$\boldsymbol{R}$: {⟨x,y⟩ for ⟨x,y⟩ ∈ A^2 if x$\boldsymbol{R}$y}

$\boldsymbol{T}\cdot\boldsymbol{R}$: {⟨a,c⟩ | ∃b ∈ B (⟨a,b⟩ ∈ $\boldsymbol{T}$ ∧ ⟨b,c⟩ ∈ $\boldsymbol{R}$)}

$\boldsymbol{R}^2$: a$\boldsymbol{R}^2$c ⇔ {⟨a,c⟩ | ∃b ∈ A (⟨a,b⟩ ∈ $\boldsymbol{R}$ ∧ ⟨b,c⟩ ∈ $\boldsymbol{R}$)}

an ordered pair ⟨a,c⟩∈$\boldsymbol{R}^2$ means there's a "middle" b∈B that satisfies ⟨a,b⟩∈$\boldsymbol{R}$ and ⟨b,c⟩∈$\boldsymbol{R}$

examples

- (a=−b)2 = $\boldsymbol{I}\mathbb{R}$
- ⟨a,b⟩ ∈ $\boldsymbol{R}^2$ ⇔ ⟨a,c⟩,⟨c,b⟩ ∈ $\boldsymbol{R}$

Empty ∅A: $\boldsymbol{R}$:=rel(A×B) = ∅

No pair ∈ A×B satisfies ⟨a,b⟩ ∈ $\boldsymbol{R}$ properties

- $\boldsymbol{S}$·∅A = ∅
- symmetric and anti-symmetric ?

examples

- {⟨x,y⟩ ∈ $\mathbb{N}^2$ | x+y < x}

Identity $\boldsymbol{I}$A

properties

- $\boldsymbol{R}\cdot\boldsymbol{I}$A = $\boldsymbol{R}$

Reflexivity: $\boldsymbol{R}$:=rel(A) is reflexive if ∀a ∈ A(⟨a,a⟩ ∈ $\boldsymbol{R}$)

$\boldsymbol{R}$ is reflexive if every a in A satisfies ⟨a,a⟩∈$\boldsymbol{R}$. In other words:
$\boldsymbol{I}$A ⊆ $\boldsymbol{R}$
A = { −1, 0, 1 }. Is ⋱ contained in $\boldsymbol{R}$? $\boldsymbol{R}$ = lambda a,b: a⊙b; all(R(x,x) for x in A)? properties

- ⇔ $\boldsymbol{R}^{-1}$ is reflexive
- → $\boldsymbol{R}$ ⊆ $\boldsymbol{R}^2$ (and $\boldsymbol{R}^2$ is reflexive)
- → $\boldsymbol{R}$⊆$\boldsymbol{R}^2$
- if $\boldsymbol{S}$⊆$\boldsymbol{R}$ then $\boldsymbol{S}$ is reflexive
- if $\boldsymbol{S}$ is reflexive then both $\boldsymbol{R}$∪$\boldsymbol{S}$ and $\boldsymbol{R}$∩$\boldsymbol{S}$ are reflexive

examples

- $\boldsymbol{U}$A: ∀a ∈ A(⟨a,a⟩ ∈ A×A = $\boldsymbol{U}$A)
- $\boldsymbol{I}$A: ∀a ∈ A(⟨a,a⟩ ∈ {⟨−1, −1⟩, ⟨0, 0⟩, ⟨1, 1⟩})
- ≤, ≥ // both contain ⋱ counter examples
- ≠ (which is $\boldsymbol{U}$A − $\boldsymbol{I}$A)
- <, >, ∅A
- a=−b ⋰

Anti-Reflexivity: $\boldsymbol{R}$:=rel(A) is anti-reflexive iff ¬∃a ∈ A(⟨a,a⟩ ∈ $\boldsymbol{R}$)

$\boldsymbol{R}$ is reflexive if every a in A satisfies ⟨a,a⟩ ∉ $\boldsymbol{R}$. In other words: $\boldsymbol{I}$A ∩ $\boldsymbol{R}$ = ∅ // just $\boldsymbol{I}$A ⊈ $\boldsymbol{R}$ isn't enough; $\boldsymbol{I}$A = {⟨1,1⟩, ⟨2,2⟩} ⊈ $\boldsymbol{R}$ = {⟨1,1⟩, ⟨1,2⟩} but ⟨1,1⟩ ∈ $\boldsymbol{R}$ so isn't anti-reflexive examples

- ≠, <, >, ∅A counter examples
- $\boldsymbol{U}$A, $\boldsymbol{I}$A, a=−b ∴, ≤, ≥

Symmetry: $\boldsymbol{R}$:=rel(A) is symmetric iff $\boldsymbol{R}$ = $\boldsymbol{R}$−1

$\boldsymbol{R}$ is symmetric if every ⟨x,y⟩ in $\boldsymbol{R}$ satisfies ⟨y,x⟩ ∈ $\boldsymbol{R}$ // assuming both x and y exist in A ∀x∀y(⟨x,y⟩∈$\boldsymbol{R}$ → ⟨y,x⟩∈$\boldsymbol{R}$) $\boldsymbol{R}$ = lambda a,b: a⊙b; all(rel(y,x) for x,y in R)?

properties

- if $\boldsymbol{S}$ is symmetric then both $\boldsymbol{R}$∪$\boldsymbol{S}$ and $\boldsymbol{R}$∩$\boldsymbol{S}$ are reflexive
- if $\boldsymbol{S}$ is symmetric then $\boldsymbol{R}$\$\boldsymbol{S}$ is symmetric examples
- ∅A // can't point at ⟨x,y⟩ and say ⟨y,x⟩ is not in ∅−1
- $\boldsymbol{U}$A, $\boldsymbol{I}$A, a=−b ∴, ≠ counter examples
- ≤, ≥, <, >

Anti-Symmetry: $\boldsymbol{R}$:=rel(A) is anti-symmetric iff $\boldsymbol{R}$ ∩ $\boldsymbol{R}$−1 = ∅

$\boldsymbol{R}$ is anti-symmetric if every ⟨x,y⟩ in $\boldsymbol{R}$ satisfies ⟨y,x⟩ ∉ $\boldsymbol{R}$ ∀x∀y((x,y) ∈ $\boldsymbol{R}$ → (y,x) ∉ $\boldsymbol{R}$) $\boldsymbol{R}$ ∩ $\boldsymbol{R}$−1 = ∅ means there can't be a ⟨x,x⟩

properties

- → $\boldsymbol{R}$ is anti-reflexive
- → $\boldsymbol{R}$−1 is anti-symmetric
- if $\boldsymbol{S}$⊆$\boldsymbol{R}$ then $\boldsymbol{S}$ is anti-symmetric
- if $\boldsymbol{S}$∪$\boldsymbol{T}$ is anti-symmetric then both $\boldsymbol{S}$ and $\boldsymbol{T}$ are anti-symmetric
- → $\boldsymbol{R}$∩$\boldsymbol{S}$ is anti-symmetric

examples

- <, >, ∅A
- b > a**2

counter examples

- ≠, ≤, ≥, $\boldsymbol{U}$A, $\boldsymbol{I}$A, a=−b ∴, ≠
- b < a**2 // ⟨3,4⟩ and ⟨4,3⟩ are symmetric

Weak Anti-Symmetry: $\boldsymbol{R}$ ∩ $\boldsymbol{R}$−1 ⊆ $\boldsymbol{I}$A

∀x∀y(⟨x,y⟩ ∈ $\boldsymbol{R}$ ∧ ⟨y,x⟩ ∈ $\boldsymbol{R}$ → x=y) if both ⟨x,y⟩ ∈ $\boldsymbol{R}$ and ⟨y,x⟩ ∈ $\boldsymbol{R}$ it's only because they're equal for x,y ∈ A: if x≠y and ⟨x,y⟩ ∈ $\boldsymbol{R}$ then must ⟨y,x⟩ ∉ $\boldsymbol{R}$

AS vs WAS: AS requires every pair's opposite to not be in $\boldsymbol{R}$, whereas WAS requires the same only for pairs that x=y

examples

- $\boldsymbol{I}$A

Transitivity: $\boldsymbol{R}$2 ⊆ $\boldsymbol{R}$

∀x∀y∀z(($\boldsymbol{R}$(x,y) ∧ $\boldsymbol{R}$(y,z)) → $\boldsymbol{R}$(x,z)) Every (x,y,z) ∈ A that satisfy ⟨x,y⟩ ∈ $\boldsymbol{R}$ and ⟨y,z⟩ ∈ $\boldsymbol{R}$ also satisfy ⟨x,z⟩ ∈ $\boldsymbol{R}$ If you see an x that leads to y that leads to z, then expect x to lead to z // this is why $\boldsymbol{R}$2 ⊆ $\boldsymbol{R}$

properties

- if $\boldsymbol{T}$ is symmetric and anti-symmetric then it's also transitive examples
- A={1,2,3}; $\boldsymbol{R}$ = {⟨**1**,2⟩, ⟨2,**3**⟩, **⟨1,3⟩**} ⇒ $\boldsymbol{R}$2 = {⟨1,3⟩} ⊆ $\boldsymbol{R}$
- A={1,2,3}; $\boldsymbol{T}$ = {⟨1,2⟩} ⇒ $\boldsymbol{T}$2 = ∅ ⊆ $\boldsymbol{T}$
- $\boldsymbol{W}$ = {⟨1,1⟩} ⇒ $\boldsymbol{W}$2 = {⟨1,1⟩} ⊆ $\boldsymbol{W}$
- $\boldsymbol{I}$A
- ∅A
- $\boldsymbol{U}$A // if ⟨a,b⟩ ∈ A2 and ⟨b,a⟩ ∈ A2 then ⟨a,c⟩ ∈ A2
- if |A| > 1 then ≠ is trans
- <, ≤

counter examples

- $\boldsymbol{P}$={⟨1,2⟩, ⟨2,1⟩} ⇒ $\boldsymbol{P}$2 = {⟨1,1⟩, ⟨2,2⟩} ⊈ $\boldsymbol{P}$ // iow: 1 leads to 2 leads to 1, but ⟨1,1⟩ ∉ $\boldsymbol{P}$
- ∃x∃y∃z($\boldsymbol{R}$(x,y) ∧ $\boldsymbol{R}$(y,z) ∧ ¬$\boldsymbol{R}$(x,z))

Equivalance: $\boldsymbol{R}$ over A is equivalence iff R is reflexive, symmetric and transitive

examples

- $\boldsymbol{U}$A, $\boldsymbol{I}$A, equality
- "Has the same absolute value" on the set of real numbers
- if A=∅ then ∅A is symmetric, transitive and reflexive

counter examples

- ≥ // reflexive and transitive but not symmetric
- if A≠∅ then ∅A is symmetric and transitive, but not reflexive

Connexivity: $\boldsymbol{R}$ over A is connexive iff ∀(x,y) ∈ A (⟨x,y⟩∈$\boldsymbol{R}$ ∨ ⟨y,x⟩∈$\boldsymbol{R}$ ∨ x = y)

properties

- $\boldsymbol{R}$ cannot be symmetric, except for $\boldsymbol{U}$A

Total Order: antireflexive, transitive, and connex

examples

- < over ℝ counter examples
- if A≠∅ then *I*A isn't total order because for all a∈A: a=a

Partial Order: ≤ is a partial order iff it's antireflexive and transitive

examples

- ⊂ over 𝒫(A)

???

for all a, b, and c:

- a ≤ a // reflex
- if a ≤ b and b ≤ a, then a = b // antisymm
- if a ≤ b and b ≤ c, then a ≤ c // trans

examples

- equality

???

Partition of A is a set of non-empty, non-overlapping subsets of A whose union = A

properties

- every a∈A is in exactly one block
- no block contains ∅
- union of blocks = A
- intersection of any two blocks = ∅
- → A is finite ⇒ rank of P is |X| − |P| ?

examples

- {A} is partition of A // trivial
- ∅'s only partition is ∅
- {1,2,3} has five partitions: {{1},{2},{3}}, {{1, 2}, {3}}, {{1, 3},{2}}, {{1},{2, 3}}, {{1, 2, 3}}

counter examples:

- not partitions of {1,2,3}:
  - {{}, {1,3}, {2}} // contains ∅
  - {{1, 2}, {2, 3}} // 2 exists ∈ more than one block
  - {{1}, {2}} // no block contains 3