גלעד ברנע ת.ז. 203627310

ממ"ן 11

היי,
אני ממש מצטער על ההגשה המאוחרת.
זה לא הולך להיות הרגל, קרה משהו ספציפי שגרר את זה.
מקווה שזה לא מאוחר מידי

שוב מתנצל, ותודה!

1

א: לא נכון
ב: נכון
ג: לא נכון
ד: נכון
ה: לא נכון
ו: לא נכון
ז: נכון
ח: לא נכון

2

א

Prove:
$(A \setminus B) \cup (B \setminus C) = (A \cup B) \setminus (B \cap C)$

First: expanding left-hand side $(A \setminus B) \cup (B \setminus C)$

$(A \cap \neg B) \cup (B \cap \neg C)$ // difference definition
$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup B) \cap (\neg B \cup \neg C)$ // distributivity
$(A \cup B) \cap (A \cup \neg C) \cap (\neg B \cup \neg C)$ // $(\neg B \cup B) \equiv T$
$\mathbf{(A \cup B) \cap [(A \cap \neg B) \cup \neg C]}$ // dist.

Second: expanding right-hand side $(A \cup B) \setminus (B \cap C)$

$(A \cup B) \cap \overline{(B \cap C)}$
$(A \cup B) \cap (\neg B \cup \neg C)$
$(A \cap \neg B) \cup (A \cap \neg C) \cup (B \cap \neg B) \cup (B \cap \neg C)$ // dist
$(A \setminus B) \cup (A \cap \neg C) \cup (B \cap \neg C)$ // $(B \cap \neg B) \equiv \varnothing$

$(A \setminus B) \cup [(A \cup B) \cap \neg C]$ // dist

$[(A \setminus B) \cup (A \cup B)] \cap [(A \cap \neg B) \cup \neg C]$ // dist

// I'll now prove that $[(A \setminus B) \cup (A \cup B)] \equiv (A \cup B)$,

// then get back to expanding the full statement

> Since $(A \setminus B) \subseteq A$ and $A \subseteq (A \cup B) \Rightarrow$
> $(A \setminus B) \subseteq (A \cup B)$
> Therefore
> $(A \setminus B) \cup (A \cup B) = (A \cup B)$

$(A \cup B) \cap [(A \cap \neg B) \cup \neg C]$

We see that left-hand side ≡ right-hand side, therefore

$(A \setminus B) \cup (B \setminus C) = (A \cup B) \setminus (B \cap C)$

ב

Prove:

if $P(A) \vee P(B) = P(C)$, then $(C=A) \vee (C=B)$

I'll be proving:

$(C \subseteq A \wedge A \subseteq C) \vee (C \subseteq B \wedge B \subseteq C)$

Since it's equivalent to

$(C=A) \vee (C=B)$

First: proof that $C \subseteq A \vee C \subseteq B$

$C \in P(C)$ // power set definition

$P(C) = P(A) \vee P(B) \Rightarrow C \in (P(A) \vee P(B))$

$C \in P(A) \vee C \in P(B)$

$C \subseteq A \vee C \subseteq B$

Second: proof that $A \subseteq C \vee B \subseteq C$

$A \in P(A)$

$P(A) \subseteq P(A) \cup P(B)$ // union definition

$A \in P(A) \cup P(B)$

Given $P(C) = (P(A) \cup P(B)) \Rightarrow A \in P(C)$

$A \subseteq C$

$B \in P(B)$

$P(B) \subseteq P(A) \cup P(B)$ // union definition

$B \in P(A) \cup P(B)$

Given $P(C) = (P(A) \cup P(B)) \Rightarrow B \in P(C)$

$B \subseteq C$

Since $C \subseteq A \lor C \subseteq B$ and $A \subseteq C$ and $B \subseteq C$,
// More formally: $(C \subseteq A \lor C \subseteq B) \land (A \subseteq C \land B \subseteq C)$
it follows that:
$(C=A) \lor (C=B)$

ג

Prove:
if A,B are finite and $|P(A)| = 2 \cdot |P(A \setminus B)|$, then $|A \cap B| = 1$

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(1)

$A \setminus B \equiv A \setminus (A \cap B)$ // by definition

(2)

We know that for any two sets X,Y, if $Y \subseteq X$ then $|X \setminus Y| = |X| - |X \cap Y|$
Certainly $(A \cap B) \subseteq A$, so
$|A \setminus (A \cap B)| = |A| - |A \cap B|$.

(3)

Assuming $|A \cap B| = 1$, if follows that:
$|A| - |A \cap B| = |A| - 1$, therefore using (1) and (2):
$|A \setminus B| = |A \setminus (A \cap B)| = |A| - |A \cap B| = |A| - 1$, so
$|P(A \setminus B)| = 2^{|A \setminus B|} = 2^{(|A| - 1)}$

(4): Expanding $2 \cdot |P(A \setminus B)|$

$2 \cdot |P(A \setminus B)| = 2 \cdot 2^{(|A| - 1)} = 2^{|A|}$

(5)

$|P(A)| = 2^{|A|}$ // by definition

(6): Putting it all together

$|P(A)| = 2 \cdot |P(A \setminus B)|$

3

א

Prove: if $(A \subset B)$, then $(A \cup \neg B) \neq U$

Since A is a **proper** subset of B, then $(B \setminus A) \neq \varnothing$.
Expanding $(B \setminus A) = (B \cap \neg A) = (\neg A \cap B) = \overline{(A \cup \neg B)}$ // DeMorgan

Therefore $\overline{(A \cup \neg B)} \neq \varnothing$

Since the complement of a given set X is the universal set (U) if and only if X=∅, it follows that the complement of a given set Y is **not** U if and only if Y≠∅.
Because $\overline{(A \cup \neg B)} \neq \varnothing$, then the complement of $\overline{(A \cup \neg B)} \neq U$,
therefore **(A ∪ ¬B) ≠ U**.

ב

Prove: if (¬A Δ B) = (¬B Δ C), then A=C.

We know that (¬A Δ B) = (¬B Δ A), because:
(¬A Δ B) = (¬A ∩ ¬B) ∪ (A ∩ ¬¬B) =
(¬A ∩ ¬B) ∪ (B ∩ A)   // double negation
Similarly,
(¬B Δ A) = (¬B ∩ ¬A) ∪ (¬¬B ∩ A) =
(¬A ∩ ¬B) ∪ (B ∩ A)   // double negation, comm.

It's given that (¬A Δ B) = (¬B Δ C), so
(¬B Δ A) = (¬B Δ C)   // replaced (¬A Δ B) with (¬B Δ A)

Since for any sets X, Y, Z:
(X Δ Y) = (X Δ Z) ⇒ X = Z
It follows that
(¬B Δ A) = (¬B Δ C) ⇒ A = C
Therefore **A = C**.