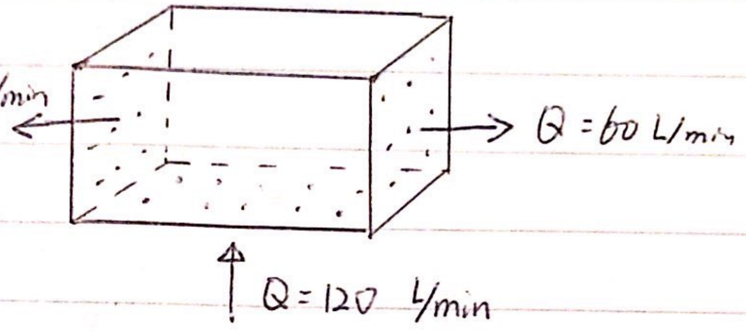
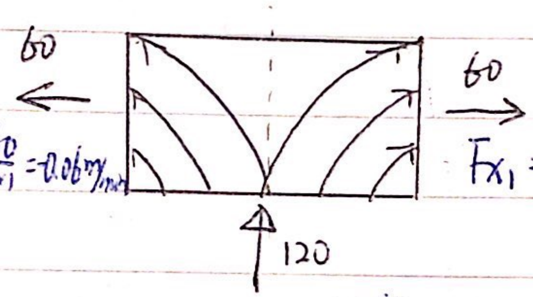
Assignment 1 : Potential Flow

In 3D.



In 2D:



$F_{x2} = \frac{-60}{1\times1} = -0.06\ m/min$

$F_{x1} = \frac{60}{1\times1} = 0.06\ m/min$

$F_y = \frac{120\ L/min}{1\times2\ m^2} = 0.06\ m/min$

(a): Stream Function at boundaries :

Solution:

$\rightarrow \quad F_y = -\frac{\partial\psi_{bot}}{\partial x} = 0.06 \quad \rightarrow \quad \psi_{bot} = -0.06x + A$

$F_y' = -\frac{\partial\psi_{top}}{\partial x} = 0 \quad \rightarrow \quad \psi_{top} = B$

$F_{x1} = \frac{\partial\psi_{right}}{\partial y} = 0.06 \rightarrow \psi_{right} = 0.06y + C$

$F_{x2} = \frac{\partial\psi_{left}}{\partial y} = -0.06 \rightarrow \psi_{left} = -0.06y + D$

$\rightarrow$ Set $\psi_{(2,0)} = 0$: $\psi_{bot} = -0.06x + 0.12 \quad \psi_{top} = 60$

$\psi_{right} = 0.06y \quad \psi_{left} = -0.06y + 0.12$

(b) Solve for Stream Function for internal grids:

SOR:

$$U_{ij}^{new} = U_{ij}^{old} - \omega \frac{\xi_{ij}}{e_{ij}}$$

See the code for details.

(C): Obtain Liquid flux: $\vec{V} = \begin{pmatrix} \frac{\partial\psi}{\partial y} \\ -\frac{\partial\psi}{\partial x} \end{pmatrix}$

See the code for details.

(d): Analytical Solution:

$$\psi = X(x)\,Y(y) \rightarrow \frac{1}{X(x)}\frac{d^2X(x)}{dx^2} + \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} = 0$$

$$\begin{cases} X = \gamma\cosh(\lambda x) + \delta\sinh(\lambda x) \\ Y = \alpha\cos(\lambda y) + \beta\sin(\lambda y) \end{cases}$$

$\psi = \psi_1 + \psi_2 + \psi_3 + \psi_4$, use $\psi_1, \psi_2, \psi_3, \psi_4$ to simulate boundaries

Use ψ_1 as a solution, & $\psi_L = \psi_1(0, y)$, while $\psi_R = \psi_T = \psi_B = 0$

Use same approach as Lecture notes. we can get:

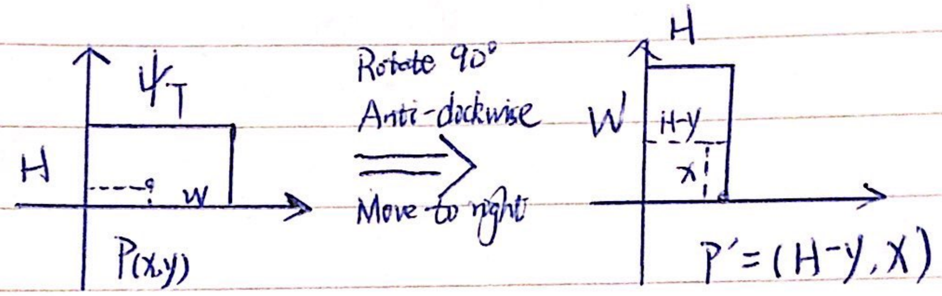
$$\psi_1 = \beta\sin\frac{n\pi y}{H}\left(\gamma\cosh\left(\frac{n\pi x}{H}\right) - \gamma\coth\left(\frac{n\pi w}{H}\right)\cdot\sinh\left(\frac{n\pi x}{H}\right)\right)$$

Since $\gamma\beta \sin\left(\frac{n\pi y}{H}\right) = \sum_{n=1}^{\infty} \gamma_n \sin\left(\frac{n\pi y}{H}\right)$:

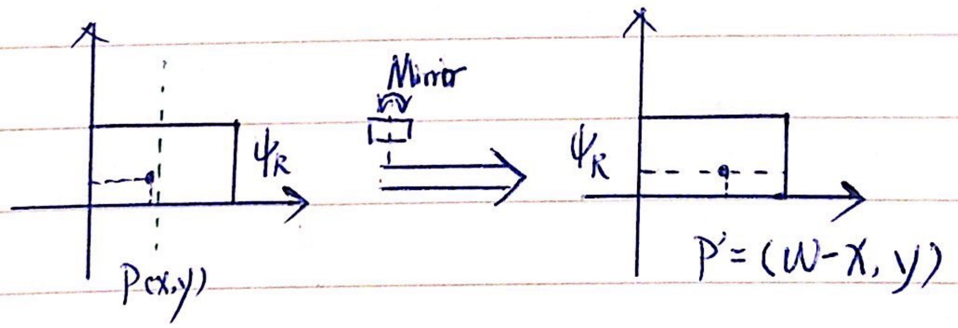
$$\begin{cases} \psi_1 = \sum_{n=1}^{\infty} \gamma_n \sin\left(\frac{n\pi y}{H}\right)\left(\cosh\frac{n\pi x}{H} - \coth\left(\frac{n\pi W}{H}\right)\sin\left(\frac{n\pi x}{H}\right)\right) \\ \gamma_n = \frac{2}{H}\int_0^H \psi_L \sin\frac{n\pi y}{H}\, dy \end{cases}$$

Use coordination transformation to get other 3 boundary conditions:

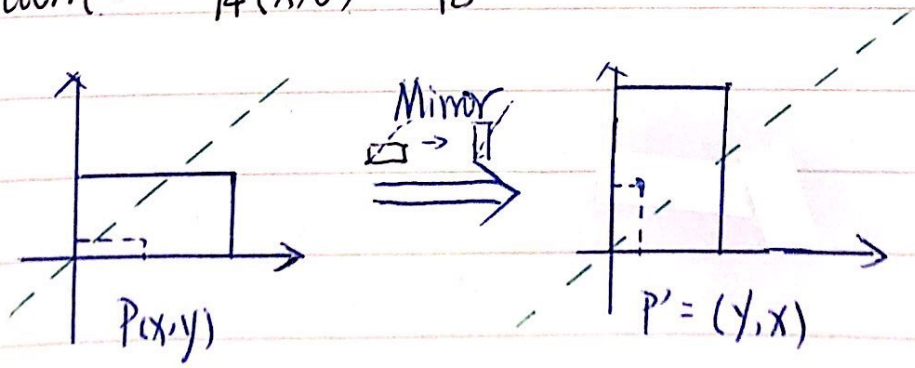
① Top : $\psi_2(x,H) = \psi_T$:



② Right: $\psi_3(W,y) = \psi_R$:



③ Bottom : $\psi_4(x,0) = \psi_B$:



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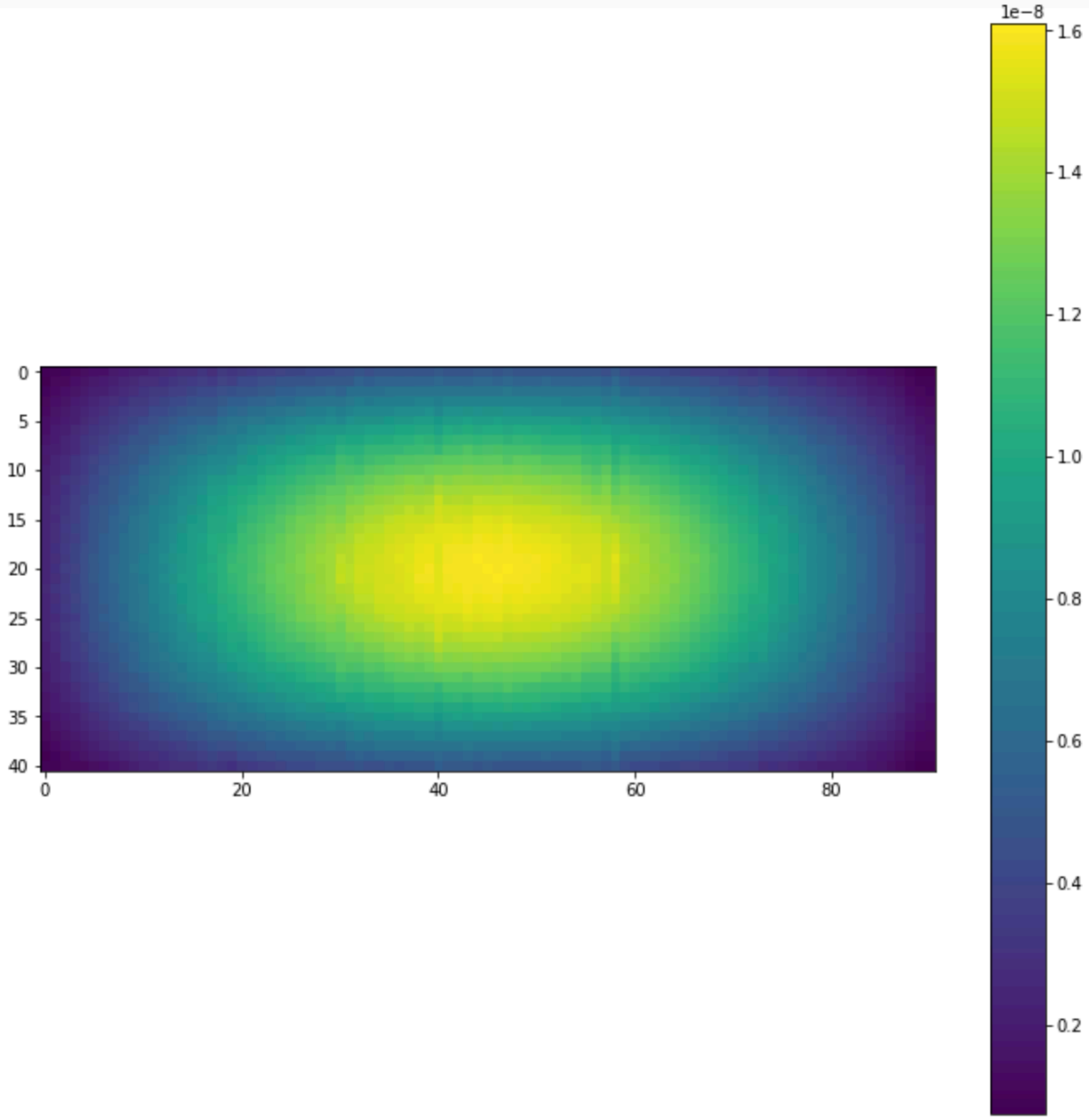
$$Y_n = \frac{2}{H}\int_0^H \psi_L \sin\left(\frac{n\pi x}{H}\right) dy$$

set ψ_L as linear :

$$\psi_L = my + c$$

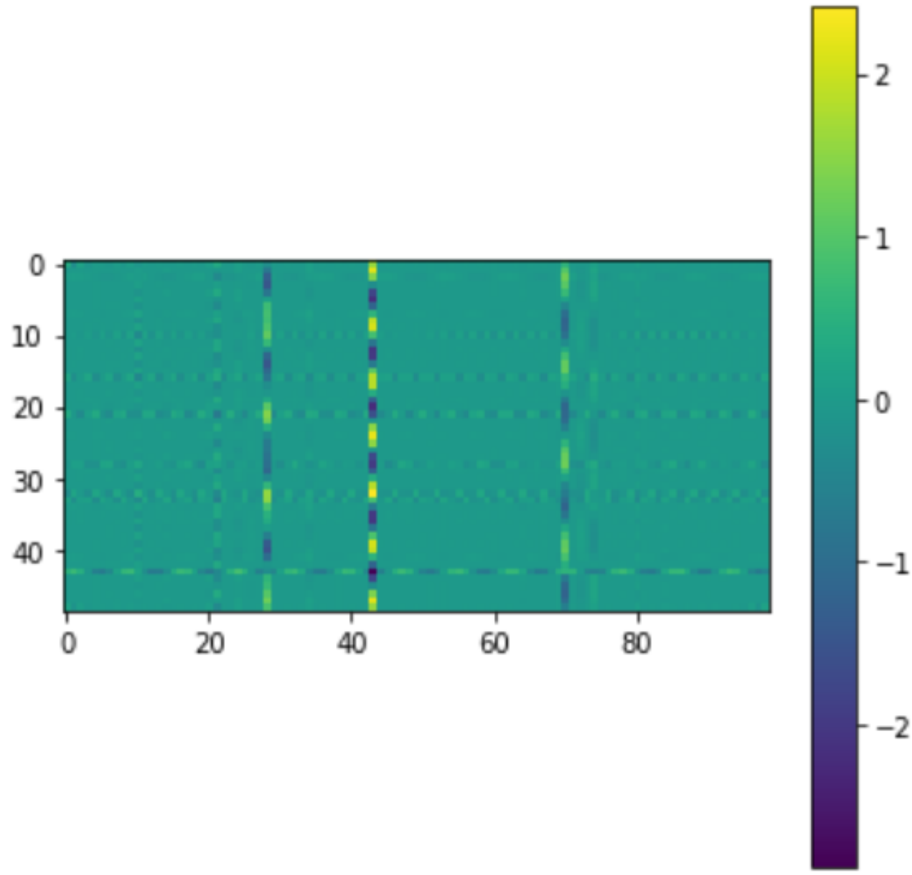
$$\therefore Y_n = \frac{2}{H}\int_0^H (my+c)\sin\left(\frac{n\pi x}{H}\right) dy$$

$$= -\frac{2}{\pi n}\left((-1)^n H m + (-1)^n c - c\right)$$



Discrepancy shows a periodic pattern

Implies the periodic error from Fourier Series



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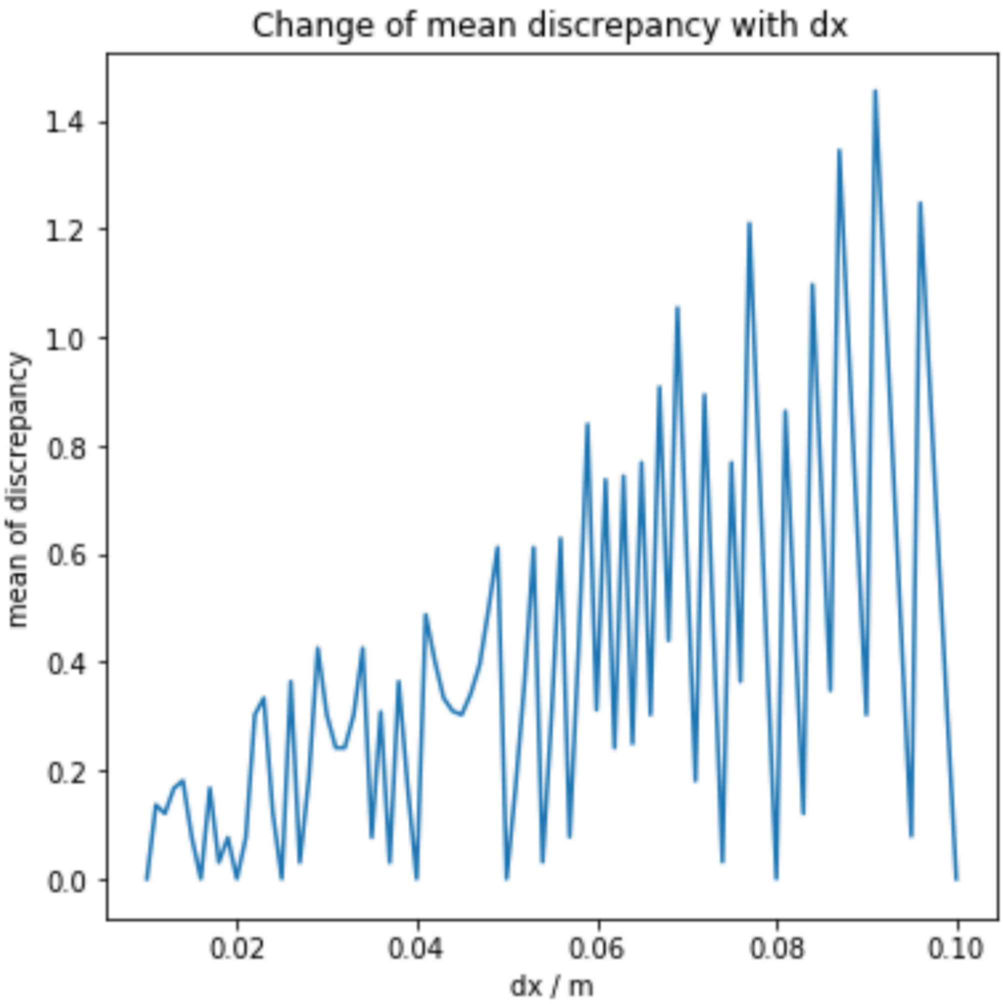

Discrepancy with changing dx
(dx in 0.01~0.1, with interval of 0.001)