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ממ"ן 11

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Prove: $A\Delta B \subseteq D \wedge B\Delta C \subseteq D \rightarrow A\Delta C \subseteq D$

Since $(X\rightarrow Z) \wedge (Y\rightarrow Z) \equiv (X\vee Y)\rightarrow Z$, also: $(A\Delta B\subseteq D \wedge B\Delta C\subseteq D) \equiv (A\Delta B \cup B\Delta C) \subseteq D$. Proof: $(X\rightarrow Z) \wedge (Y\rightarrow Z)$ $Z\vee\neg X \wedge Z\vee\neg Y$ $Z\vee(X\vee Y)$ $(X\vee Y) \rightarrow Z$

$A\Delta B\subseteq D \wedge B\Delta C\subseteq D$ $(x\in(A\Delta B) \rightarrow x\in D) \wedge (x\in(B\Delta C) \rightarrow x\in D)$ $x\in(A\Delta B \cup B\Delta C) \rightarrow x\in D$ $(A\Delta B \cup B\Delta C) \subseteq D$ Therefore: $(A\Delta B\subseteq D \wedge B\Delta C\subseteq D) \equiv (A\Delta B \cup B\Delta C) \subseteq D$ // (1) I'll prove that $A\Delta C \subseteq (A\Delta B \cup B\Delta C)$, then it would follow by transience that $A\Delta C\subseteq C$ Expanding $(A\Delta B \cup B\Delta C)$: $(B\cap A) \cup (B\cap C) \cup (B\cap A) \cup (B\cap C)$ $[Bn(A\cup C)] \cup [Bn(AuC)]$ **$(A\Delta B \cup B\Delta C) \equiv [Bn(A\cup C)] \cup [Bn(AuC)]$** // (2)

Expanding $A\Delta C$: (AnC) u (AnC) $(A\cup C)$ n (AuC) **$A\Delta C \equiv (A\cup C)$ n (AuC)** // (3)

Define: $P = B$; $Q = (A\cup C)$; $R = (AuC)$ So proving $Q\wedge R \rightarrow (\neg P\vee Q \vee P\wedge R)$, will prove that $A\Delta C \subseteq (A\Delta B \cup B\Delta C)$ Proving $Q\wedge R \rightarrow (\neg P\vee Q \vee P\wedge R)$ is always true: Premise: $(Q\wedge R)$; Conclusion: $(\neg P\vee Q \vee P\wedge R)$. Premise $\rightarrow$ Conclusion is always true if the following holds: Whenever the Premise is true, also the Conclusion is true. Assuming that Premise is true: $Q\wedge R \equiv \mathbf{T} \Longrightarrow Q\equiv\mathbf{T} \wedge R\equiv\mathbf{T}$ Using that in the Conclusion: $(\neg P\vee Q \vee P\wedge R) \equiv (\neg P\vee\mathbf{T} \vee P\wedge\mathbf{T}) \equiv \neg P\vee P \equiv \mathbf{T}$ Therefore the conclusion is dependent upon the premise, therefore **$Q\wedge R \rightarrow (\neg P\vee Q \vee P\wedge R) \equiv T$** // (4) P, Q and R are placeholders (defined above), so I'll use their "real" values: $Q\wedge R \rightarrow (\neg P\vee Q \vee P\wedge R) \equiv (A\cup C)\cap(AuC) \rightarrow [Bn(A\cup C)] \cup [Bn(AuC)]$ // Using (3): $A\Delta C \subseteq [Bn(A\cup C)] \cup [Bn(AuC)]$ // Using (2): **$A\Delta C \subseteq (A\Delta B \cup B\Delta C)$** // (5) Since it's given that: $A\Delta B\subseteq D \wedge B\Delta C\subseteq D$ Using (1), it's equivalent to: $(A\Delta B \cup B\Delta C) \subseteq D$ And because of (5), we know that $A\Delta C \subseteq (A\Delta B \cup B\Delta C)$ Together with the transience of $\subseteq$, // $X\subseteq Y$ and $Y\subseteq Z \Rightarrow X\subseteq Z$ **$A\Delta B\subseteq D \wedge B\Delta C\subseteq D \rightarrow A\Delta C\subseteq D$**