גלעד ברנע ת.ז. 203627310

ממ"ן 11

Note : I'll be using e.g. *¬(A ∪ B)*
to represent the *complement of (A ∪ B)*
(My editor doesn't support superscript or overline)

2

א

Prove:
$(A\backslash B) \cup (B\backslash C) = (A \cup B) \backslash (B \cap C)$

First: expand left-hand side

$(A \cap \neg B) \cup (B \cap \neg C)$ // diff
$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup B) \cap (\neg B \cup \neg C)$ // distributivity
$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup \neg C)$ // $(\neg B \cup B) \equiv T$
$\mathbf{(A \cup B) \cap [(A \cap \neg B) \cup \neg C]}$ // dist.

Second: expand right-hand side

$(A \cup B) \cap \overline{(B \cap C)}$
$(A \cup B) \cap (\neg B \cup \neg C)$
$(A \cap \neg B) \cup (A \cap \neg C) \cup (B \cap \neg B) \cup (B \cap \neg C)$
$(A\backslash B) \cup (A \cap \neg C) \cup (B \cap \neg C)$ // $(B \cap \neg B) \equiv \varnothing$
$(A\backslash B) \cup [(A \cup B) \cap \neg C]$ // dist
$[(A\backslash B) \cup (A \cup B)] \cap [(\neg A \cap \neg B) \cup \neg C]$ // dist
// I'll now prove that $[(A\backslash\neg B) \cup (A \cup B)] \equiv (A \cup B)$,
// then get back to expanding the full statement

Since $(A\backslash B) \subseteq A$ and $A \subseteq (A \cup B) \Rightarrow$
$(A\backslash B) \subseteq (A \cup B)$
Therefore
$(A\backslash B) \cup (A \cup B) = (A \cup B)$

$(A \cup B) \cap [(\neg A \cap \neg B) \cup \neg C]$

ב

Prove:

if P(A) ∨ P(B) = P(C), then (C=A) ∨ (C=B)

I'll be proving:
(C⊆A ∧ A⊆C) ∨ (C⊆B ∧ B⊆C)
Since it's equivalent to
(C=A) ∨ (C=B)

First: proof that C⊆A ∨ C⊆B

C ∈ P(C) // power set definition
P(C) = P(A) ∨ P(B) ⇒ C ∈ (P(A) ∨ P(B))
C ∈ P(A) ∨ C ∈ P(B)
C⊆A ∨ C⊆B

Second: proof that A⊆C ∨ B⊆C

A ∈ P(A)
P(A) ⊆ P(A) ∪ P(B) // union definition
A ∈ P(A) ∪ P(B)
Given P(C) = P(A) ∪ P(B) ⇒ A ∈ P(C)
A⊆C

B ∈ P(B)
P(B) ⊆ P(A) ∪ P(B) // union definition
B ∈ P(A) ∪ P(B)
Given P(C) = P(A) ∪ P(B) ⇒ B ∈ P(C)
B⊆C

Since C⊆A ∨ C⊆B and A⊆C and B⊆C,
we conclude that:
C⊆A ∨ C⊆B ∧ A⊆C ∧ B⊆C
Therefore
(C=A) ∨ (C=B)