Logic

For any P, Q:
$(P \to Q) \lor (Q \to P)$ *(always True)*

An arg is valid iff:
If premises are True, then conclusion is True
So a counter-example is when premises are True but conclusion is False

Ideompotence
$P \equiv (P \lor P)$
$P \equiv (P \land P)$

Commutativity
$(P \lor Q) \equiv (Q \lor P)$
$(P \land Q) \equiv (Q \land P)$

Associativity
$(P \lor Q) \lor R \equiv P \lor (Q \lor R)$
$(P \land Q) \land R \equiv P \land (Q \land R)$

DeMorgan
$\neg(P \lor Q) \equiv (\neg P) \land (\neg Q)$
$\neg(P \land Q) \equiv (\neg P) \lor (\neg Q)$

Distributivity
$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$
$P \lor (Q \land R) \equiv (P \lor Q) \land (P \lor R)$

With True / False
$(P \lor \text{True}) \equiv \text{True}$
$(P \land \text{False}) \equiv \text{False}$
$(P \lor \text{False}) \equiv P$
$(P \land \text{True}) \equiv P$
$(P \lor \neg P) \equiv \text{True}$
$(P \land \neg P) \equiv \text{False}$

Double Negation
$P \equiv \neg(\neg P)$

Implication
$(P \to Q) \equiv (\neg P \lor Q) \equiv \neg(P \land \neg Q)$ *DM*
$(P \to Q) \equiv (\neg Q \to \neg P)$ *Contrapos.*
$\neg(P \to Q) \equiv (P \land \neg Q)$

Equivalence
$(P \leftrightarrow Q) \equiv (P \to Q) \land (Q \to P)$
$(P \leftrightarrow Q) \equiv (P \land Q) \lor [(\neg P) \land (\neg Q)]$ *DM*
$\neg(P \leftrightarrow Q) \equiv (P \land \neg Q) \lor (\neg P \land Q)$

Exportation
$(P \land Q) \to R \equiv P \to (Q \to R)$

Absurdity
$(P \to Q) \land (P \to \neg Q) \equiv \neg P$

Modal Logic

$\forall x\varphi \equiv \neg[\exists x \neg\varphi]$
$\exists x\varphi \equiv \neg[\forall x \neg\varphi]$

DeMorgan
$\neg\forall x\varphi \equiv \exists x\neg\varphi$
$\forall x\neg\varphi \equiv \neg\exists x\varphi$

If x is the set of {a,b,c}, then
$\forall x P(x) \equiv P(a) \land P(b) \land P(c)$
$\exists x P(x) \equiv P(a) \lor P(b) \lor P(c)$

Applying DeMorgan:
$P(a) \land P(b) \land P(c) \equiv \neg(\neg P(a) \lor \neg P(b) \lor \neg P(c))$
$P(a) \lor P(b) \lor P(c) \equiv \neg(\neg P(a) \land \neg P(b) \land \neg P(c))$