$\boldsymbol{R}$: {⟨x,y⟩ for ⟨x,y⟩ ∈ A^2 if x$\boldsymbol{R}$y}

$\boldsymbol{T}\cdot\boldsymbol{R}$: {⟨a,c⟩ | ∃b ∈ B (⟨a,b⟩ ∈ $\boldsymbol{T}$ ∧ ⟨b,c⟩ ∈ $\boldsymbol{R}$)}

$\boldsymbol{R}^2$: aR2c ⟺ {⟨a,c⟩ | ∃b ∈ A (⟨a,b⟩ ∈ $\boldsymbol{R}$ ∧ ⟨b,c⟩ ∈ $\boldsymbol{R}$)}

an ordered pair ⟨a,c⟩∈R2 means there's a "middle" b∈B that satisfies ⟨a,b⟩∈$\boldsymbol{R}$ and ⟨b,c⟩∈$\boldsymbol{R}$

properties

- whw

examples

- $(a=-b)^2 = \mathbf{I}_{\mathbb{R}}$
- ⟨a,b⟩ ∈ $\boldsymbol{R}^2$ ⟺ ⟨a,c⟩,⟨c,b⟩ ∈ $\boldsymbol{R}$

Reflexivity: $\boldsymbol{R}$:=relation(A) is reflexive if ∀a ∈ A(⟨a,a⟩ ∈ $\boldsymbol{R}$)

$\boldsymbol{R}$ is reflexive if every a in A satisfies ⟨a,a⟩∈$\boldsymbol{R}$. In other words:

$\mathbf{I}_A \subseteq \boldsymbol{R}$

A = { −1, 0, 1 }. Is the eq oblique contained in $\boldsymbol{R}$?

properties

- ⟺ $\boldsymbol{R}^{-1}$ is reflexive
- → $\boldsymbol{R} \subseteq \boldsymbol{R}^2$ (and $\boldsymbol{R}^2$ is reflexive)
- if $\boldsymbol{S} \subseteq \boldsymbol{R}$ → $\boldsymbol{S}$ is reflexive
- $\boldsymbol{S}$ is reflexive → both $\boldsymbol{R} \cup \boldsymbol{S}$ and $\boldsymbol{R} \cap \boldsymbol{S}$ are reflexive

examples

- $\mathbf{U}_A$: ∀a ∈ A(⟨a,a⟩ ∈ A×A = $\mathbf{U}_A$)
- $\mathbf{I}_A$: ∀a ∈ A(⟨a,a⟩ ∈ {⟨-1, −1⟩, ⟨0, 0⟩, ⟨1, 1⟩})
- ≤, ≥ // both have eq oblique

counter examples

- ≠ (which is $\mathbf{U}_A - \mathbf{I}_A$)
- ⊂, >, ∅
- a=−b (forward oblique)

Anti-Reflexivity: $\boldsymbol{R}$:=relation(A) is anti-reflexive iff ¬∃a ∈ A(⟨a,a⟩ ∈ $\boldsymbol{R}$)

$\boldsymbol{R}$ is reflexive if every a in A satisfies ⟨a,a⟩ ∉ $\boldsymbol{R}$. In other words:

$\mathbf{I}_A \cap \boldsymbol{R} = \varnothing$ (just $\mathbf{I}_A \not\subseteq \boldsymbol{R}$ isn't enough; $\mathbf{I}_A = \{\langle 1,1\rangle, \langle 2,2\rangle\} \not\subseteq \boldsymbol{R} = \{\langle 1,1\rangle, \langle 1,2\rangle\}$ but $\langle 1,1\rangle \in \boldsymbol{R}$ so isn't anti-reflexive)

examples

- $\neq$, $\subset$, $>$, $\varnothing$

counter examples

- $\mathbf{U}_A$, $\mathbf{I}_A$, a=−b, $\leq$, $\geq$

Symmetry: $\boldsymbol{R}$:=relation(A) is symmetric iff $\boldsymbol{R} = \boldsymbol{R}^{-1}$

$\boldsymbol{R}$ is reflexive if every $\langle x,y\rangle$ in $\boldsymbol{R}$ satisfies $\langle y,x\rangle \in \boldsymbol{R}$. In other words:

examples

- $\varnothing$ (can't point at $\langle x,y\rangle$ and say $\langle y,x\rangle$ is not in $\varnothing^{-1}$)
- $\mathbf{U}_A$, $\mathbf{I}_A$, a=−b, $\neq$

counter examples

- $\leq$, $\geq$, $\subset$, $>$