Lesson 2

$\in$ vs $\subseteq$

Definition: $A \subseteq B \Leftrightarrow \forall x[\ (x \in A) \rightarrow (x \in B)\ ]$
In other wrds: A is a subset of B, iff, every x in A is a member of B
$\in$ left_side in right_side
$\subseteq$ all(x in right_side for x in left_side)

Examples
$A = \{ 1, 2, \{1,2,3\} \}$
$\{1,2,3\} \in A$
$\{1,2,3\} \not\subseteq A$

$\varnothing \notin \varnothing \equiv \varnothing \notin \{\}$ no items on right side, nothing belongs to it
$\varnothing \in \{\varnothing\}$ right side contains 1 item: $\varnothing$, so $\varnothing$ in right side
$\varnothing \subseteq \varnothing$ every set is a subset of itself
$\varnothing \subseteq \{\varnothing\}$ empty set is a subset of every set
$\varnothing \subseteq \{1\}$ “
$\varnothing \notin \{1\}$

$\{\varnothing\} \subseteq \{\varnothing\}$ all(x in right_side for x in left_side)
$\{\varnothing\} \notin \{\varnothing\}$ [foo] in [foo]? no
$\{\varnothing\} \in \{ \{\varnothing\} \}$ [foo] in [[foo]]? yes
$\{\varnothing\} \not\subseteq \{ \{\varnothing\} \}$ foo in [[foo]]? no

$\{ \{\varnothing,1\} \} \notin \{ \{\varnothing\}, 1 \}$
$\{ \{\varnothing,1\} \} \not\subseteq \{ \{\varnothing\}, 1 \}$

$|\varnothing| = 0$
$|\{\varnothing\}| = 1$

Power Set

Definition: $X \in P(A) \Leftrightarrow X \subseteq A$
In other words: X belongs to the set of all A's subgroups, iff X itself is a subset of A.

Always true: $\varnothing \in P(A)$ because $\varnothing$ is a subset of every set
|P(A)| = 2**|A|

Examples
$A = \{ \varnothing, 0, \{0\} \}$;
$\{\varnothing\} \in P(A)$?
$\{\varnothing\} \in P(A) \Leftrightarrow \{\varnothing\} \subseteq A$ power set definition
$\{\varnothing\} \subseteq A \Leftrightarrow \varnothing \in A$ subset definition
Since $\varnothing$ is a member of A, $\varnothing \in A$
and we only used $\Leftrightarrow$, Truth value "propagates" backwards
$\{\varnothing\} \in P(A)$ The set containing $\varnothing$ is a member of the set containing all of A's subsets.