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ממ"ן 11

1

Prove:

$A\Delta B\subseteq D \land B\Delta C\subseteq D \rightarrow A\Delta C\subseteq D$

Since $(X\rightarrow Z) \land (Y\rightarrow Z) \equiv (X\lor Y)\rightarrow Z$, also:

$(A\Delta B\subseteq D \land B\Delta C\subseteq D) \equiv (A\Delta B \cup B\Delta C) \subseteq D$. Proof:

$(X\rightarrow Z) \land (Y\rightarrow Z)$
$Z\lor\neg X \land Z\lor\neg Y$
$Z\lor\neg(X\lor Y)$
$(X\lor Y) \rightarrow Z$

$A\Delta B\subseteq D \land B\Delta C\subseteq D$
$(x\in(A\Delta B) \rightarrow x\in D) \land (x\in(B\Delta C) \rightarrow x\in D)$
$x\in(A\Delta B \cup B\Delta C) \rightarrow x\in D$
$(A\Delta B \cup B\Delta C) \subseteq D$
Therefore:
$(A\Delta B\subseteq D \land B\Delta C\subseteq D) \equiv (A\Delta B \cup B\Delta C) \subseteq D$ // (1)

I'll prove that $A\Delta C \subseteq (A\Delta B \cup B\Delta C)$.

Expanding $(A\Delta B \cup B\Delta C)$:

$(\overline{B}\cap A) \cup (\overline{B}\cap C) \cup (B\cap \overline{A}) \cup (B\cap \overline{C})$

$[\overline{B}\cap(A \cup C)] \cup [B\cap(\overline{A} \cup \overline{C})]$

$\mathbf{(A\Delta B \cup B\Delta C) \equiv [\overline{B}\cap(A \cup C)] \cup [B\cap(\overline{A} \cup \overline{C})]}$ // (2)

Expanding $A\Delta C$:

$(A\cap \overline{C}) \cup (\overline{A}\cap C)$

$(A\cup C) \cap (\overline{A} \cup \overline{C})$

$\mathbf{A\Delta C \equiv (A\cup C) \cap (\overline{A} \cup \overline{C})}$ // (3)

Define:

$P = B$; $Q = (A \cup C)$; $R = (\overline{A} \cup \overline{C})$

So proving $Q\land R \rightarrow (\neg P\lor Q \lor P\land R)$,

will prove that $A\Delta C \subseteq (A\Delta B \cup B\Delta C)$

Proving $Q\land R \rightarrow (\neg P\lor Q \lor P\land R)$ is always true:

Premise: $(Q \wedge R)$; Conclusion: $(\neg P \vee Q \vee P \wedge R)$.

Premise → Conclusion is always true if the following holds:

Whenever the Premise is true, also the Conclusion is true.

Assuming that Premise is true:

$Q \wedge R \equiv \mathbf{T} \implies$

$Q \equiv \mathbf{T} \wedge R \equiv \mathbf{T}$

Using that in the Conclusion:

$(\neg P \vee Q \vee P \wedge R) \equiv (\neg P \vee \mathbf{T} \vee P \wedge \mathbf{T}) \equiv \neg P \vee P \equiv \mathbf{T}$

Therefore the conclusion is dependent upon the premise, therefore

$\mathbf{Q \wedge R \rightarrow (\neg P \vee Q \vee P \wedge R) \equiv T}$ // (4)

P, Q and R are placeholders (defined above), so I'll use their "real" values:

$Q \wedge R \rightarrow (\neg P \vee Q \vee P \wedge R) \equiv$

$(A \cup C) \cap (\overline{A} \cup \overline{C}) \rightarrow [\overline{B} \cap (A \cup C)] \cup [B \cap (\overline{A} \cup \overline{C})]$

// Using (3):

$A \Delta C \subseteq [\overline{B} \cap (A \cup C)] \cup [B \cap (\overline{A} \cup \overline{C})]$

// Using (2):

$\mathbf{A \Delta C \subseteq (A \Delta B \cup B \Delta C)}$ // (5)

Since it's given that:

$A \Delta B \subseteq D \wedge B \Delta C \subseteq D$

Using (1), it's equivalent to:

$(A \Delta B \cup B \Delta C) \subseteq D$

And because of (5), we know that

$A \Delta C \subseteq (A \Delta B \cup B \Delta C)$

Together with the transience of $\subseteq$, // $X \subseteq Y$ and $Y \subseteq Z \Rightarrow X \subseteq Z$

$\mathbf{A \Delta B \subseteq D \wedge B \Delta C \subseteq D \rightarrow A \Delta C \subseteq D}$