11 ממ"ן

Note: I'll be using e.g. $\neg(A \cup B)$
to represent the complement of $(A \cup B)$
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Prove:
if $P(A) \vee P(B) = P(C)$, then $(C=A) \vee (C=B)$

I'll be proving:
$(C \subseteq A \wedge A \subseteq C) \vee (C \subseteq B \wedge B \subseteq C)$
Since it's equivalent to
$(C=A) \vee (C=B)$

First: proof that $C \subseteq A \vee C \subseteq B$

$C \in P(C)$ // power set definition
$P(C) = P(A) \vee P(B) \Rightarrow C \in (P(A) \vee P(B))$
$C \in P(A) \vee C \in P(B)$
$C \subseteq A \vee C \subseteq B$

Second: proof that $A \subseteq C \vee B \subseteq C$

$A \in P(A)$
$P(A) \subseteq P(A) \cup P(B)$ // union definition
$A \in P(A) \cup P(B)$
Given $P(C) = P(A) \cup P(B) \Rightarrow A \in P(C)$
$A \subseteq C$

$B \in P(B)$
$P(B) \subseteq P(A) \cup P(B)$ // union definition
$B \in P(A) \cup P(B)$
Given $P(C) = P(A) \cup P(B) \Rightarrow B \in P(C)$
$B \subseteq C$

Since $C \subseteq A \vee C \subseteq B$ and $A \subseteq C$ and $B \subseteq C$, we conclude that:

$C \subseteq A \vee C \subseteq B \wedge A \subseteq C \wedge B \subseteq C$

Therefore

$\mathbf{(C=A) \vee (C=B)}$