their "real" values:
$Q \wedge R \rightarrow (\neg P \vee Q \vee P \wedge R) \equiv$
$(A \cup C) \cap (\overline{A} \cup \overline{C}) \rightarrow [\overline{B} \cap (A \cup C)] \cup [B \cap (\overline{A} \cup \overline{C})]$
// Using (3):
$A \Delta C \subseteq [\overline{B} \cap (A \cup C)] \cup [B \cap (\overline{A} \cup \overline{C})]$
// Using (2):
$\boldsymbol{A \Delta C \subseteq (A \Delta B \cup B \Delta C)}$ // (5)

Since it's given that:
$A \Delta B \subseteq D \wedge B \Delta C \subseteq D$
Using (1), it's equivalent to:
$(A \Delta B \cup B \Delta C) \subseteq D$
And because of (5), we know that
$A \Delta C \subseteq (A \Delta B \cup B \Delta C)$
Together with the transience of $\subseteq$, // $X \subseteq Y$ and $Y \subseteq Z \Rightarrow X \subseteq Z$
$\boldsymbol{A \Delta B \subseteq D \wedge B \Delta C \subseteq D \rightarrow A \Delta C \subseteq D}$