ממ"ך 11

Note : Sometimes I'll be using e.g. ¬(A ∪ B)
to represent the complement of (A ∪ B)
(My script doesn't fully support superscript or overline)

2

א

Prove:
(A\B) ∪ (B\C) = (A ∪ B) \ (B ∩ C)

First: expand left-hand side (A\B) ∪ (B\C)

(A ∩ ¬B) ∪ (B ∩ ¬C)   // diff
(A ∪ B) ∩ (A ∪ ¬C) ∩ (¬B ∪ B) ∩ (¬B ∪ ¬C)   // distributivity
(A ∪ B) ∩ (A ∪ ¬C) ∩ (¬B ∪ ¬C)   // (¬B ∪ B) ≡ T
 (A ∪ B) ∩ [(A ∩ ¬B) ∪ ¬C]   // dist.

Second: expand right-hand side (A ∪ B) \ (B ∩ C)

(A ∪ B) ∩ $\overline{(B ∩ C)}$
(A ∪ B) ∩ (¬B ∪ ¬C)
(A ∩ ¬B) ∪ (A ∩ ¬C) ∪ (B ∩ ¬B) ∪ (B ∩ ¬C)
(A\B) ∪ (A ∩ ¬C) ∪ (B ∩ ¬C)   // (B ∩ ¬B) ≡ ∅
(A\B) ∪ [(A ∪ B) ∩ ¬C]   // dist
[(A\B) ∪ (A ∪ B)] ∩ [(A ∩ ¬B) ∪ ¬C]   // dist
 // I'll now prove that [(A\B) ∪ (A ∪ B)] ≡ (A ∪ B),
 // then get back to expanding the full statement
Since (A\B) ⊆ A and A ⊆ (A ∪ B) ⇒
(A\B) ⊆ (A ∪ B)
Therefore
(A\B) ∪ (A ∪ B) = (A ∪ B)
 (A ∪ B) ∩ [(A ∩ ¬B) ∪ ¬C]

We see that left-hand side ≡ right-hand side, Therefore
 (A\B) ∪ (B\C) = (A ∪ B) \ (B ∩ C)

ב

Prove:
if P(A) ∨ P(B) = P(C), then (C=A) ∨ (C=B)

I'll be proving:
(C⊆A ∧ A⊆C) ∨ (C⊆B ∧ B⊆C)
Since it's equivalent to
(C=A) ∨ (C=B)

First: proof that C⊆A ∨ C⊆B

C ∈ P(C)   // power set definition
P(C) = P(A) ∨ P(B) ⇒ C ∈ (P(A) ∨ P(B))
C ∈ P(A) ∨ C ∈ P(B)
 C⊆A ∨ C⊆B

Second: proof that A⊆C ∨ B⊆C

A ∈ P(A)
P(A) ⊆ P(A) ∪ P(B)   // union definition
A ∈ P(A) ∪ P(B)
Given P(C) = P(A) ∪ P(B) ⇒ A ∈ P(C)
 A⊆C

B ∈ P(B)
P(B) ⊆ P(A) ∪ P(B)   // union definition
B ∈ P(A) ∪ P(B)
Given P(C) = P(A) ∪ P(B) ⇒ B ∈ P(C)
 B⊆C

Since C⊆A ∨ C⊆B and A⊆C and B⊆C,
we conclude that:
C⊆A ∨ C⊆B ∧ A⊆C ∧ B⊆C
Therefore
 (C=A) ∨ (C=B)