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ממ"ן 11

Note : Sometimes I'll be using e.g. ¬A to represent the complement of A (My editor doesn't fully support superscript or overline)

1

א: לא נכון

ב: נכון

ג: לא נכון

ד: נכון

ה: לא נכון

ו: לא נכון

ז: נכון

ח: לא נכון

2

א

Prove:

$(A\backslash B) \cup (B\backslash C) = (A \cup B) \backslash (B \cap C)$

First: expanding left-hand side $(A\backslash B) \cup (B\backslash C)$

$(A \cap \neg B) \cup (B \cap \neg C)$ // difference definition

$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup B) \cap (\neg B \cup \neg C)$ // distributivity

$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup \neg C)$ // $(\neg B \cup B) \equiv T$

$\mathbf{(A \cup B) \cap [(A \cap \neg B) \cup \neg C]}$ // dist.

Second: expanding right-hand side $(A \cup B) \backslash (B \cap C)$

$(A \cup B) \cap \overline{(B \cap C)}$

$(A \cup B) \cap (\neg B \cup \neg C)$

$(A \cap \neg B) \cup (A \cap \neg C) \cup (B \cap \neg B) \cup (B \cap \neg C)$ // dist

$(A\backslash B) \cup (A \cap \neg C) \cup (B \cap \neg C)$ // $(B \cap \neg B) \equiv \varnothing$

$(A\backslash B) \cup [(A \cup B) \cap \neg C]$ // dist

$[(A\backslash B) \cup (A \cup B)] \cap [(A \cap \neg B) \cup \neg C]$ // dist

// I'll now prove that $[(A\backslash B) \cup (A \cup B)] \equiv (A \cup B)$,

// then get back to expanding the full statement

Since $(A \setminus B) \subseteq A$ and $A \subseteq (A \cup B) \Rightarrow$
$(A \setminus B) \subseteq (A \cup B)$
Therefore
$(A \setminus B) \cup (A \cup B) = (A \cup B)$

$\mathbf{(A \cup B) \cap [(A \cap \neg B) \cup \neg C]}$

We see that left-hand side ≡ right-hand side, therefore
$\mathbf{(A \setminus B) \cup (B \setminus C) = (A \cup B) \setminus (B \cap C)}$

ב

Prove:
if $P(A) \vee P(B) = P(C)$, then $(C=A) \vee (C=B)$

I'll be proving:
$(C \subseteq A \wedge A \subseteq C) \vee (C \subseteq B \wedge B \subseteq C)$
Since it's equivalent to
$(C=A) \vee (C=B)$

First: proof that $C \subseteq A \vee C \subseteq B$

$C \in P(C)$ // power set definition
$P(C) = P(A) \vee P(B) \Rightarrow C \in (P(A) \vee P(B))$
$C \in P(A) \vee C \in P(B)$
$\mathbf{C \subseteq A \vee C \subseteq B}$

Second: proof that $A \subseteq C \vee B \subseteq C$

$A \in P(A)$
$P(A) \subseteq P(A) \cup P(B)$ // union definition
$A \in P(A) \cup P(B)$
Given $P(C) = (P(A) \cup P(B)) \Rightarrow A \in P(C)$
$\mathbf{A \subseteq C}$
$B \in P(B)$
$P(B) \subseteq P(A) \cup P(B)$ // union definition
$B \in P(A) \cup P(B)$
Given $P(C) = (P(A) \cup P(B)) \Rightarrow B \in P(C)$
$\mathbf{B \subseteq C}$

Since $C \subseteq A \vee C \subseteq B$ and $A \subseteq C$ and $B \subseteq C$,
// More formally: $(C \subseteq A \vee C \subseteq B) \wedge (A \subseteq C \wedge B \subseteq C)$
it follows that:
$\mathbf{(C=A) \vee (C=B)}$

ג

Prove:
if A,B are finite and $|P(A)| = 2\cdot|P(A\setminus B)|$, then $|A \cap B| = 1$

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(1)

$A\setminus B \equiv A \setminus (A \cap B)$ // by definition

(2)

We know that for any two sets X,Y, if $Y \subseteq X$ then $|X\setminus Y| = |X| - |X \cap Y|$
Certainly $(A \cap B) \subseteq A$, so
$|A \setminus (A \cap B)| = |A| - |A \cap B|$.

(3)

Assuming $|A \cap B| = 1$, if follows that:
$|A| - |A \cap B| = |A| - 1$, therefore using (1) and (2):
$|A\setminus B| = |A \setminus (A \cap B)| = |A| - |A \cap B| = |A| - 1$, so
$|P(A\setminus B)| = 2^{|A\setminus B|} = 2^{(|A| - 1)}$

(4): Expanding 2·|P(A\B)|

$2\cdot|P(A\setminus B)| = 2\cdot 2^{(|A| - 1)} = 2^{|A|}$

(5)

$|P(A)| = 2^{|A|}$ // by definition

(6): Putting it all together

$\mathbf{|P(A)| = 2\cdot|P(A\setminus B)|}$

3

א

Prove: if $(A \subset B)$, then $(A \cup \neg B) \neq U$

Since A is a **proper** subset of B, then $(B\setminus A) \neq \varnothing$.
Expanding $(B\setminus A) = (B \cap \neg A) = (\neg A \cap B) = \overline{(A \cup \neg B)}$ // DeMorgan
Therefore $\overline{(A \cup \neg B)} \neq \varnothing$

Since the complement of a given set X is the universal set (U) if and only if $X=\varnothing$, it follows that the complement of a given set Y is **not** U if and only if $Y\neq\varnothing$.
Because $\overline{(A \cup \neg B)} \neq \varnothing$, then the complement of $\overline{(A \cup \neg B)} \neq U$,
therefore $\mathbf{(A \cup \neg B) \neq U}$.

ב

Prove: if $(\neg A \,\Delta\, B) = (\neg B \,\Delta\, C)$, then $A=C$.

We know that $(\neg A \,\Delta\, B) = (\neg B \,\Delta\, A)$, because:

$(\neg A \,\Delta\, B) = (\neg A \cap \neg B) \cup (A \cap \neg\neg B) =$

$(\neg A \cap \neg B) \cup (B \cap A)$ // double negation

Similarly,

$(\neg B \,\Delta\, A) = (\neg B \cap \neg A) \cup (\neg\neg B \cap A) =$

$(\neg A \cap \neg B) \cup (B \cap A)$ // double negation, comm.

It's given that $(\neg A \,\Delta\, B) = (\neg B \,\Delta\, C)$, so

$(\neg B \,\Delta\, A) = (\neg B \,\Delta\, C)$ // replaced $(\neg A \,\Delta\, B)$ with $(\neg B \,\Delta\, A)$

Since for any sets X, Y, Z:

$(X \,\Delta\, Y) = (X \,\Delta\, Z) \Rightarrow X = Z$

It follows that

$(\neg B \,\Delta\, A) = (\neg B \,\Delta\, C) \Rightarrow A = C$

Therefore **A = C** .