גלעד ברנע ת.ז. 203627310

ממ"ן 11

Note : Sometimes I'll be using e.g. ¬A to represent the complement of A
(My editor doesn't fully support superscript or overline)

2

א

Prove:
$(A \setminus B) \cup (B \setminus C) = (A \cup B) \setminus (B \cap C)$

First: expand left-hand side $(A \setminus B) \cup (B \setminus C)$

$(A \cap \neg B) \cup (B \cap \neg C)$ // diff
$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup B) \cap (\neg B \cup \neg C)$ // distributivity
$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup \neg C)$ // $(\neg B \cup B) \equiv T$
$\mathbf{(A \cup B) \cap [(A \cap \neg B) \cup \neg C]}$ // dist.

Second: expand right-hand side $(A \cup B) \setminus (B \cap C)$

$(A \cup B) \cap \overline{(B \cap C)}$
$(A \cup B) \cap (\neg B \cup \neg C)$
$(A \cap \neg B) \cup (A \cap \neg C) \cup (B \cap \neg B) \cup (B \cap \neg C)$ // dist
$(A \setminus B) \cup (A \cap \neg C) \cup (B \cap \neg C)$ // $(B \cap \neg B) \equiv \varnothing$
$(A \setminus B) \cup [(A \cup B) \cap \neg C]$ // dist
$[(A \setminus B) \cup (A \cup B)] \cap [(A \cap \neg B) \cup \neg C]$ // dist
// I'll now prove that $[(A \setminus B) \cup (A \cup B)] \equiv (A \cup B)$,
// then get back to expanding the full statement

> Since $(A \setminus B) \subseteq A$ and $A \subseteq (A \cup B) \Rightarrow$
> $(A \setminus B) \subseteq (A \cup B)$
> Therefore
> $(A \setminus B) \cup (A \cup B) = (A \cup B)$

$\mathbf{(A \cup B) \cap [(A \cap \neg B) \cup \neg C]}$

We see that left-hand side ≡ right-hand side, therefore
$\mathbf{(A \setminus B) \cup (B \setminus C) = (A \cup B) \setminus (B \cap C)}$

ב

Prove:
if $P(A) \vee P(B) = P(C)$, then $(C=A) \vee (C=B)$

I'll be proving:
$(C\subseteq A \wedge A\subseteq C) \vee (C\subseteq B \wedge B\subseteq C)$
Since it's equivalent to
$(C=A) \vee (C=B)$

First: proof that $C\subseteq A \vee C\subseteq B$

$C \in P(C)$ // power set definition
$P(C) = P(A) \vee P(B) \Rightarrow C \in (P(A) \vee P(B))$
$C \in P(A) \vee C \in P(B)$
$C\subseteq A \vee C\subseteq B$

Second: proof that $A\subseteq C \vee B\subseteq C$

$A \in P(A)$
$P(A) \subseteq P(A) \cup P(B)$ // union definition
$A \in P(A) \cup P(B)$
Given $P(C) = (P(A) \cup P(B)) \Rightarrow A \in P(C)$
$A\subseteq C$

$B \in P(B)$
$P(B) \subseteq P(A) \cup P(B)$ // union definition
$B \in P(A) \cup P(B)$
Given $P(C) = (P(A) \cup P(B)) \Rightarrow B \in P(C)$
$B\subseteq C$

Since $C\subseteq A \vee C\subseteq B$ and $A\subseteq C$ and $B\subseteq C$,
we conclude that:
$C\subseteq A \vee C\subseteq B \wedge A\subseteq C \wedge B\subseteq C$
Therefore
$(C=A) \vee (C=B)$

ג

Prove:
if A,B are finite and $|P(A)| = 2\cdot|P(A\setminus B)|$, then $|A \cap B| = 1$

---

(1)

$A \setminus B \equiv A \setminus (A \cap B)$ // by definition

(2)

We know that for any two sets X,Y, if $Y \subseteq X$ then $|X \setminus Y| = |X| - |X \cap Y|$
Certainly $(A \cap B) \subseteq A$, so
$|A \setminus (A \cap B)| = |A| - |A \cap B|$.

(3)

Assuming $|A \cap B| = 1$, if follows that:
$|A| - |A \cap B| = |A| - 1$, therefore using (1) and (2):
$|A \setminus B| = |A \setminus (A \cap B)| = |A| - |A \cap B| = |A| - 1$, so
$|P(A \setminus B)| = 2\text{^}|A \setminus B| = 2\text{^}(|A| - 1)$

(4): Expanding 2·|P(A\B)|

$2 \cdot |P(A \setminus B)| = 2 \cdot 2\text{^}(|A| - 1) = 2\text{^}|A|$

(5)

$|P(A)| = 2\text{^}|A|$ // by definition

(6)

$|P(A)| = 2 \cdot |P(A \setminus B)|$

3

א

Prove: if $(A \subset B)$, then $(A \cup \neg B) \neq U$

Since A is a **proper** subset of B, then $(B \setminus A) \neq \varnothing$.
Expanding $(B \setminus A)$:
$(B \setminus A) =$
$(B \cap \neg A) =$
$(\neg A \cap B) =$ // comm.
$\overline{(A \cup \neg B)}$ // DeMorgan
Therefore $\overline{(A \cup \neg B)} \neq \varnothing$

Since the complement of a given set X is the universal set (U) if and only if $X=\varnothing$, it follows that the complement of a given set Y is **not** U if and only if $Y \neq \varnothing$.
Because $\overline{(A \cup \neg B)} \neq \varnothing$, then the complement of $\overline{(A \cup \neg B)} \neq U$,
therefore $(A \cup \neg B) \neq U$.

ב

Prove: if $(\neg A \,\Delta\, B) = (\neg B \,\Delta\, C)$, then $A=C$

We know that $(\neg A \,\Delta\, B) = (\neg B \,\Delta\, A)$, because:
$(\neg A \cap \neg B) \cup (B \cap A) = (\neg B \cap \neg A) \cup (A \cap B)$ // symm diff definition

---

It's given that $(\neg A \,\Delta\, B) = (\neg B \,\Delta\, C)$, so
$(\neg B \,\Delta\, A) = (\neg B \,\Delta\, C)$ // $(\neg B \,\Delta\, A) = (\neg A \,\Delta\, B)$

Since for any sets X, Y, Z; if $(X \,\Delta\, Y) = (X \,\Delta\, Z)$, then $X = Z$; if follows that $A=C$