גלעד ברנע ת.ז. 203627310

ממ"ן 11

Note : Sometimes I'll be using e.g. $\neg(A \cup B)$
to represent the complement of $(A \cup B)$
(My script doesn't fully support superscript or overline)

2

א

Prove:
$(A \setminus B) \cup (B \setminus C) = (A \cup B) \setminus (B \cap C)$

First: expand left-hand side $(A \setminus B) \cup (B \setminus C)$

$(A \cap \neg B) \cup (B \cap \neg C)$ // diff
$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup B) \cap (\neg B \cup \neg C)$ // distributivity
$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup \neg C)$ // $(\neg B \cup B) \equiv T$
$\mathbf{(A \cup B) \cap [(A \cap \neg B) \cup \neg C]}$ // dist.

Second: expand right-hand side $(A \cup B) \setminus (B \cap C)$

$(A \cup B) \cap \overline{(B \cap C)}$
$(A \cup B) \cap (\neg B \cup \neg C)$
$(A \cap \neg B) \cup (A \cap \neg C) \cup (B \cap \neg B) \cup (B \cap \neg C)$ // dist
$(A \setminus B) \cup (A \cap \neg C) \cup (B \cap \neg C)$ // $(B \cap \neg B) \equiv \varnothing$
$(A \setminus B) \cup [(A \cup B) \cap \neg C]$ // dist
$[(A \setminus B) \cup (A \cup B)] \cap [(A \cap \neg B) \cup \neg C]$ // dist
// I'll now prove that $[(A \setminus B) \cup (A \cup B)] \equiv (A \cup B)$,
// then get back to expanding the full statement

Since $(A \setminus B) \subseteq A$ and $A \subseteq (A \cup B) \Rightarrow$
$(A \setminus B) \subseteq (A \cup B)$
Therefore
$(A \setminus B) \cup (A \cup B) = (A \cup B)$

$\mathbf{(A \cup B) \cap [(A \cap \neg B) \cup \neg C]}$

We see that left-hand side $\equiv$ right-hand side, therefore
$\mathbf{(A \setminus B) \cup (B \setminus C) = (A \cup B) \setminus (B \cap C)}$

ב

Prove:
if $P(A) \lor P(B) = P(C)$, then $(C=A) \lor (C=B)$

I'll be proving:
$(C \subseteq A \land A \subseteq C) \lor (C \subseteq B \land B \subseteq C)$
Since it's equivalent to
$(C=A) \lor (C=B)$

First: proof that $C \subseteq A \lor C \subseteq B$

$C \in P(C)$ // power set definition
$P(C) = P(A) \lor P(B) \Rightarrow C \in (P(A) \lor P(B))$
$C \in P(A) \lor C \in P(B)$
$C \subseteq A \lor C \subseteq B$

Second: proof that $A \subseteq C \lor B \subseteq C$

$A \in P(A)$
$P(A) \subseteq P(A) \cup P(B)$ // union definition
$A \in P(A) \cup P(B)$
Given $P(C) = (P(A) \cup P(B)) \Rightarrow A \in P(C)$
$A \subseteq C$

$B \in P(B)$
$P(B) \subseteq P(A) \cup P(B)$ // union definition
$B \in P(A) \cup P(B)$
Given $P(C) = (P(A) \cup P(B)) \Rightarrow B \in P(C)$
$B \subseteq C$

Since $C \subseteq A \lor C \subseteq B$ and $A \subseteq C$ and $B \subseteq C$,
we conclude that:
$C \subseteq A \lor C \subseteq B \land A \subseteq C \land B \subseteq C$
Therefore
$(C=A) \lor (C=B)$