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```

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```

# 1 Basic

## 1.1 Default code [f616de]

```

#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
#define X first
#define Y second
#define SZ(a) ((int)a.size())
#define ALL(v) v.begin(), v.end()
#define pb push_back

```

## 1.2 test [e6a98d]

```

#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
#define int ll
#define pii pair<int, int>
#define X first
#define Y second
#define F first
#define S second
#define vi vector<int>
#define SZ(a) ((int)a.size())
#define ALL(v) v.begin(), v.end()
#define pb push_back
#define eb emplace_back
#define push emplace
#define lb(x, v) lower_bound(ALL(x), v)
#define ub(x, v) upper_bound(ALL(x), v)
#define re(x) reverse(ALL(x))
#define uni(x) x.resize(unique(ALL(x)) - x.begin())
#define inf 1000000000
#define INF 1000000000000000000
#define mod 1000000007
#define MOD 998244353
#define get_bit(x, y) ((x>>y)&1)
#define mkp make_pair
#define IO ios_base::sync_with_stdio(0); cin.tie(0);
void abc() {cerr << endl;}
template
<typename T, typename ...U> void abc(T a, U ...b) {
    cerr << a << ' ', abc(b...);
}
#ifdef debug
#define
    test(args...) abc("[ " + string(#args) + "]", args)
#else
#define test(args...) void(0)
#endif

template<class T> bool ckmin
(T& a, const T& b) { return b<a ? a=b, 1 : 0; }
template<class T> bool ckmax
(T& a, const T& b) { return a<b ? a=b, 1 : 0; }

inline void solve() {
}

signed main() {
    IO;
    solve();
}

```

## 1.3 Shell script [5dd9f6]

```

g++ -O2 -
std=c++17 -Dbbq -Wall -Wextra -Wshadow -o $1 $1.cpp
chmod +x compile.sh

```

## 1.4 Pragma [5feeb8]

```

#pragma GCC optimize("Ofast,no-stack-protector")
#pragma GCC optimize("no-math-errno,unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target("popcnt,abm,mmx,avx,arch=skylake")
__builtin_ia32_ldmxcsr(__builtin_ia32_stmxcsr())|0x8040)

```

## 1.5 readchar [dacef1]

```

inline char readchar() {
    static const size_t bufsize = 65536;
    static char buf[bufsize];
    static char *p = buf, *end = buf;
    if (p == end) end = buf +
        fread_unlocked(buf, 1, bufsize, stdin), p = buf;
    return *p++;
}

```

## 1.6 vimrc [152334]

```

"This file should be placed at ~/.vimrc"
se nu ai hls et ru ic is sc cul
se re=1 ts=4 sts=4 sw=4 ls=2 mouse=a
syntax on
hi cursorline cterm=none ctermbg=89
set bg=dark
inoremap {<CR> {<CR><Esc>ko<tab>
"Select region and type :Hash to hash your selection."
ca Hash w !cpp -dD -P -fpreprocessed
    \ | tr -d '[:space:]' \ | md5sum \ | cut -c-6
testing

```

## 1.7 Texas holdem [61f99b]

```

char suit[4]={'C','D','H','S'}, ranks[13]={'2',
    '3','4','5','6','7','8','9','T','J','Q','K','A'};
int rk[256];
/*
    for(int i=0;i<13;++i)
        rk[ranks[i]]=i;
    for(int i=0;i<4;++i)
        rk[suit[i]]=i;
*/
struct cards{
    vector<pii> v;
    int suit_count[4],hands;
    void reset(){v.clear(),FILL(suit_count,0),hands=-1;}
    void insert(char a,char b){//suit,rank
        ++suit_count[rk[a]];
        int flag=0;
        for(auto &i:v)
            if(i.Y==rk[b])
                {
                    ++i.X,flag=1;
                    break;
                }
        if(!flag) v.pb(pii(1,rk[b]));
    }
    void insert(string s){insert(s[0],s[1]);}
    void ready(){
        int Straight=0,Flush
            =(max_element(suit_count,suit_count+4)==5);
        sort(ALL(v),[](ii a,ii b){return a>b;});
        if(SZ(v)==5&&v[0].Y==v[1].Y+1&&v[1].Y
            ==v[2].Y+1&&v[2].Y==v[3].Y+1&&v[3].Y==v[4].Y+1)
            Straight=1;
        else if(SZ(v)==5&&v[0].Y==12&&
            v[1].Y==3&&v[2].Y==2&&v[3].Y==1&&v[4].Y==0)
            v[0].Y=3,v[1].Y=2,v[2].Y=1,v[3].Y=0,v[4].Y=-1,Straight=1;
        if(Straight&&Flush) hands=1;
        else if(v[0].X==4) hands=2;
        else if(v[0].X==3&&v[1].X==2) hands=3;
        else if(Flush) hands=4;
        else if(Straight) hands=5;
        else if(v[0].X==3) hands=6;
        else if(v[0].X==2&&v[1].X==2) hands=7;
        else if(v[0].X==2) hands=8;
        else hands=9;
    }
    bool operator>(const cards &a)const{
        if(hands==a.hands) return v>a.v;
        return hands<a.hands;
    }
};

```

## 1.8 black magic [9612b4]

```

#include <ext/pb_ds/priority_queue.hpp>
#include <ext/pb_ds/assoc_container.hpp> // rb_tree
#include <ext/rope> // rope
using namespace __gnu_pbds;
using namespace __gnu_cxx; // rope
typedef __gnu_pbds::priority_queue<int> heap;
int main() {

```

```

heap h1, h2; // max heap
h1.push(1), h1.push(3), h2.push(2), h2.push(4);
h1.join(h2); // h1 = {1, 2, 3, 4}, h2 = {};
tree<ll, null_type, less<ll>, rb_tree_tag
    , tree_order_statistics_node_update> st;
tree<ll, ll, less<ll>, rb_tree_tag
    , tree_order_statistics_node_update> mp;
for (int x : {0, 3, 20, 50}) st.insert(x);
assert(st.
    order_of_key(3) == 1 && st.order_of_key(4) == 2);
assert(*st.find_by_order
    (2) == 20 && *st.lower_bound(4) == 20);
rope<char> *root[10]; // nsqrt(n)
root[0] = new rope<char>();
root[1] = new rope<char>(*root[0]);
// root[1]->insert(pos, 'a');
// root[1]->at(pos); 0-base
// root[1]->erase(pos, size);
}
// __int128_t, __float128_t
// for (int i = bs._Find_first
    ()); i < bs.size(); i = bs._Find_next(i));

```

## 2 Graph

### 2.1 SCC [517e91]

```

struct SCC { // 0-base
    int n, dft, nsc;
    vector<int> low, dfn, bln, instack, stk;
    vector<vector<int>> G;
    void dfs(int u) {
        low[u] = dfn[u] = ++dft;
        instack[u] = 1, stk.pb(u);
        for (int v : G[u])
            if (!dfn[v])
                dfs(v), low[u] = min(low[u], low[v]);
            else if (instack[v] && dfn[v] < dfn[u])
                low[u] = min(low[u], dfn[v]);
        if (low[u] == dfn[u]) {
            for (; stk.back() != u; stk.pop_back())
                bln[stk
                    .back()] = nsc, instack[stk.back()] = 0;
            instack[u] = 0, bln[u] = nsc++, stk.pop_back();
        }
    }
    SCC(int _n): n(_n), dft(), nsc
        (), low(n), dfn(n), bln(n), instack(n), G(n) {}
    void add_edge(int u, int v) {
        G[u].pb(v);
    }
    void solve() {
        for (int i = 0; i < n; ++i)
            if (!dfn[i]) dfs(i);
    }
}; // scc_id(i): bln[i]

```

### 2.2 Minimum Arborescence [4c8d8d]

```

struct zhu_liu { // O(VE)
    struct edge {
        int u, v;
        ll w;
    };
    vector<edge> E; // 0-base
    int pe[N], id[N], vis[N];
    ll in[N];
    void init() { E.clear(); }
    void add_edge(int u, int v, ll w) {
        if (u != v) E.pb(edge{u, v, w});
    }
    ll build(int root, int n) {
        ll ans = 0;
        for (;;) {
            fill_n(in, n, INF);
            for (int i = 0; i < SZ(E); ++i)
                if (E[i].u != E[i].v && E[i].w < in[E[i].v])
                    pe[E[i].v] = i, in[E[i].v] = E[i].w;
            for (int u = 0; u < n; ++u) // no solution
                if (u != root && in[u] == INF) return -INF;
            int cntnode = 0;
            fill_n(id, n, -1), fill_n(vis, n, -1);
            for (int u = 0; u < n; ++u) {
                if (u != root) ans += in[u];
                int v = u;
                while (vis[v] != u && !~id[v] && v != root)
                    vis[v] = u, v = E[pe[v]].u;
            }
        }
    }
};

```

```

    if (v != root && !~id[v]) {
        for (int x = E[pe[v]].u; x != v;
            x = E[pe[x]].u)
            id[x] = cntnode;
        id[v] = cntnode++;
    }
}
if (!cntnode) break; // no cycle
for (int u = 0; u < n; ++u)
    if (!~id[u]) id[u] = cntnode++;
for (int i = 0; i < SZ(E); ++i) {
    int v = E[i].v;
    E[i].u = id[E[i].u], E[i].v = id[E[i].v];
    if (E[i].u != E[i].v) E[i].w -= in[v];
}
n = cntnode, root = id[root];
}
return ans;
}
};

```

### 2.3 Dominator Tree [915f9c]

```

struct dominator_tree { // 1-base
    vector<int> G[N], rG[N];
    int n, pa[N], dfn[N], id[N], Time;
    int semi[N], idom[N], best[N];
    vector<int> tree[N]; // dominator_tree
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), rG[i].clear();
    }
    void add_edge(int u, int v) {
        G[u].pb(v), rG[v].pb(u);
    }
    void dfs(int u) {
        id[dfn[u] = ++Time] = u;
        for (auto v : G[u])
            if (!dfn[v]) dfs(v), pa[dfn[v]] = dfn[u];
    }
    int find(int y, int x) {
        if (y <= x) return y;
        int tmp = find(pa[y], x);
        if (semi[best[y]] > semi[best[pa[y]]])
            best[y] = best[pa[y]];
        return pa[y] = tmp;
    }
    void tarjan(int root) {
        Time = 0;
        for (int i = 1; i <= n; ++i) {
            dfn[i] = idom[i] = 0;
            tree[i].clear();
            best[i] = semi[i] = i;
        }
        dfs(root);
        for (int i = Time; i > 1; --i) {
            int u = id[i];
            for (auto v : rG[u])
                if (v = dfn[v]) {
                    find(v, i);
                    semi[i] = min(semi[i], semi[best[v]]);
                }
            tree[semi[i]].pb(i);
            for (auto v : tree[pa[i]]) {
                find(v, pa[i]);
                idom[v] =
                    semi[best[v]] == pa[i] ? pa[i] : best[v];
            }
            tree[pa[i]].clear();
        }
        for (int i = 2; i <= Time; ++i) {
            if (idom[i] != semi[i]) idom[i] = idom[idom[i]];
            tree[id[idom[i]]].pb(id[i]);
        }
    }
};

```

### 2.4 MinimumMeanCycle [e8ed41]

```

ll road[N][N]; // input here
struct MinimumMeanCycle {
    ll dp[N + 5][N], n;
    pll solve() {
        ll a = -1, b = -1, L = n + 1;
        for (int i = 2; i <= L; ++i)
            for (int k = 0; k < n; ++k)

```

```

    for (int j = 0; j < n; ++j)
        dp[i][j] = min(dp[i - 1][k] + road[k][j], dp[i][j]);
    for (int i = 0; i < n; ++i) {
        if (dp[L][i] >= INF) continue;
        ll ta = 0, tb = 1;
        for (int j = 1; j < n; ++j)
            if (dp[j][i] < INF &&
                ta * (L - j) < (dp[L][i] - dp[j][i]) * tb)
                ta = dp[L][i] - dp[j][i], tb = L - j;
        if (ta == 0) continue;
        if (a == -1 || a * tb > ta * b) a = ta, b = tb;
    }
    if (a != -1) {
        ll g = __gcd(a, b);
        return pll(a / g, b / g);
    }
    return pll(-1LL, -1LL);
}
void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j) dp[i + 2][j] = INF;
}
};

```

## 2.5 Minimum Clique Cover [745700]

```

struct Clique_Cover { // 0-base, O(n^2n)
    int co[1 << N], n, E[N];
    int dp[1 << N];
    void init(int _n) {
        n = _n, fill_n(dp, 1 << n, 0);
        fill_n(E, n, 0), fill_n(co, 1 << n, 0);
    }
    void add_edge(int u, int v) {
        E[u] |= 1 << v, E[v] |= 1 << u;
    }
    int solve() {
        for (int i = 0; i < n; ++i)
            co[1 << i] = E[i] | (1 << i);
        co[0] = (1 << n) - 1;
        dp[0] = (n & 1) * 2 - 1;
        for (int i = 1; i < (1 << n); ++i) {
            int t = i & -i;
            dp[i] = -dp[i ^ t];
            co[i] = co[i ^ t] & co[t];
        }
        for (int i = 0; i < (1 << n); ++i)
            co[i] = (co[i] & i) == i;
        fwt(co, 1 << n, 1);
        for (int ans = 1; ans < n; ++ans) {
            int sum = 0; // probabilistic
            for (int i = 0; i < (1 << n); ++i)
                sum += (dp[i] * co[i]);
            if (sum) return ans;
        }
        return n;
    }
};

```

## 2.6 Maximum Clique Dyn [09472e]

```

struct MaxClique { // fast when N <= 100
    bitset<N> G[N], cs[N];
    int ans, sol[N], q, cur[N], d[N], n;
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].reset();
    }
    void add_edge(int u, int v) {
        G[u][v] = G[v][u] = 1;
    }
    void pre_dfs(vector<int> &r, int l, bitset<N> mask) {
        if (l < 4) {
            for (int i : r) d[i] = (G[i] & mask).count();
            sort(ALL(r), [&](int x, int y) { return d[x] > d[y]; });
        }
        vector<int> c(SZ(r));
        int lft = max(ans - q + 1, 1), rgt = 1, tp = 0;
        cs[1].reset(), cs[2].reset();
        for (int p : r) {
            int k = 1;
            while ((cs[k] & G[p]).any()) ++k;
            if (k > rgt) cs[++rgt + 1].reset();
            cs[k][p] = 1;
        }
    }
};

```

```

        if (k < lft) r[tp++] = p;
    }
    for (int k = lft; k <= rgt; ++k)
        for (int p = cs[k]._Find_first(); p < N; p = cs[k]._Find_next(p))
            r[tp] = p, c[tp] = k, ++tp;
    dfs(r, c, l + 1, mask);
}
void dfs(vector<int> &r, vector<int> &c, int l, bitset<N> mask) {
    while (!r.empty()) {
        int p = r.back();
        r.pop_back(), mask[p] = 0;
        if (q + c.back() <= ans) return;
        cur[q++] = p;
        vector<int> nr;
        for (int i : r) if (G[p][i]) nr.pb(i);
        if (!nr.empty()) pre_dfs(nr, l, mask & G[p]);
        else if (q > ans) ans = q, copy_n(cur, q, sol);
        c.pop_back(), --q;
    }
}
int solve() {
    vector<int> r(n);
    ans = q = 0, iota(ALL(r), 0);
    pre_dfs(r, 0, bitset<N>(string(n, '1')));
    return ans;
}
};

```

## 2.7 Minimum Arborescence fast [48ee1b]

```

/* TODO
DSU: disjoint set
- DSU(n), .boss(x), .Union(x, y)
min_heap<
    T, Info>: min heap for type {T, Info} with lazy tag
- .push({w, i}),
  .top(), .join(heap), .pop(), .empty(), .add_lazy(v)
*/
struct E { int s, t; ll w; }; // 0-base
vector<int> dmst(const vector<E> &e, int n, int root) {
    vector<min_heap<ll, int>> h(n * 2);
    for (int i = 0; i < SZ(e); ++i)
        h[e[i].t].push({e[i].w, i});
    DSU dsu(n * 2);
    vector<int> v(n * 2, -1), pa(n * 2, -1), r(n * 2);
    v[root] = n + 1;
    int pc = n;
    for (int i = 0; i < n; ++i) if (v[i] == -1) {
        for (int p = i; v[p] == -1 || v[p] == i; p = dsu.boss(e[r[p]].s)) {
            if (v[p] == i) {
                int q = p; p = pc++;
                do {
                    h[q].add_lazy(-h[q].top().X);
                    pa[q] = p, dsu.Union(p, q), h[p].join(h[q]);
                } while ((q = dsu.boss(e[r[q]].s)) != p);
            }
            v[p] = i;
            while (!h[p].empty() && dsu.boss(e[h[p].top().Y].s) == p)
                h[p].pop();
            if (h[p].empty()) return {}; // no solution
            r[p] = h[p].top().Y;
        }
    }
    vector<int> ans;
    for (int i = pc
        - 1; i >= 0; i--) if (i != root && v[i] != n) {
        for (int f = e[r[i]].t; ~f && v[f] != n; f = pa[f])
            v[f] = n;
        ans.pb(r[i]);
    }
    return ans; // default minimize, returns edgeid array
} // O(Ef(E)), f(E) from min_heap

```

## 2.8 BCC Vertex [f56bab]

```

struct BCC { // 0-base
    int n, dft, nbcc;
    vector<int> low, dfn, bln, stk, is_ap, cir;
    vector<vector<int>> G, bcc, nG;
    void make_bcc(int u) {
        bcc.emplace_back(1, u);
        for (; stk.back() != u; stk.pop_back())
            bln[stk.back()] = nbcc, bcc[nbcc].pb(stk.back());
    }
};

```

```

    stk.pop_back(), bln[u] = nbcc++;
}
void dfs(int u, int f) {
    int child = 0;
    low[u] = dfn[u] = ++dft, stk.pb(u);
    for (int v : G[u])
        if (!dfn[v]) {
            dfs(v, u), ++child;
            low[u] = min(low[u], low[v]);
            if (dfn[u] <= low[v]) {
                is_ap[u] = 1, bln[u] = nbcc;
                make_bcc(v), bcc.back().pb(u);
            }
        } else if (dfn[v] < dfn[u] && v != f)
            low[u] = min(low[u], dfn[v]);
    if (f == -1 && child < 2) is_ap[u] = 0;
    if (f == -1 && child == 0) make_bcc(u);
}
BCC(int _n): n(_n), dft(),
    nbcc(), low(n), dfn(n), bln(n), is_ap(n), G(n) {}
void add_edge(int u, int v) {
    G[u].pb(v), G[v].pb(u);
}
void solve() {
    for (int i = 0; i < n; ++i)
        if (!dfn[i]) dfs(i, -1);
}
void block_cut_tree() {
    cir.resize(nbcc);
    for (int i = 0; i < n; ++i)
        if (is_ap[i])
            bln[i] = nbcc++;
    cir.resize(nbcc, 1), nG.resize(nbcc);
    for (int i = 0; i < nbcc && !cir[i]; ++i)
        for (int j : bcc[i])
            if (is_ap[j])
                nG[i].pb(bln[j]), nG[bln[j]].pb(i);
} // up to 2 * n - 2 nodes!! bln[i] for id
};

```

## 2.9 NumberofMaximalClique [66fef5]

```

struct BronKerbosch { // 1-base
    int n, a[N], g[N][N];
    int S, all[N][N], some[N][N], none[N][N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j) g[i][j] = 0;
    }
    void add_edge(int u, int v) {
        g[u][v] = g[v][u] = 1;
    }
    void dfs(int d, int an, int sn, int nn) {
        if (S > 1000) return; // pruning
        if (sn == 0 && nn == 0) ++S;
        int u = some[d][0];
        for (int i = 0; i < sn; ++i) {
            int v = some[d][i];
            if (g[u][v]) continue;
            int tsn = 0, tnn = 0;
            copy_n(all[d], an, all[d + 1]);
            all[d + 1][an] = v;
            for (int j = 0; j < sn; ++j)
                if (g[v][some[d][j]])
                    some[d + 1][tsn++] = some[d][j];
            for (int j = 0; j < nn; ++j)
                if (g[v][none[d][j]])
                    none[d + 1][tnn++] = none[d][j];
            dfs(d + 1, an + 1, tsn, tnn);
            some[d][i] = 0, none[d][nn++] = v;
        }
    }
    int solve() {
        iota(some[0], some[0] + n, 1);
        S = 0, dfs(0, 0, n, 0);
        return S;
    }
};

```

## 2.10 2SAT [d0abc7]

```

struct SAT { // 0-base
    int n;
    vector<bool> istrue;
    SCC scc;
    SAT(int _n): n(_n), istrue(n + n), scc(n + n) {}

```

```

    int rv(int a) {
        return a >= n ? a - n : a + n;
    }
    void add_clause(int a, int b) {
        scc.add_edge(rv(a), b), scc.add_edge(rv(b), a);
    }
    bool solve() {
        scc.solve();
        for (int i = 0; i < n; ++i) {
            if (scc.bln[i] == scc.bln[i + n]) return false;
            istrue[i] = scc.bln[i] < scc.bln[i + n];
            istrue[i + n] = !istrue[i];
        }
        return true;
    }
};

```

## 2.11 Virtual Tree [551777]

```

vector<int> vG[N];
int top, st[N];

void insert(int u) {
    if (top == -1) return st[++top] = u, void();
    int p = LCA(st[top], u);
    if (p == st[top]) return st[++top] = u, void();
    while (top >= 1 && dep[st[top - 1]] >= dep[p])
        vG[st[top - 1]].pb(st[top]), --top;
    if (st[top] != p)
        vG[p].pb(st[top]), --top, st[++top] = p;
    st[++top] = u;
}

void reset(int u) {
    for (int i : vG[u]) reset(i);
    vG[u].clear();
}

void solve(vector<int> &v) {
    top = -1;
    sort(ALL(v),
        [&](int a, int b) { return dfn[a] < dfn[b]; });
    for (int i : v) insert(i);
    while (top > 0) vG[st[top - 1]].pb(st[top]), --top;
    // do something
    reset(v[0]);
}

```

## 2.12 Bridge [f72ae7]

```

struct ECC { // 0-base
    int n, dft, ecnt, necc;
    vector<int> low, dfn, bln, is_bridge, stk;
    vector<vector<pii>> G;
    void dfs(int u, int f) {
        dfn[u] = low[u] = ++dft, stk.pb(u);
        for (auto [v, e] : G[u])
            if (!dfn[v])
                dfs(v, e), low[u] = min(low[u], low[v]);
            else if (e != f)
                low[u] = min(low[u], dfn[v]);
        if (low[u] == dfn[u]) {
            if (f != -1) is_bridge[f] = 1;
            for (; stk.back() != u; stk.pop_back())
                bln[stk.back()] = necc;
            bln[u] = necc++, stk.pop_back();
        }
    }
    ECC(int _n): n(_n), dft(),
        ecnt(), necc(), low(n), dfn(n), bln(n), G(n) {}
    void add_edge(int u, int v) {
        G[u].pb(pii(v, ecnt)), G[v].pb(pii(u, ecnt++));
    }
    void solve() {
        is_bridge.resize(ecnt);
        for (int i = 0; i < n; ++i)
            if (!dfn[i]) dfs(i, -1);
    }
}; // ecc_id(i): bln[i]

```

## 2.13 MinimumSteinerTree [e6662f]

```

struct SteinerTree { // 0-base
    int n, dst[N][N], dp[1 << T][N], tdst[N];
    int vcst[N]; // the cost of vertices
    void init(int _n) {
        n = _n;

```

```

    for (int i = 0; i < n; ++i) {
        fill_n(dst[i], n, INF);
        dst[i][i] = vcst[i] = 0;
    }
}
void chmin(int &x, int val) {
    x = min(x, val);
}
void add_edge(int ui, int vi, int wi) {
    chmin(dst[ui][vi], wi);
}
void shortest_path() {
    for (int k = 0; k < n; ++k)
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                chmin(dst[i][j], dst[i][k] + dst[k][j]);
}
int solve(const vector<int>& ter) {
    shortest_path();
    int t = SZ(ter), full = (1 << t) - 1;
    for (int i = 0; i <= full; ++i)
        fill_n(dp[i], n, INF);
    copy_n(vcst, n, dp[0]);
    for (int msk = 1; msk <= full; ++msk) {
        if (!(msk & (msk - 1))) {
            int who = __lg(msk);
            for (int i = 0; i < n; ++i)
                dp[msk]
                    [i] = vcst[ter[who]] + dst[ter[who]][i];
        }
        for (int i = 0; i < n; ++i)
            for (int sub = (
                msk - 1) & msk; sub; sub = (sub - 1) & msk)
                chmin(dp[msk][i],
                    dp[sub][i] + dp[msk ^ sub][i] - vcst[i]);
        for (int i = 0; i < n; ++i) {
            tdst[i] = INF;
            for (int j = 0; j < n; ++j)
                chmin(tdst[i], dp[msk][j] + dst[j][i]);
        }
        copy_n(tdst, n, dp[msk]);
    }
    return *min_element(dp[full], dp[full] + n);
}
}; // O(V 3^T + V^2 2^T)

```

## 2.14 Vizing [a24f68]

```

namespace vizing { // returns
    edge coloring in adjacent matrix G. 1 - based
const int N = 105;
int C[N][N], G[N][N], X[N], vst[N], n;
void init(int _n) { n = _n;
    for (int i = 0; i <= n; ++i)
        for (int j = 0; j <= n; ++j)
            C[i][j] = G[i][j] = 0;
}
void solve(vector<pii> &E) {
    auto update = [&](int u) {
        for (X[u] = 1; C[u][X[u]]; ++X[u]);
    };
    auto color = [&](int u, int v, int c) {
        int p = G[u][v];
        G[u][v] = G[v][u] = c;
        C[u][c] = v, C[v][c] = u;
        C[u][p] = C[v][p] = 0;
        if (p) X[u] = X[v] = p;
        else update(u), update(v);
        return p;
    };
};
auto flip = [&](int u, int c1, int c2) {
    int p = C[u][c1];
    swap(C[u][c1], C[u][c2]);
    if (p) G[u][p] = G[p][u] = c2;
    if (!C[u][c1]) X[u] = c1;
    if (!C[u][c2]) X[u] = c2;
    return p;
};
fill_n(X + 1, n, 1);
for (int t = 0; t < SZ(E); ++t) {
    int u = E[t]
        ].X, v0 = E[t].Y, v = v0, c0 = X[u], c = c0, d;
    vector<pii> L;
    fill_n(vst + 1, n, 0);
    while (!G[u][v0]) {
        L.emplace_back(v, d = X[v]);
        if (!C[v][c]) for (int a = SZ(
            L) - 1; a >= 0; --a) c = color(u, L[a].X, c);
    }
}

```

```

    else if (!C[u][d]) for (int a = SZ(L)
        ) - 1; a >= 0; --a) color(u, L[a].X, L[a].Y);
    else if (vst[d]) break;
    else vst[d] = 1, v = C[u][d];
}
if (!G[u][v0]) {
    for (; v; v = flip(v, c, d), swap(c, d));
    if (int a; C[u][c0]) {
        for (
            a = SZ(L) - 2; a >= 0 && L[a].Y != c; --a);
        for (; a >= 0; --a) color(u, L[a].X, L[a].Y);
    }
    else --t;
}
}
} // namespace vizing

```

## 2.15 Maximum Clique [03ff71]

```

struct Maximum_Clique {
    typedef bitset<MAXN> bst;
    bst N[MAXN], empty;
    int p[MAXN], n, ans;
    void BronKerbosch2(bst R, bst P, bst X) {
        if (P == empty && X == empty)
            return ans = max(ans, (int)R.count()), void();
        bst tmp = P | X;
        int u;
        if ((R | P | X).count() <= ans) return;
        for (int uu = 0; uu < n; ++uu) {
            u = p[uu];
            if (tmp[u] == 1) break;
        }
        // if (double(clock())/CLOCKS_PER_SEC > .999)
        // return;
        bst now2 = P & ~N[u];
        for (int vv = 0; vv < n; ++vv) {
            int v = p[vv];
            if (now2[v] == 1) {
                R[v] = 1;
                BronKerbosch2(R, P & N[v], X & N[v]);
                R[v] = 0, P[v] = 0, X[v] = 1;
            }
        }
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) N[i].reset();
    }
    void add_edge(int u, int v) {
        N[u][v] = N[v][u] = 1;
    }
    int solve() { // remember srand
        bst R, P, X;
        ans = 0, P.flip();
        for (int i = 0; i < n; ++i) p[i] = i;
        random_shuffle(p, p + n), BronKerbosch2(R, P, X);
        return ans;
    }
};

```

## 3 Data Structure

### 3.1 2D Segment Tree [6985fc]

```

int num[501][501], N, M; // input here
struct seg_2D {
    struct node {
        int data;
        node *lc, *rc;
    } * root;
    node *merge(node *a, node *b, int l, int r) {
        node *p = new node;
        p->data = max(a->data, b->data);
        if (l == r) return p;
        int m = l + r >> 1;
        p->lc = merge(a->lc, b->lc, l, m);
        p->rc = merge(a->rc, b->rc, m + 1, r);
        return p;
    }
    node *build(int l, int r, int x) {
        node *p = new node;
        if (l == r) return p->data = num[x][l], p;
        int m = l + r >> 1;
        p->lc = build(l, m, x), p->rc = build(m + 1, r, x);
        p->data = max(p->lc->data, p->rc->data);
    }
}

```

```

    return p;
}
int query(int L, int R, int l, int r, node *p) {
    if (L <= l && R >= r) return p->data;
    int m = l + r >> 1, re = 0;
    if (L <= m) re = query(L, R, l, m, p->lc);
    if (R > m)
        re = max(re, query(L, R, m + 1, r, p->rc));
    return re;
}
};
struct seg_1D {
    struct node {
        seg_2D data;
        node *lc, *rc;
    } * root;
    node *s_build(int l, int r) {
        node *p = new node;
        if (l == r)
            return p->data.root = p->data.build(1, M, l), p;
        int m = l + r >> 1;
        p->lc = s_build(l, m), p->rc = s_build(m + 1, r);
        p->data.root = p->data.merge(
            p->lc->data.root, p->rc->data.root, 1, M);
        return p;
    }
    int s_query(int L, int R, int l, int r, node *p,
        int yl, int yr) {
        if (L <= l && R >= r)
            return p->data.query(yl, yr, 1, M, p->data.root);
        int m = l + r >> 1, re = 0;
        if (L <= m)
            re = s_query(L, R, l, m, p->lc, yl, yr);
        if (R > m)
            re = max(
                re, s_query(L, R, m + 1, r, p->rc, yl, yr));
        return re;
    }
    void init() { root = s_build(1, N); }
    int query(int xl, int xr, int yl, int yr) {
        return s_query(xl, xr, 1, N, root, yl, yr);
    }
};

```

### 3.2 Sparse table [cef484]

```

struct Sparse_table {
    int st[__lg(MAXN) + 1][MAXN], n;
    void init(int _n, int *data) {
        n = _n;
        for (int i = 0; i < n; ++i) st[0][i] = data[i];
        for (int i = 1, t = 2; t < n; t <= 1, i++)
            for (int j = 0; j + t <= n; j++)
                st[i][j]
                    = max(st[i - 1][j], st[i - 1][j + t / 2]);
    }
    int query(int a, int b) {
        int t = __lg(b - a + 1);
        return max(st[t][a], st[t][b - (1 << t) + 1]);
    }
};

```

### 3.3 Binary Index Tree [18be78]

```

struct Binary_Index_Tree {
    int bit[MAXN + 1], lazy[MAXN + 1], n;
    int lb(int x) { return x & -x; }
    void init(int _n, int *data) {
        n = _n;
        for (int i = 1, t; i <= n; ++i) {
            bit[i] = data[i], lazy[i] = 0, t = i - lb(i);
            for (int j = i - 1; j > t; j -= lb(j))
                bit[i] += bit[j];
        }
    }
    void suf_modify(int x, int v) {
        for (int t = x; t; t -= lb(t)) lazy[t] += v;
        for (int t = x + lb(x); t && t <= n; t += lb(t))
            bit[t] += v * (x - t + lb(t));
    }
    void modify(int x, int v) {
        for (; x; x -= lb(x)) bit[x] += v;
    }
    int query(int x) {
        int re = 0;
        for (int t = x; t; t -= lb(t))
            re += lazy[t] * lb(t) + bit[t];
    }
};

```

```

    for (int t = x + lb(x); t && t <= n; t += lb(t))
        re += lazy[t] * (x - t + lb(t));
    return re;
}
};

```

### 3.4 Segment Tree [0f243e]

```

struct Segment_Tree {
    struct node {
        int data, lazy;
        node *l, *r;
        node() : data(0), lazy(0), l(0), r(0) {}
        void up() {
            if (l) data = max(l->data, r->data);
        }
        void down() {
            if (l) {
                l->data += lazy, l->lazy += lazy;
                r->data += lazy, r->lazy += lazy;
            }
            lazy = 0;
        }
    } * root;
    int l, r;
    node *build(int l, int r, int *data) {
        node *p = new node();
        if (l == r) return p->data = data[l], p;
        int m = (l + r) / 2;
        p->l = build(l, m, data),
        p->r = build(m + 1, r, data);
        return p->up(), p;
    }
    void s_modify(
        int L, int R, int l, int r, node *p, int x) {
        if (r < L || l > R) return;
        p->down();
        if (L <= l && R >= r)
            return p->data += x, p->lazy += x, void();
        int m = (l + r) / 2;
        s_modify(L, R, l, m, p->l, x);
        s_modify(L, R, m + 1, r, p->r, x);
        p->up();
    }
    int s_query(int L, int R, int l, int r, node *p) {
        p->down();
        if (L <= l && R >= r) return p->data;
        int m = (l + r) / 2;
        if (R <= m) return s_query(L, R, l, m, p->l);
        if (L > m) return s_query(L, R, m + 1, r, p->r);
        return max(s_query(L, R, l, m, p->l),
            s_query(L, R, m + 1, r, p->r));
    }
    void init(int L, int R, int *data) {
        l = L, r = R;
        root = build(l, r, data);
    }
    void modify(int L, int R, int x) {
        s_modify(L, R, l, r, root, x);
    }
    int query(int L, int R) {
        return s_query(L, R, l, r, root);
    }
};

```

### 3.5 BIT kth [7de9a0]

```

int bit[N + 1]; // N = 2 ^ k
int query_kth(int k) {
    int res = 0;
    for (int i = N >> 1; i >= 1; i >>= 1)
        if (bit[res + i] < k)
            k -= bit[res + i];
    return res + 1;
}

```

### 3.6 Centroid Decomposition [6971c7]

```

struct Cent_Dec { // 1-base
    vector<pll> G[N];
    pll info[N]; // store info. of itself
    pll upinfo[N]; // store info. of climbing up
    int n, pa[N], layer[N], sz[N], done[N];
    ll dis[__lg(N) + 1][N];
    void init(int _n) {
        n = _n, layer[0] = -1;
        fill_n(pa + 1, n, 0), fill_n(done + 1, n, 0);
    }
};

```



```

    for (int i = 1; i <= n; ++i) G[i].clear();
}
void add_edge(int a, int b, int w) {
    G[a].pb(pll(b, w)), G[b].pb(pll(a, w));
}
void get_cent(
    int u, int f, int &mx, int &c, int num) {
    int mxsz = 0;
    sz[u] = 1;
    for (pll e : G[u])
        if (!done[e.X] && e.X != f) {
            get_cent(e.X, u, mx, c, num);
            sz[u] += sz[e.X], mxsz = max(mxsz, sz[e.X]);
        }
    if (mx > max(mxsz, num - sz[u]))
        mx = max(mxsz, num - sz[u]), c = u;
}
void dfs(int u, int f, ll d, int org) {
    // if required, add self info or climbing info
    dis[layer[org]][u] = d;
    for (pll e : G[u])
        if (!done[e.X] && e.X != f)
            dfs(e.X, u, d + e.Y, org);
}
int cut(int u, int f, int num) {
    int mx = 1e9, c = 0, lc;
    get_cent(u, f, mx, c, num);
    done[c] = 1, pa[c] = f, layer[c] = layer[f] + 1;
    for (pll e : G[c])
        if (!done[e.X]) {
            if (sz[e.X] > sz[c])
                lc = cut(e.X, c, num - sz[c]);
            else lc = cut(e.X, c, sz[e.X]);
            upinfo[lc] = pll(), dfs(e.X, c, e.Y, c);
        }
    return done[c] = 0, c;
}
void build() { cut(1, 0, n); }
void modify(int u) {
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        info[a].X += dis[ly][u], ++info[a].Y;
        if (pa[a])
            upinfo[a].X += dis[ly - 1][u], ++upinfo[a].Y;
    }
}
ll query(int u) {
    ll rt = 0;
    for (int a = u, ly = layer[a]; a;
        a = pa[a], --ly) {
        rt += info[a].X + info[a].Y * dis[ly][u];
        if (pa[a])
            rt -=
                upinfo[a].X + upinfo[a].Y * dis[ly - 1][u];
    }
    return rt;
}
};

```

### 3.7 DSU [e8502d]

```

struct DSU {
    vector<int> arr;
    DSU(int n = 0): arr(n) {
        iota(ALL(arr), 0);
    }
    int boss(int x) {
        if (arr[x] == x) return x;
        return arr[x] = boss(arr[x]);
    }
    bool Union(int x, int y) {
        x = boss(x), y = boss(y);
        if (x == y) return 0;
        arr[y] = x;
        return 1;
    }
};

```

### 3.8 Smart Pointer [7f0fff]

```

#ifndef REFERENCE_POINTER
#define REFERENCE_POINTER
template <typename T> struct _RefCounter {
    T data;
    int ref;
    _RefCounter(const T &d = 0) : data(d), ref(0) {}
};

```

```

template <typename T> struct reference_pointer {
    _RefCounter<T> *p;
    T *operator->() { return &p->data; }
    T &operator*() { return p->data; }
    operator _RefCounter<T> *() { return p; }
    reference_pointer &operator=(
        const reference_pointer &t) {
        if (p && !--p->ref) delete p;
        p = t.p;
        p && ++p->ref;
        return *this;
    }
    reference_pointer(_RefCounter<T> *t = 0) : p(t) {
        p && ++p->ref;
    }
    reference_pointer(const reference_pointer &t)
        : p(t.p) {
        p && ++p->ref;
    }
    ~reference_pointer() {
        if (p && !--p->ref) delete p;
    }
};
template <typename T>
inline reference_pointer<T> new_reference(
    const T &nd) {
    return reference_pointer<T>(new _RefCounter<T>(nd));
}
#endif
// note:
reference_pointer<int> a;
a = new_reference(5);
a = new_reference<int>(5);
a = new_reference((int)5);
reference_pointer<int> b = a;

struct P {
    int a, b;
    P(int _a, int _b) : a(_a), b(_b) {}
} p(2, 3);
reference_pointer<P> a;
c = new_reference(P(1, 2));
c = new_reference<P>(P(1, 2));
c = new_reference(p);

```

### 3.9 IntervalContainer [dbcccd]

```

/* Add and
   remove intervals from a set of disjoint intervals.
   * Will merge the added interval with
   any overlapping intervals in the set when adding.
   * Intervals are [inclusive, exclusive). */
set<pii>::
    iterator addInterval(set<pii>& is, int L, int R) {
        if (L == R) return is.end();
        auto it = is.lower_bound({L, R}), before = it;
        while (it != is.end() && it->X <= R) {
            R = max(R, it->Y);
            before = it = is.erase(it);
        }
        if (it != is.begin() && (--it)->Y >= L) {
            L = min(L, it->X);
            R = max(R, it->Y);
            is.erase(it);
        }
        return is.insert(before, pii(L, R));
    }
void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->Y;
    if (it->X == L) is.erase(it);
    else (int&)it->Y = L;
    if (R != r2) is.emplace(R, r2);
}

```

### 3.10 KDTree useful [22a1d3]

```

template <typename T, size_t kd> // kd???????
class kd_tree {
public:
    struct point {
        T d[kd];
        inline T dist(const point &x) const {
            T ret = 0;
            for (size_t i = 0; i < kd; ++i)
                ret += std::abs(d[i] - x.d[i]);
        }
    };
};

```



```

    return ret;
}
inline bool operator==(const point &p) {
    for (size_t i = 0; i < kd; ++i) {
        if (d[i] != p.d[i]) return 0;
    }
    return 1;
}
inline bool operator<(const point &b) const {
    return d[0] < b.d[0];
}
};

private:
struct node {
    node *l, *r;
    point pid;
    int s;
    node(const point &p) : l(0), r(0), pid(p), s(1) {}
    inline void up() {
        s = (l ? l->s : 0) + 1 + (r ? r->s : 0);
    }
} * root;
const double alpha, loga;
const T INF; //????INF,????
int maxn;
struct __cmp {
    int sort_id;
    inline bool operator()(
        const node *x, const node *y) const {
        return operator()(x->pid, y->pid);
    }
    inline bool operator()(
        const point &x, const point &y) const {
        if (x.d[sort_id] != y.d[sort_id])
            return x.d[sort_id] < y.d[sort_id];
        for (size_t i = 0; i < kd; ++i) {
            if (x.d[i] != y.d[i]) return x.d[i] < y.d[i];
        }
        return 0;
    }
} cmp;
void clear(node *o) {
    if (!o) return;
    clear(o->l);
    clear(o->r);
    delete o;
}
inline int size(node *o) { return o ? o->s : 0; }
std::vector<node *> A;
node *build(int k, int l, int r) {
    if (l > r) return 0;
    if (k == kd) k = 0;
    int mid = (l + r) / 2;
    cmp.sort_id = k;
    std::nth_element(A.begin() + l, A.begin() + mid,
        A.begin() + r + 1, cmp);
    node *ret = A[mid];
    ret->l = build(k + 1, l, mid - 1);
    ret->r = build(k + 1, mid + 1, r);
    ret->up();
    return ret;
}
inline bool isbad(node *o) {
    return size(o->l) > alpha * o->s ||
        size(o->r) > alpha * o->s;
}
void flatten(node *u,
    typename std::vector<node *>::iterator &it) {
    if (!u) return;
    flatten(u->l, it);
    *it = u;
    flatten(u->r, ++it);
}
inline void rebuild(node *u, int k) {
    if ((int)A.size() < u->s) A.resize(u->s);
    typename std::vector<node *>::iterator it =
        A.begin();
    flatten(u, it);
    u = build(k, 0, u->s - 1);
}
bool insert(
    node *u, int k, const point &x, int dep) {
    if (!u) {
        u = new node(x);
        return dep <= 0;
    }

```

```

    }
    ++u->s;
    cmp.sort_id = k;
    if (insert(cmp(x, u->pid) ? u->l : u->r,
        (k + 1) % kd, x, dep - 1)) {
        if (!isbad(u)) return 1;
        rebuild(u, k);
    }
    return 0;
}
node *findmin(node *o, int k) {
    if (!o) return 0;
    if (cmp.sort_id == k)
        return o->l ? findmin(o->l, (k + 1) % kd) : o;
    node *l = findmin(o->l, (k + 1) % kd);
    node *r = findmin(o->r, (k + 1) % kd);
    if (l && !r) return cmp(l, o) ? l : o;
    if (!l && r) return cmp(r, o) ? r : o;
    if (!l && !r) return o;
    if (cmp(l, r)) return cmp(l, o) ? l : o;
    return cmp(r, o) ? r : o;
}
bool erase(node *u, int k, const point &x) {
    if (!u) return 0;
    if (u->pid == x) {
        if (u->r)
            ;
        else if (u->l) {
            u->r = u->l;
            u->l = 0;
        } else {
            delete u;
            u = 0;
            return 1;
        }
        --u->s;
        cmp.sort_id = k;
        u->pid = findmin(u->r, (k + 1) % kd)->pid;
        return erase(u->r, (k + 1) % kd, u->pid);
    }
    cmp.sort_id = k;
    if (erase(cmp(x, u->pid) ? u->l : u->r,
        (k + 1) % kd, x)) {
        --u->s;
        return 1;
    } else return 0;
}
inline T heuristic(const T h[]) const {
    T ret = 0;
    for (size_t i = 0; i < kd; ++i) ret += h[i];
    return ret;
}
int qM;
std::priority_queue<std::pair<T, point>> pQ;
void nearest(
    node *u, int k, const point &x, T *h, T &mndist) {
    if (u == 0 || heuristic(h) >= mndist) return;
    T dist = u->pid.dist(x), old = h[k];
    /*mndist=std::min(mndist,dist);*/
    if (dist < mndist) {
        pQ.push(std::make_pair(dist, u->pid));
        if ((int)pQ.size() == qM + 1) {
            mndist = pQ.top().first, pQ.pop();
        }
    }
    if (x.d[k] < u->pid.d[k]) {
        nearest(u->l, (k + 1) % kd, x, h, mndist);
        h[k] = std::abs(x.d[k] - u->pid.d[k]);
        nearest(u->r, (k + 1) % kd, x, h, mndist);
    } else {
        nearest(u->r, (k + 1) % kd, x, h, mndist);
        h[k] = std::abs(x.d[k] - u->pid.d[k]);
        nearest(u->l, (k + 1) % kd, x, h, mndist);
    }
    h[k] = old;
}
std::vector<point> in_range;
void range(
    node *u, int k, const point &mi, const point &ma) {
    if (!u) return;
    bool is = 1;
    for (int i = 0; i < kd; ++i)
        if (u->pid.d[i] < mi.d[i] ||
            ma.d[i] < u->pid.d[i]) {
            is = 0;
            break;
        }
}

```

```

    }
    if (is) in_range.push_back(u->pid);
    if (mi.d[k] <= u->pid.d[k])
        range(u->l, (k + 1) % kd, mi, ma);
    if (ma.d[k] >= u->pid.d[k])
        range(u->r, (k + 1) % kd, mi, ma);
}

public:
kd_tree(const T &INF, double a = 0.75)
: root(0), alpha(a), loga(log2(1.0 / a)), INF(INF),
  maxn(1) {}
inline void clear() {
    clear(root), root = 0, maxn = 1;
}
inline void build(int n, const point *p) {
    clear(root), A.resize(maxn = n);
    for (int i = 0; i < n; ++i) A[i] = new node(p[i]);
    root = build(0, 0, n - 1);
}
inline void insert(const point &x) {
    insert(root, 0, x, std::lg(size(root)) / loga);
    if (root->s > maxn) maxn = root->s;
}
inline bool erase(const point &p) {
    bool d = erase(root, 0, p);
    if (root && root->s < alpha * maxn) rebuild();
    return d;
}
inline void rebuild() {
    if (root) rebuild(root, 0);
    maxn = root->s;
}
inline T nearest(const point &x, int k) {
    qM = k;
    T mndist = INF, h[kd] = {};
    nearest(root, 0, x, h, mndist);
    mndist = pQ.top().first;
    pQ = std::priority_queue<std::pair<T, point>>();
    return mndist; /*???x?k????*/
}
inline const std::vector<point> &range(
    const point &mi, const point &ma) {
    in_range.clear();
    range(root, 0, mi, ma);
    return in_range; /*???mi?ma???vector*/
}
inline int size() { return root ? root->s : 0; }
};

```

### 3.11 min heap [b3de3d]

```

template<class T, class Info>
struct min_heap {
    priority_queue<pair<T, Info>, vector<
        pair<T, Info>>, greater<pair<T, Info>>> pq;
    T lazy = 0;
    void push(pair<T, Info> v) {
        pq.emplace(v.X - lazy, v.Y);
    }
    pair<T, Info> top() {
        return make_pair(pq.top().X + lazy, pq.top().Y);
    }
    void join(min_heap &rgt) {
        if (SZ(pq) < SZ(rgt.pq)) {
            swap(pq, rgt.pq);
            swap(lazy, rgt.lazy);
        }
        while (!rgt.pq.empty()) {
            push(rgt.top());
            rgt.pop();
        }
    }
    void pop() {
        pq.pop();
    }
    bool empty() {
        return pq.empty();
    }
    void add_lazy(T v) {
        lazy += v;
    }
};

```

### 3.12 LiChaoST [2c55c3]

```

struct L {

```

```

    ll m, k, id;
    L() : id(-1) {}
    L(ll a, ll b, ll c) : m(a), k(b), id(c) {}
    ll at(ll x) { return m * x + k; }
};
class LiChao { // maintain max
private:
    int n; vector<L> nodes;
    void insert(int l, int r, int rt, L ln) {
        int m = (l + r) >> 1;
        if (nodes[rt].id == -1)
            return nodes[rt] = ln, void();
        bool atLeft = nodes[rt].at(l) < ln.at(l);
        if (nodes[rt].at(m) < ln.at(m))
            atLeft ^= 1, swap(nodes[rt], ln);
        if (r - l == 1) return;
        if (atLeft) insert(l, m, rt << 1, ln);
        else insert(m, r, rt << 1 | 1, ln);
    }
    ll query(int l, int r, int rt, ll x) {
        int m = (l + r) >> 1; ll ret = -INF;
        if (nodes[rt].id != -1) ret = nodes[rt].at(x);
        if (r - l == 1) return ret;
        if (x
            < m) return max(ret, query(l, m, rt << 1, x));
        return max(ret, query(m, r, rt << 1 | 1, x));
    }
public:
    LiChao(int n_) : n(n_), nodes(n * 4) {}
    void insert(L ln) { insert(0, n, 1, ln); }
    ll query(ll x) { return query(0, n, 1, x); }
};

```

### 3.13 Treap [4a5ee3]

```

struct node {
    int data, sz;
    node *l, *r;
    node(int k) : data(k), sz(1), l(0), r(0) {}
    void up() {
        sz = 1;
        if (l) sz += l->sz;
        if (r) sz += r->sz;
    }
    void down() {}
};
int sz(node *a) { return a ? a->sz : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (rand() % (sz(a) + sz(b)) < sz(a))
        return a->down(), a->r = merge(a->r, b), a->up(),
            a;
    return b->down(), b->l = merge(a, b->l), b->up(), b;
}
void split(node *o, node *&a, node *&b, int k) {
    if (!o) return a = b = 0, void();
    o->down();
    if (o->data <= k)
        a = o, split(o->r, a->r, b, k), a->up();
    else b = o, split(o->l, a, b->l, k), b->up();
}
void split2(node *o, node *&a, node *&b, int k) {
    if (sz(o) <= k) return a = o, b = 0, void();
    o->down();
    if (sz(o->l) + 1 <= k)
        a = o, split2(o->r, a->r, b, k - sz(o->l) - 1);
    else b = o, split2(o->l, a, b->l, k);
    o->up();
}
node *kth(node *o, int k) {
    if (k <= sz(o->l)) return kth(o->l, k);
    if (k == sz(o->l) + 1) return o;
    return kth(o->r, k - sz(o->l) - 1);
}
int Rank(node *o, int key) {
    if (!o) return 0;
    if (o->data < key)
        return sz(o->l) + 1 + Rank(o->r, key);
    else return Rank(o->l, key);
}
bool erase(node *o, int k) {
    if (!o) return 0;
    if (o->data == k) {
        node *t = o;
        o->down(), o = merge(o->l, o->r);
        delete t;
        return 1;
    }
}

```

```

}
node *t = k < o->data ? o->l : o->r;
return erase(t, k) ? o->up(), 1 : 0;
}
void insert(node *o, int k) {
    node *a, *b;
    split(o, a, b, k),
    o = merge(a, merge(new node(k), b));
}
void interval(node *o, int l, int r) {
    node *a, *b, *c;
    split2(o, a, b, l - 1), split2(b, b, c, r);
    // operate
    o = merge(a, merge(b, c));
}

```

### 3.14 link cut tree [703f02]

```

struct Splay { // xor-sum
    static Splay nil;
    Splay *ch[2], *f;
    int val, sum, rev, size;
    Splay(int
        _val = 0) : val(_val), sum(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr()
    { return f->ch[0] != this && f->ch[1] != this; }
    int dir()
    { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void give_tag(int r) {
        if (r) swap(ch[0], ch[1]), rev ^= 1;
    }
    void push() {
        if (ch[0] != &nil) ch[0]->give_tag(rev);
        if (ch[1] != &nil) ch[1]->give_tag(rev);
        rev = 0;
    }
    void pull() {
        // take care of the nil!
        size = ch[0]->size + ch[1]->size + 1;
        sum = ch[0]->sum ^ ch[1]->sum ^ val;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(), x->pull();
}
void splay(Splay *x) {
    vector<Splay*> splayVec;
    for (Splay *q = x;; q = q->f) {
        splayVec.pb(q);
        if (q->isr()) break;
    }
    reverse(ALL(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir() == x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
Splay* access(Splay *x) {
    Splay *q = nil;
    for (; x != nil; x = x->f)
        splay(x), x->setCh(q, 1), q = x;
    return q;
}
void root_path(Splay *x) { access(x), splay(x); }
void chroot(Splay *x) {
    root_path(x), x->give_tag(1);
    x->push(), x->pull();
}
void split(Splay *x, Splay *y) {

```

```

    chroot(x), root_path(y);
}
void link(Splay *x, Splay *y) {
    root_path(x), chroot(y);
    x->setCh(y, 1);
}
void cut(Splay *x, Splay *y) {
    split(x, y);
    if (y->size != 5) return;
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
Splay* get_root(Splay *x) {
    for (root_path(x); x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    return get_root(x) == get_root(y);
}
Splay* lca(Splay *x, Splay *y) {
    access(x), root_path(y);
    if (y->f == nil) return y;
    return y->f;
}
void change(Splay *x, int val) {
    splay(x), x->val = val, x->pull();
}
int query(Splay *x, Splay *y) {
    split(x, y);
    return y->sum;
}

```

### 3.15 Heavy light Decomposition [b91cf9]

```

struct Heavy_light_Decomposition { // 1-base
    int n, ulink[N], deep[N], mxson[N], w[N], pa[N];
    int t, pl[N], data[N], val[N]; // val: vertex data
    vector<int> G[N];
    void init(int _n) {
        n = _n;
        for (int i = 1; i <= n; ++i)
            G[i].clear(), mxson[i] = 0;
    }
    void add_edge(int a, int b) {
        G[a].pb(b), G[b].pb(a);
    }
    void dfs(int u, int f, int d) {
        w[u] = 1, pa[u] = f, deep[u] = d++;
        for (int &i : G[u])
            if (i != f) {
                dfs(i, u, d), w[u] += w[i];
                if (w[mxson[u]] < w[i]) mxson[u] = i;
            }
    }
    void cut(int u, int link) {
        data[pl[u] = ++t] = val[u], ulink[u] = link;
        if (!mxson[u]) return;
        cut(mxson[u], link);
        for (int i : G[u])
            if (i != pa[u] && i != mxson[u])
                cut(i, i);
    }
    void build() { dfs(1, 1, 1), cut(1, 1), /*build*/; }
    int query(int a, int b) {
        int ta = ulink[a], tb = ulink[b], res = 0;
        while (ta != tb) {
            if (deep[ta] > deep[tb]) swap(ta, tb), swap(a, b);
            // query(pl[ta], pl[tb])
            tb = ulink[b = pa[tb]];
        }
        if (pl[a] > pl[b]) swap(a, b);
        // query(pl[a], pl[b])
    }
}

```

### 3.16 Leftist Tree [2201dc]

```

struct node {
    ll v, data, sz, sum;
    node *l, *r;
    node(ll k)
        : v(0), data(k), sz(1), l(0), r(0), sum(k) {}
};
ll sz(node *p) { return p ? p->sz : 0; }

```

```

ll V(node *p) { return p ? p->v : -1; }
ll sum(node *p) { return p ? p->sum : 0; }
node *merge(node *a, node *b) {
    if (!a || !b) return a ? a : b;
    if (a->data < b->data) swap(a, b);
    a->r = merge(a->r, b);
    if (V(a->r) > V(a->l)) swap(a->r, a->l);
    a->v = V(a->r) + 1, a->sz = sz(a->l) + sz(a->r) + 1;
    a->sum = sum(a->l) + sum(a->r) + a->data;
    return a;
}
void pop(node *&o) {
    node *tmp = o;
    o = merge(o->l, o->r);
    delete tmp;
}

```

### 3.17 KDTree [85f231]

```

namespace kdt {
int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn],
    yl[maxn], yr[maxn];
point p[maxn];
int build(int l, int r, int dep = 0) {
    if (l == r) return -1;
    function<bool(const point &, const point &)> f =
        [dep](const point &a, const point &b) {
            if (dep & 1) return a.x < b.x;
            else return a.y < b.y;
        };
    int m = (l + r) >> 1;
    nth_element(p + l, p + m, p + r, f);
    xl[m] = xr[m] = p[m].x;
    yl[m] = yr[m] = p[m].y;
    lc[m] = build(l, m, dep + 1);
    if (~lc[m]) {
        xl[m] = min(xl[m], xl[lc[m]]);
        xr[m] = max(xr[m], xr[lc[m]]);
        yl[m] = min(yl[m], yl[lc[m]]);
        yr[m] = max(yr[m], yr[lc[m]]);
    }
    rc[m] = build(m + 1, r, dep + 1);
    if (~rc[m]) {
        xl[m] = min(xl[m], xl[rc[m]]);
        xr[m] = max(xr[m], xr[rc[m]]);
        yl[m] = min(yl[m], yl[rc[m]]);
        yr[m] = max(yr[m], yr[rc[m]]);
    }
    return m;
}
bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
        q.y < yl[o] - ds || q.y > yr[o] + ds)
        return false;
    return true;
}
long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
}
void dfs(
    const point &q, long long &d, int o, int dep = 0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x ||
        !(dep & 1) && q.y < p[o].y) {
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
        if (~rc[o]) dfs(q, d, rc[o], dep + 1);
        if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
}
void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
}
long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
}
} // namespace kdt

```

### 3.18 RangeChmin ChmaxAdd Range Sum [cd19b2]

```

#include <algorithm>
#include <iostream>
using namespace std;
typedef long long ll;

const int MAXC = 200005;
const ll INF = 1e18;

struct node {
    ll sum;
    ll mx, mxcnt, smx;
    ll mi, micnt, smi;
    ll lazymax, lazymin, lazyadd;
    node(ll k = 0)
        : sum(k), mxcnt(1), smx(-INF), mi(k),
          micnt(1), smi(INF), lazymax(-INF), lazymin(INF),
          lazyadd(0) {}
    node operator+(const node &a) const {
        node rt;
        rt.sum = sum + a.sum;
        rt.mx = max(mx, a.mx);
        rt.mi = min(mi, a.mi);
        if (mx == a.mx) {
            rt.mxcnt = mxcnt + a.mxcnt;
            rt.smx = max(smx, a.smx);
        } else if (mx > a.mx) {
            rt.mxcnt = mxcnt;
            rt.smx = max(smx, a.mx);
        } else {
            rt.mxcnt = a.mxcnt;
            rt.smx = max(mx, a.smx);
        }
        if (mi == a.mi) {
            rt.micnt = micnt + a.micnt;
            rt.smi = min(smi, a.smi);
        } else if (mi < a.mi) {
            rt.micnt = micnt;
            rt.smi = min(smi, a.mi);
        } else {
            rt.micnt = a.micnt;
            rt.smi = min(mi, a.smi);
        }
        rt.lazymax = -INF;
        rt.lazymin = INF;
        rt.lazyadd = 0;
        return rt;
    }
} seg[MAXC << 2];

ll a[MAXC];

void give_tag_min(int rt, ll t) {
    if (t >= seg[rt].mx) return;
    seg[rt].lazymin = t;
    seg[rt].lazymax = min(seg[rt].lazymax, t);
    seg[rt].sum -= seg[rt].mxcnt * (seg[rt].mx - t);
    if (seg[rt].mx == seg[rt].smi) seg[rt].smi = t;
    if (seg[rt].mx == seg[rt].mi) seg[rt].mi = t;
    seg[rt].mx = t;
}

void give_tag_max(int rt, ll t) {
    if (t <= seg[rt].mi) return;
    seg[rt].lazymax = t;
    seg[rt].sum += seg[rt].micnt * (t - seg[rt].mi);
    if (seg[rt].mi == seg[rt].smx) seg[rt].smx = t;
    if (seg[rt].mi == seg[rt].mx) seg[rt].mx = t;
    seg[rt].mi = t;
}

void give_tag_add(int l, int r, int rt, ll t) {
    seg[rt].lazyadd += t;
    if (seg[rt].lazymax != -INF) seg[rt].lazyadd += t;
    if (seg[rt].lazymin != INF) seg[rt].lazyadd += t;
    seg[rt].mx += t;
    if (seg[rt].smx != -INF) seg[rt].smx += t;
    seg[rt].mi += t;
    if (seg[rt].smi != INF) seg[rt].smi += t;
    seg[rt].sum += (ll)(r - l + 1) * t;
}

void tag_down(int l, int r, int rt) {
    if (seg[rt].lazyadd != 0) {
        int mid = (l + r) >> 1;

```

```

    give_tag_add(l, mid, rt << 1, seg[rt].lazyadd);
    give_tag_add(
        mid + 1, r, rt << 1 | 1, seg[rt].lazyadd);
    seg[rt].lazyadd = 0;
}
if (seg[rt].lazymin != INF) {
    give_tag_min(rt << 1, seg[rt].lazymin);
    give_tag_min(rt << 1 | 1, seg[rt].lazymin);
    seg[rt].lazymin = INF;
}
if (seg[rt].lazymax != -INF) {
    give_tag_max(rt << 1, seg[rt].lazymax);
    give_tag_max(rt << 1 | 1, seg[rt].lazymax);
    seg[rt].lazymax = -INF;
}
}

void build(int l, int r, int rt) {
    if (l == r) return seg[rt] = node(a[l]), void();
    int mid = (l + r) >> 1;
    build(l, mid, rt << 1);
    build(mid + 1, r, rt << 1 | 1);
    seg[rt] = seg[rt << 1] + seg[rt << 1 | 1];
}

void modifymax(
    int L, int R, int l, int r, int rt, ll t) {
    if (L <= l && R >= r && t < seg[rt].smi)
        return give_tag_max(rt, t);
    if (l != r) tag_down(l, r, rt);
    int mid = (l + r) >> 1;
    if (L <= mid) modifymax(L, R, l, mid, rt << 1, t);
    if (R > mid)
        modifymax(L, R, mid + 1, r, rt << 1 | 1, t);
    seg[rt] = seg[rt << 1] + seg[rt << 1 | 1];
}

void modifymin(
    int L, int R, int l, int r, int rt, ll t) {
    if (L <= l && R >= r && t > seg[rt].smx)
        return give_tag_min(rt, t);
    if (l != r) tag_down(l, r, rt);
    int mid = (l + r) >> 1;
    if (L <= mid) modifymin(L, R, l, mid, rt << 1, t);
    if (R > mid)
        modifymin(L, R, mid + 1, r, rt << 1 | 1, t);
    seg[rt] = seg[rt << 1] + seg[rt << 1 | 1];
}

void modifyadd(
    int L, int R, int l, int r, int rt, ll t) {
    if (L <= l && R >= r)
        return give_tag_add(l, r, rt, t);
    if (l != r) tag_down(l, r, rt);
    int mid = (l + r) >> 1;
    if (L <= mid) modifyadd(L, R, l, mid, rt << 1, t);
    if (R > mid)
        modifyadd(L, R, mid + 1, r, rt << 1 | 1, t);
    seg[rt] = seg[rt << 1] + seg[rt << 1 | 1];
}

ll query(int L, int R, int l, int r, int rt) {
    if (L <= l && R >= r) return seg[rt].sum;
    if (l != r) tag_down(l, r, rt);
    int mid = (l + r) >> 1;
    if (R <= mid) return query(L, R, l, mid, rt << 1);
    if (L > mid)
        return query(L, R, mid + 1, r, rt << 1 | 1);
    return query(L, R, l, mid, rt << 1) +
        query(L, R, mid + 1, r, rt << 1 | 1);
}

int main() {
    ios::sync_with_stdio(0), cin.tie(0);
    int n, m;
    cin >> n >> m;
    for (int i = 1; i <= n; ++i) cin >> a[i];
    build(1, n, 1);
    while (m--) {
        int k, x, y;
        ll t;
        cin >> k >> x >> y, ++x;
        if (k == 0) cin >> t, modifymin(x, y, 1, n, 1, t);
        else if (k == 1)
            cin >> t, modifymax(x, y, 1, n, 1, t);
        else if (k == 2)

```

```

        cin >> t, modifyadd(x, y, 1, n, 1, t);
    } else cout << query(x, y, 1, n, 1) << "\n";
}
}

```

### 3.19 discrete trick [2062d6]

```

vector<int> val;
// build
sort(ALL
    (val)), val.resize(unique(ALL(val)) - val.begin());
// index of x
upper_bound(ALL(val), x) - val.begin();
// max idx <= x
upper_bound(ALL(val), x) - val.begin();
// max idx < x
lower_bound(ALL(val), x) - val.begin();

```

## 4 Flow Matching

### 4.1 Maximum Simple Graph Matching [390d20]

```

struct Matching { // 0-base
    queue<int> q; int n;
    vector<int> fa, s, vis, pre, match;
    vector<vector<int>>> G;
    int Find(int u)
    { return u == fa[u] ? u : fa[u] = Find(fa[u]); }
    int LCA(int x, int y) {
        static int tk = 0; tk++; x = Find(x); y = Find(y);
        for (; swap(x, y)) if (x != n) {
            if (vis[x] == tk) return x;
            vis[x] = tk;
            x = Find(pre[match[x]]);
        }
    }
    void Blossom(int x, int y, int l) {
        for (; Find(x) != l; x = pre[y]) {
            pre[x] = y, y = match[x];
            if (s[y] == 1) q.push(y), s[y] = 0;
            for (int z: {x, y}) if (fa[z] == z) fa[z] = l;
        }
    }
    bool Bfs(int r) {
        iota(ALL(fa), 0); fill(ALL(s), -1);
        q = queue<int>(); q.push(r); s[r] = 0;
        for (; !q.empty(); q.pop()) {
            for (int x = q.front(); int u : G[x])
                if (s[u] == -1) {
                    if (pre[u] = x, s[u] = 1, match[u] == n) {
                        for (int a = u, b = x, last;
                            b != n; a = last, b = pre[a])
                            last = match[b], match[b] = a, match[a] = b;
                        return true;
                    }
                    q.push(match[u]); s[match[u]] = 0;
                } else if (!s[u] && Find(u) != Find(x)) {
                    int l = LCA(u, x);
                    Blossom(x, u, l); Blossom(u, x, l);
                }
            }
        }
        return false;
    }
    Matching(int _n) : n(_n), fa(n + 1), s(n + 1), vis
        (n + 1), pre(n + 1, n), match(n + 1, n), G(n) {}
    void add_edge(int u, int v)
    { G[u].pb(v), G[v].pb(u); }
    int solve() {
        int ans = 0;
        for (int x = 0; x < n; ++x)
            if (match[x] == n) ans += Bfs(x);
        return ans;
    }
    // match[x] == n means not matched
};

```

### 4.2 Kuhn Munkres [61bbd0]

```

struct KM { // 0-base, maximum matching
    ll w[N][N], hl[N], hr[N], slk[N];
    int fl[N], fr[N], pre[N], qu[N], ql, qr, n;
    bool vl[N], vr[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i)
            fill_n(w[i], n, -INF);
    }
};

```

```

void add_edge(int a, int b, ll wei) {
    w[a][b] = wei;
}
bool Check(int x) {
    if (vl[x] == 1, ~fl[x])
        return vr[qu[qr++]] = fl[x] = 1;
    while (~x) swap(x, fr[fl[x] = pre[x]]);
    return 0;
}
void bfs(int s) {
    fill_n(slack, n, INF), fill_n(vl, n, 0), fill_n(vr, n, 0);
    ql = qr = 0, qu[qr++] = s, vr[s] = 1;
    for (ll d;;) {
        while (ql < qr)
            for (int x = 0, y = qu[ql++]; x < n; ++x)
                if (!vl[x] && slack[x] >= (d = hl[x] + hr[y] - w[x][y])) {
                    if (pre[x] = y, d) slack[x] = d;
                    else if (!Check(x)) return;
                }
        d = INF;
        for (int x = 0; x < n; ++x)
            if (!vl[x] && d > slack[x]) d = slack[x];
        for (int x = 0; x < n; ++x) {
            if (vl[x]) hl[x] += d;
            else slack[x] -= d;
            if (vr[x]) hr[x] -= d;
        }
        for (int x = 0; x < n; ++x)
            if (!vl[x] && !slack[x] && !Check(x)) return;
    }
}
ll solve() {
    fill_n(fl, n, -1), fill_n(fr, n, -1), fill_n(hr, n, 0);
    for (int i = 0; i < n; ++i) hl[i] = *max_element(w[i], w[i] + n);
    for (int i = 0; i < n; ++i) bfs(i);
    ll res = 0;
    for (int i = 0; i < n; ++i) res += w[i][fl[i]];
    return res;
}
};

```

### 4.3 Model

- Maximum/Minimum flow with lower bound / Circulation problem
  - Construct super source  $S$  and sink  $T$ .
  - For each edge  $(x, y, l, u)$ , connect  $x \rightarrow y$  with capacity  $u - l$ .
  - For each vertex  $v$ , denote by  $in(v)$  the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds.
  - If  $in(v) > 0$ , connect  $S \rightarrow v$  with capacity  $in(v)$ , otherwise, connect  $v \rightarrow T$  with capacity  $-in(v)$ .
    - To maximize, connect  $t \rightarrow s$  with capacity  $\infty$  (skip this in circulation problem), and let  $f$  be the maximum flow from  $S$  to  $T$ . If  $f \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise, the maximum flow from  $s$  to  $t$  is the answer.
    - To minimize, let  $f$  be the maximum flow from  $S$  to  $T$ . Connect  $t \rightarrow s$  with capacity  $\infty$  and let the flow from  $S$  to  $T$  be  $f'$ . If  $f + f' \neq \sum_{v \in V, in(v) > 0} in(v)$ , there's no solution. Otherwise,  $f'$  is the answer.
  - The solution of each edge  $e$  is  $l_e + f_e$ , where  $f_e$  corresponds to the flow of edge  $e$  on the graph.
- Construct minimum vertex cover from maximum matching  $M$  on bipartite graph  $(X, Y)$ 
  - Redirect every edge:  $y \rightarrow x$  if  $(x, y) \in M$ ,  $x \rightarrow y$  otherwise.
  - DFS from unmatched vertices in  $X$ .
  - $x \in X$  is chosen iff  $x$  is unvisited.
  - $y \in Y$  is chosen iff  $y$  is visited.
- Minimum cost cyclic flow
  - Construct super source  $S$  and sink  $T$
  - For each edge  $(x, y, c)$ , connect  $x \rightarrow y$  with  $(cost, cap) = (c, 1)$  if  $c > 0$ , otherwise connect  $y \rightarrow x$  with  $(cost, cap) = (-c, 1)$
  - For each edge with  $c < 0$ , sum these cost as  $K$ , then increase  $d(y)$  by 1, decrease  $d(x)$  by 1
  - For each vertex  $v$  with  $d(v) > 0$ , connect  $S \rightarrow v$  with  $(cost, cap) = (0, d(v))$
  - For each vertex  $v$  with  $d(v) < 0$ , connect  $v \rightarrow T$  with  $(cost, cap) = (0, -d(v))$
  - Flow from  $S$  to  $T$ , the answer is the cost of the flow  $C + K$
- Maximum density induced subgraph
  - Binary search on answer, suppose we're checking answer  $T$
  - Construct a max flow model, let  $K$  be the sum of all weights
  - Connect source  $s \rightarrow v, v \in G$  with capacity  $K$
  - For each edge  $(u, v, w)$  in  $G$ , connect  $u \rightarrow v$  and  $v \rightarrow u$  with capacity  $w$

- For  $v \in G$ , connect it with sink  $v \rightarrow t$  with capacity  $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
  - $T$  is a valid answer if the maximum flow  $f < K|V|$
- Minimum weight edge cover
    - For each  $v \in V$  create a copy  $v'$ , and connect  $u' \rightarrow v'$  with weight  $w(u, v)$ .
    - Connect  $v \rightarrow v'$  with weight  $2\mu(v)$ , where  $\mu(v)$  is the cost of the cheapest edge incident to  $v$ .
    - Find the minimum weight perfect matching on  $G'$ .
  - Project selection problem
    - If  $p_v > 0$ , create edge  $(s, v)$  with capacity  $p_v$ ; otherwise, create edge  $(v, t)$  with capacity  $-p_v$ .
    - Create edge  $(u, v)$  with capacity  $w$  with  $w$  being the cost of choosing  $u$  without choosing  $v$ .
    - The mincut is equivalent to the maximum profit of a subset of projects.
  - Dual of minimum cost maximum flow
    - Capacity  $c_{uv}$ , Flow  $f_{uv}$ , Cost  $w_{uv}$ , Required Flow difference for vertex  $b_u$ .
    - If all  $w_{uv}$  are integers, then optimal solution can happen when all  $p_u$  are integers.

$$\begin{aligned}
 & \min \sum_{uv} w_{uv} f_{uv} \\
 & -f_{vu} \geq -c_{uv} \Leftrightarrow \min \sum_u b_u p_u + \sum_{uv} c_{uv} \max(0, p_v - p_u - w_{uv}) \\
 & \sum_v f_{vu} - \sum_v f_{uv} = -b_u \quad p_u \geq 0
 \end{aligned}$$

### 4.4 MincostMaxflow dijkstra [94c520]

```

struct MinCostMaxFlow { // 0-base
    struct Edge {
        ll from, to, cap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    int inq[N], n, s, t;
    ll dis[N], up[N], pot[N];
    bool BellmanFord() {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, ll cap, Edge *e) {
            if (cap > 0 && dis[u] > d) {
                dis[u] = d, up[u] = cap, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, INF, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u]) {
                ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
                relax(e.to, d2, min(up[u], e.cap - e.flow), &e);
            }
        }
        return dis[t] != INF;
    }
    bool Dijkstra() {
        fill_n(dis, n, INF);
        priority_queue<pll, vector<pll>, greater<pll>> pq;
        auto relax = [&](int u, ll d, ll cap, Edge *e) {
            if (cap > 0 && dis[u] > d) {
                dis[u] = d, up[u] = cap, past[u] = e;
                pq.push(pll(d, u));
            }
        };
        relax(s, 0, INF, 0);
        while (!pq.empty()) {
            auto [d, u] = pq.top();
            pq.pop();
            if (dis[u] != d) continue;
            for (auto &e : G[u]) {
                ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
                relax(e.to, d2, min(up[u], e.cap - e.flow), &e);
            }
        }
        return dis[t] != INF;
    }
    void solve(int _s, int _t, ll &flow, ll &cost, bool neg = true) {
        s = _s, t = _t, flow = 0, cost = 0;
        if (neg) BellmanFord(), copy_n(dis, n, pot);
        for (; Dijkstra(); copy_n(dis, n, pot)) {
            for (int i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
            flow += up[t], cost += up[t] * dis[t];
        }
    }
};

```



```

    for (int i = t; past[i]; i = past[i]->from) {
        auto &e = *past[i];
        e.flow += up[t], G[e.to][e.rev].flow -= up[t];
    }
}
void init(int _n) {
    n = _n, fill_n(pot, n, 0);
    for (int i = 0; i < n; ++i) G[i].clear();
}
void add_edge(ll a, ll b, ll cap, ll cost) {
    G[a].pb(Edge{a, b, cap, 0, cost, SZ(G[b])});
    G[b].pb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
}
};

```

#### 4.5 isap [a2dc77]

```

struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r)
            : v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV * 2];
    int iter[MAXV * 2], d[MAXV * 2], gap[MAXV * 2], tot;
    void init(int x) {
        tot = x + 2;
        s = x + 1, t = x + 2;
        for (int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
    int dfs(int p, int flow) {
        if (p == t) return flow;
        for (int &i = iter[p]; i < SZ(G[p]); i++) {
            Edge &e = G[p][i];
            if (e.c > 0 && d[p] == d[e.v] + 1) {
                int f = dfs(e.v, min(flow, e.c));
                if (f) {
                    e.c -= f;
                    G[e.v][e.r].c += f;
                    return f;
                }
            }
        }
        if ((--gap[d[p]]) == 0) d[s] = tot;
        else {
            d[p]++;
            iter[p] = 0;
            ++gap[d[p]];
        }
        return 0;
    }
    int solve() {
        int res = 0;
        gap[0] = tot;
        for (res = 0; d[s] < tot; res += dfs(s, INF))
            ;
        return res;
    }
} flow;

```

#### 4.6 Gomory Hu tree [62c88c]

```

MaxFlow Dinic;
int g[MAXN];
void GomoryHu(int n) { // 0-base
    fill_n(g, n, 0);
    for (int i = 1; i < n; ++i) {
        Dinic.reset();
        add_edge(i, g[i], Dinic.maxflow(i, g[i]));
        for (int j = i + 1; j <= n; ++j)
            if (g[j] == g[i] && ~Dinic.dis[j])
                g[j] = i;
    }
}

```

#### 4.7 MincostMaxflow [0722e9]

```

struct MinCostMaxFlow { // 0-base
    struct Edge {
        ll from, to, cap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    int inq[N], n, s, t;
    ll dis[N], up[N], pot[N];
    bool BellmanFord() {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, ll cap, Edge *e) {
            if (cap > 0 && dis[u] > d) {
                dis[u] = d, up[u] = cap, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, INF, 0);
        while (!q.empty()) {
            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u]) {
                ll d2 = dis[u] + e.cost + pot[u] - pot[e.to];
                relax(e.to, d2, min(up[u], e.cap - e.flow), &e);
            }
        }
        return dis[t] != INF;
    }
    void solve(int _s, int _t, ll &flow, ll &cost, bool neg = true) {
        s = _s, t = _t, flow = 0, cost = 0;
        if (neg) BellmanFord(), copy_n(dis, n, pot);
        for (; BellmanFord(); copy_n(dis, n, pot)) {
            for (int i = 0; i < n; ++i) dis[i] += pot[i] - pot[s];
            flow += up[t], cost += up[t] * dis[t];
            for (int i = t; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                e.flow += up[t], G[e.to][e.rev].flow -= up[t];
            }
        }
    }
    void init(int _n) {
        n = _n, fill_n(pot, n, 0);
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(ll a, ll b, ll cap, ll cost) {
        G[a].pb(Edge{a, b, cap, 0, cost, SZ(G[b])});
        G[b].pb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
    }
};

```

#### 4.8 SW-mincut [8e90f0]

```

struct SW{ // global min cut, O(V^3)
#define REP for (int i = 0; i < n; ++i)
    static const int MXN = 514, INF = 2147483647;
    int vst[MXN], edge[MXN][MXN], wei[MXN];
    void init(int n) {
        REP fill_n(edge[i], n, 0);
    }
    void addEdge(int u, int v, int w){
        edge[u][v] += w; edge[v][u] += w;
    }
    int search(int &s, int &t, int n){
        fill_n(vst, n, 0), fill_n(wei, n, 0);
        s = t = -1;
        int mx, cur;
        for (int j = 0; j < n; ++j) {
            mx = -1, cur = 0;
            REP if (wei[i] > mx) cur = i, mx = wei[i];
            vst[cur] = 1, wei[cur] = -1;
            s = t; t = cur;
            REP if (!vst[i]) wei[i] += edge[cur][i];
        }
        return mx;
    }
    int solve(int n) {
        int res = INF;
        for (int x, y; n > 1; n--){
            res = min(res, search(x, y, n));
            REP edge[i][x] = (edge[x][i] += edge[y][i]);
            REP {
                edge[y][i] = edge[n - 1][i];
                edge[i][y] = edge[i][n - 1];
            }
        }
    }
};

```



```

    } // edge[y][y] = 0;
}
return res;
}
} sw;

```

## 4.9 Maximum Weight Matching [a10467]

```

#define REP(i, l, r) for (int i=l; i<=(r); ++i)
struct WeightGraph { // 1-based
    struct edge { int u, v, w; }; int n, nx;
    vector<int> lab; vector<vector<edge>> g;
    vector<int> slk, match, st, pa, S, vis;
    vector<vector<int>> flo, flo_from; queue<int> q;
    WeightGraph(int n_) : n(n_), nx(n * 2), lab(nx + 1),
        g(nx + 1, vector<edge>(nx + 1)), slk(nx + 1),
        flo(nx + 1), flo_from(nx + 1, vector(n + 1, 0)) {
        match = st = pa = S = vis = slk;
        REP(u, 1, n) REP(v, 1, n) g[u][v] = {u, v, 0};
    }
    int E(edge e)
    { return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2; }
    void update_slk(int u, int x, int &s)
    { if (!s || E(g[u][x]) < E(g[s][x])) s = u; }
    void set_slk(int x) {
        slk[x] = 0;
        REP(u, 1, n)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
                update_slk(u, x, slk[x]);
    }
    void q_push(int x) {
        if (x <= n) q.push(x);
        else for (int y : flo[x]) q_push(y);
    }
    void set_st(int x, int b) {
        st[x] = b;
        if (x > n) for (int y : flo[x]) set_st(y, b);
    }
    vector<int> split_flo(auto &f, int xr) {
        auto it = find(ALL(f), xr);
        if (auto pr = it - f.begin(); pr % 2 == 1)
            reverse(1 + ALL(f), it = f.end() - pr);
        auto res = vector(f.begin(), it);
        return f.erase(f.begin(), it), res;
    }
    void set_match(int u, int v) {
        match[u] = g[u][v].v;
        if (u <= n) return;
        int xr = flo_from[u][g[u][v].u];
        auto &f = flo[u], z = split_flo(f, xr);
        REP(i, 0, SZ(z) - 1) set_match(z[i], z[i ^ 1]);
        set_match(xr, v); f.insert(f.end(), ALL(z));
    }
    void augment(int u, int v) {
        for (;;) {
            int xnv = st[match[u]]; set_match(u, v);
            if (!xnv) return;
            set_match(v = xnv, u = st[pa[xnv]]);
        }
    }
    int lca(int u, int v) {
        static int t = 0; ++t;
        for (++t; u || v; swap(u, v)) if (u) {
            if (vis[u] == t) return u;
            vis[u] = t, u = st[match[u]];
            if (u) u = st[pa[u]];
        }
        return 0;
    }
    void add_blossom(int u, int o, int v) {
        int b = find(n + 1 + ALL(st), 0) - begin(st);
        lab[b] = 0, S[b] = 0, match[b] = match[o];
        vector<int> f = {o};
        for (int t : {u, v}) {
            reverse(1 + ALL(f));
            for (int x = t, y; x != o; x = st[pa[y]])
                f.pb(x), f.pb(y = st[match[x]]), q_push(y);
        }
        flo[b] = f; set_st(b, b);
        REP(x, 1, nx) g[b][x].w = g[x][b].w = 0;
        fill(ALL(flo_from[b]), 0);
        for (int xs : flo[b]) {
            REP(x, 1, nx)
                if (g[b][x].w == 0 || E(g[xs][x]) < E(g[b][x]))
                    g[b][x] = g[xs][x], g[x][b] = g[x][xs];
            REP(x, 1, n)
                if (flo_from[xs][x]) flo_from[b][x] = xs;
        }
    }

```

```

    }
    set_slk(b);
}
void expand_blossom(int b) {
    for (int x : flo[b]) set_st(x, x);
    int xr = flo_from[b][g[b][pa[b]].u], xs = -1;
    for (int x : split_flo(flo[b], xr)) {
        if (xs == -1) { xs = x; continue; }
        pa[xs] = g[x][xs].u, S[xs] = 1, S[x] = 0;
        slk[xs] = 0, set_slk(x), q_push(x), xs = -1;
    }
    for (int x : flo[b])
        if (x == xr) S[x] = 1, pa[x] = pa[b];
        else S[x] = -1, set_slk(x);
    st[b] = 0;
}
bool on_found_edge(const edge &e) {
    if (int u = st[e.u], v = st[e.v]; S[v] == -1) {
        int nu = st[match[v]]; pa[v] = e.u; S[v] = 1;
        slk[v] = slk[nu] = S[nu] = 0; q_push(nu);
    } else if (S[v] == 0) {
        if (int o = lca(u, v)) add_blossom(u, o, v);
        else return augment(u, v), augment(v, u), true;
    }
    return false;
}
bool matching() {
    fill(ALL(S), -1), fill(ALL(slk), 0);
    q = queue<int>();
    REP(x, 1, nx) if (st[x] == x && !match[x])
        pa[x] = S[x] = 0, q_push(x);
    if (q.empty()) return false;
    for (;;) {
        while (SZ(q)) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            REP(v, 1, n)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (E(g[u][v]) != 0)
                        update_slk(u, st[v], slk[st[v]]);
                    else if
                        (on_found_edge(g[u][v])) return true;
                }
        }
        int d = INF;
        REP(b, n + 1, nx) if (st[b] == b && S[b] == 1)
            d = min(d, lab[b] / 2);
        REP(x, 1, nx)
            if (int
                s = slk[x]; st[x] == x && s && S[x] <= 0)
                d = min(d, E(g[s][x]) / (S[x] + 2));
        REP(u, 1, n)
            if (S[st[u]] == 1) lab[u] += d;
            else if (S[st[u]] == 0) {
                if (lab[u] <= d) return false;
                lab[u] -= d;
            }
        REP(b, n + 1, nx) if (st[b] == b && S[b] >= 0)
            lab[b] += d * (2 - 4 * S[b]);
        REP(x, 1, nx)
            if (int
                s = slk[x]; st[x] == x &&
                s && st[s] != x && E(g[s][x]) == 0)
                if (on_found_edge(g[s][x])) return true;
        REP(b, n + 1, nx)
            if (st[b] == b && S[b] == 1 && lab[b] == 0)
                expand_blossom(b);
    }
    return false;
}
pair<ll, int> solve() {
    fill(ALL(match), 0);
    REP(u, 0, n) st[u] = u, flo[u].clear();
    int w_max = 0;
    REP(u, 1, n) REP(v, 1, n) {
        flo_from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
    fill(ALL(lab), w_max);
    int n_matches = 0; ll tot_weight = 0;
    while (matching()) ++n_matches;
    REP(u, 1, n) if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int u, int v, int w)
{ g[u][v].w = g[v][u].w = w; }

```

```
};
```

#### 4.10 Minimum Weight Matching wrong [f27d66]

```
struct Graph { // 0-base (Perfect Match), n is even
    int n, match[N], onstk[N], stk[N], tp;
    ll edge[N][N], dis[N];
    void init(int _n) {
        n = _n, tp = 0;
        for (int i = 0; i < n; ++i) fill_n(edge[i], n, 0);
    }
    void add_edge(int u, int v, ll w) {
        edge[u][v] = edge[v][u] = w;
    }
    bool SPFA(int u) {
        stk[tp++] = u, onstk[u] = 1;
        for (int v = 0; v < n; ++v)
            if (!onstk[v] && match[u] != v) {
                int m = match[v];
                if (dis[m] >
                    dis[u] - edge[v][m] + edge[u][v]) {
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1, stk[tp++] = v;
                    if (onstk[m] || SPFA(m)) return 1;
                    --tp, onstk[v] = 0;
                }
            }
        onstk[u] = 0, --tp;
        return 0;
    }
    ll solve() { // find a match
        for (int i = 0; i < n; ++i) match[i] = i ^ 1;
        while (1) {
            int found = 0;
            fill_n(dis, n, 0);
            fill_n(onstk, n, 0);
            for (int i = 0; i < n; ++i)
                if (tp == 0, !onstk[i] && SPFA(i))
                    for (found = 1; tp >= 2; ) {
                        int u = stk[--tp];
                        int v = stk[--tp];
                        match[u] = v, match[v] = u;
                    }
            if (!found) break;
        }
        ll ret = 0;
        for (int i = 0; i < n; ++i)
            ret += edge[i][match[i]];
        return ret >> 1;
    }
};
```

#### 4.11 Bipartite Matching [623c76]

```
struct Bipartite_Matching { // 0-base
    int mp[N], mq[N], dis[N + 1], cur[N], l, r;
    vector<int> G[N + 1];
    bool dfs(int u) {
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            int e = G[u][i];
            if (mq[e] == l
                || (dis[mq[e]] == dis[u] + 1 && dfs(mq[e])))
                return mp[mq[e] = u] = e, 1;
        }
        return dis[u] = -1, 0;
    }
    bool bfs() {
        queue<int> q;
        fill_n(dis, l + 1, -1);
        for (int i = 0; i < l; ++i)
            if (!mp[i])
                q.push(i), dis[i] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int e : G[u])
                if (!dis[mq[e]])
                    q.push(mq[e]), dis[mq[e]] = dis[u] + 1;
        }
        return dis[l] != -1;
    }
    int matching() {
        int res = 0;
        fill_n(mp, l, -1), fill_n(mq, r, l);
        while (bfs()) {
            fill_n(cur, l, 0);
            for (int i = 0; i < l; ++i)
```

```
        res += (!mp[i] && dfs(i));
    }
    return res; // (i, mp[i] != -1)
}
void add_edge(int s, int t) { G[s].pb(t); }
void init(int _l, int _r) {
    l = _l, r = _r;
    for (int i = 0; i <= l; ++i)
        G[i].clear();
}
};
```

#### 4.12 BoundedFlow [e8670b]

```
struct BoundedFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[N];
    int n, s, t, dis[N], cur[N], cnt[N];
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n + 2; ++i)
            G[i].clear(), cnt[i] = 0;
    }
    void add_edge(int u, int v, int lcap, int rcap) {
        cnt[u] -= lcap, cnt[v] += lcap;
        G[u].pb(edge{v, rcap, lcap, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, SZ(G[v])});
        G[v].pb(edge{u, 0, 0, SZ(G[u]) - 1});
    }
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < SZ(G[u]); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.cap != e.flow) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df, G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n + 3, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (edge &e : G[u])
                if (!dis[e.to] && e.flow != e.cap)
                    q.push(e.to), dis[e.to] = dis[u] + 1;
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n + 3, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    bool solve() {
        int sum = 0;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)
                add_edge(n + 1, i, cnt[i]), sum += cnt[i];
            else if (cnt[i] < 0) add_edge(i, n + 2, -cnt[i]);
        if (sum != maxflow(n + 1, n + 2)) sum = -1;
        for (int i = 0; i < n; ++i)
            if (cnt[i] > 0)
                G[n + 1].pop_back(), G[i].pop_back();
            else if (cnt[i] < 0)
                G[i].pop_back(), G[n + 2].pop_back();
        return sum != -1;
    }
    int solve(int _s, int _t) {
        add_edge(_t, _s, INF);
```

```

    if (!solve()) return -1; // invalid flow
    int x = G[_t].back().flow;
    return G[_t].pop_back(), G[_s].pop_back(), x;
}
};

```

### 4.13 Dinic [ba0999]

```

struct MaxFlow { // 0-base
    struct edge {
        int to, cap, flow, rev;
    };
    vector<edge> G[MAXN];
    int s, t, dis[MAXN], cur[MAXN], n;
    int dfs(int u, int cap) {
        if (u == t || !cap) return cap;
        for (int &i = cur[u]; i < (int)G[u].size(); ++i) {
            edge &e = G[u][i];
            if (dis[e.to] == dis[u] + 1 && e.flow != e.cap) {
                int df = dfs(e.to, min(e.cap - e.flow, cap));
                if (df) {
                    e.flow += df;
                    G[e.to][e.rev].flow -= df;
                    return df;
                }
            }
        }
        dis[u] = -1;
        return 0;
    }
    bool bfs() {
        fill_n(dis, n, -1);
        queue<int> q;
        q.push(s), dis[s] = 0;
        while (!q.empty()) {
            int tmp = q.front();
            q.pop();
            for (auto &u : G[tmp])
                if (!dis[u.to] && u.flow != u.cap) {
                    q.push(u.to);
                    dis[u.to] = dis[tmp] + 1;
                }
        }
        return dis[t] != -1;
    }
    int maxflow(int _s, int _t) {
        s = _s, t = _t;
        int flow = 0, df;
        while (bfs()) {
            fill_n(cur, n, 0);
            while ((df = dfs(s, INF))) flow += df;
        }
        return flow;
    }
    void init(int _n) {
        n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void reset() {
        for (int i = 0; i < n; ++i)
            for (auto &j : G[i]) j.flow = 0;
    }
    void add_edge(int u, int v, int cap) {
        G[u].pb(edge{v, cap, 0, (int)G[v].size()});
        G[v].pb(edge{u, 0, 0, (int)G[u].size() - 1});
    }
};

```

### 4.14 MinCostCirculation [86e6a8]

```

struct MinCostCirculation { // 0-base
    struct Edge {
        ll from, to, cap, fcap, flow, cost, rev;
    } *past[N];
    vector<Edge> G[N];
    ll dis[N], inq[N], n;
    void BellmanFord(int s) {
        fill_n(dis, n, INF), fill_n(inq, n, 0);
        queue<int> q;
        auto relax = [&](int u, ll d, Edge *e) {
            if (dis[u] > d) {
                dis[u] = d, past[u] = e;
                if (!inq[u]) inq[u] = 1, q.push(u);
            }
        };
        relax(s, 0, 0);
        while (!q.empty()) {

```

```

            int u = q.front();
            q.pop(), inq[u] = 0;
            for (auto &e : G[u])
                if (e.cap > e.flow)
                    relax(e.to, dis[u] + e.cost, &e);
        }
    }
    void try_edge(Edge &cur) {
        if (cur.cap > cur.flow) return ++cur.cap, void();
        BellmanFord(cur.to);
        if (dis[cur.from] + cur.cost < 0) {
            ++cur.flow, --G[cur.to][cur.rev].flow;
            for (int i = cur.from; past[i]; i = past[i]->from) {
                auto &e = *past[i];
                ++e.flow, --G[e.to][e.rev].flow;
            }
        }
        ++cur.cap;
    }
    void solve(int mxlg) {
        for (int b = mxlg; b >= 0; --b) {
            for (int i = 0; i < n; ++i)
                for (auto &e : G[i])
                    e.cap *= 2, e.flow *= 2;
            for (int i = 0; i < n; ++i)
                for (auto &e : G[i])
                    if (e.fcap >> b & 1)
                        try_edge(e);
        }
    }
    void init(int _n) { n = _n;
        for (int i = 0; i < n; ++i) G[i].clear();
    }
    void add_edge(ll a, ll b, ll cap, ll cost) {
        G[a].pb(Edge{b, a, cap, 0, cost, SZ(G[b]) + (a == b)});
        G[b].pb(Edge{b, a, 0, 0, -cost, SZ(G[a]) - 1});
    }
} mcmf; // O(VE * ElogC)

```

## 5 String

### 5.1 Smallest Rotation [d69462]

```

string mcp(string s) {
    int n = SZ(s), i = 0, j = 1;
    s += s;
    while (i < n && j < n) {
        int k = 0;
        while (k < n && s[i + k] == s[j + k]) ++k;
        if (s[i + k] <= s[j + k]) j += k + 1;
        else i += k + 1;
        if (i == j) ++j;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

### 5.2 Manacher [11ebce]

```

int z[MAXN]; // 0-base
/* center i: radius z[i * 2 + 1] / 2
   center i, i + 1: radius z[i * 2 + 2] / 2
   both aba, abba have radius 2 */
void Manacher(string tmp) {
    string s = "%";
    int l = 0, r = 0;
    for (char c : tmp) s.pb(c), s.pb('%');
    for (int i = 0; i < SZ(s); ++i) {
        z[i] = r > i ? min(z[2 * l - i], r - i) : 1;
        while (i - z[i] >= 0 && i + z[i] < SZ(s)
            && s[i + z[i]] == s[i - z[i]]) ++z[i];
        if (z[i] + i > r) r = z[i] + i, l = i;
    }
}

```

### 5.3 De Bruijn sequence [151f80]

```

constexpr int MAXC = 10, MAXN = 1e5 + 10;
struct DBSeq {
    int C, N, K, L, buf[MAXC * MAXN]; // K <= C^N
    void dfs(int *out, int t, int p, int &ptr) {
        if (ptr >= L) return;
        if (t > N) {
            if (N % p) return;
            for (int i = 1; i <= p && ptr < L; ++i)

```

```

    out[ptr++] = buf[i];
} else {
    buf[t] = buf[t - p], dfs(out, t + 1, p, ptr);
    for (int j = buf[t - p] + 1; j < C; ++j)
        buf[t] = j, dfs(out, t + 1, t, ptr);
}
}
void solve(int _c, int _n, int _k, int *out) {
    int p = 0;
    C = _c, N = _n, K = _k, L = N + K - 1;
    dfs(out, 1, 1, p);
    if (p < L) fill(out + p, out + L, 0);
}
} dbs;

```

## 5.4 SAM [4d0baa]

```

const int MAXM = 1000010;
struct SAM {
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int nxt[MAXM][33], cnt[MAXM], in[MAXM];
    int newNode() {
        int res = ++tot;
        fill(nxt[res], nxt[res] + 33, 0);
        mom[res] = mx[res] = cnt[res] = in[res] = 0;
        return res;
    }
    void init() {
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c) {
        int p = lst;
        int np = newNode();
        mx[np] = mx[p] + 1;
        for (; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if (p == 0) mom[np] = root;
        else {
            int q = nxt[p][c];
            if (mx[p] + 1 == mx[q]) mom[np] = q;
            else {
                int nq = newNode();
                mx[nq] = mx[p] + 1;
                for (int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for (; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np, cnt[np] = 1;
    }
    void push(char *str) {
        for (int i = 0; str[i]; i++)
            push(str[i] - 'a' + 1);
    }
    void count() {
        for (int i = 1; i <= tot; ++i)
            ++in[mom[i]];
        queue<int> q;
        for (int i = 1; i <= tot; ++i)
            if (!in[i]) q.push(i);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            cnt[mom[u]] += cnt[u];
            if (!--in[mom[u]])
                q.push(mom[u]);
        }
    }
} sam;

```

## 5.5 Aho-Corasick Automatan [8c56e8]

```

struct AC_Automatan {
    int nx[len][sigma], fl[len], cnt[len], ord[len], top;
    int rnx[len][sigma]; // node actually be reached
    int newnode() {
        fill_n(nx[top], sigma, -1);
        return top++;
    }
    void init() {top = 1, newnode();}
}

```

```

int input(string &s) {
    int X = 1;
    for (char c : s) {
        if (!nx[X][c - 'A']) nx[X][c - 'A'] = newnode();
        X = nx[X][c - 'A'];
    }
    return X; // return the end node of string
}
void make_fl() {
    queue<int> q;
    q.push(1), fl[1] = 0;
    for (int t = 0; !q.empty(); ) {
        int R = q.front();
        q.pop(), ord[t++] = R;
        for (int i = 0; i < sigma; ++i)
            if (~nx[R][i]) {
                int X = rnx[R][i] = nx[R][i], Z = fl[R];
                for (; Z && !~nx[Z][i]; ) Z = fl[Z];
                fl[X] = Z ? nx[Z][i] : 1, q.push(X);
            }
            else rnx[R][i] = R > 1 ? rnx[fl[R]][i] : 1;
    }
}
void solve() {
    for (int i = top - 2; i > 0; --i)
        cnt[fl[ord[i]]] += cnt[ord[i]];
}
} ac;

```

## 5.6 SAIS-old [ea9200]

```

class SAIS {
public:
    int *SA, *H;
    // zero based, string content MUST > 0
    // result height H[i] is LCP(SA[i - 1], SA[i])
    // string, length, |sigma|
    void build(int *s, int n, int m = 128) {
        copy_n(s, n, _s);
        _h[0] = _s[n++] = 0;
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
        SA = _sa + 1;
        H = _h + 1;
    }
private:
    bool _t[N * 2];
    int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2],
        r[N], _sa[N * 2], _h[N];
    void mkhei(int n) {
        for (int i = 0; i < n; i++) r[_sa[i]] = i;
        for (int i = 0; i < n; i++)
            if (r[i]) {
                int ans = i > 0 ? max(_h[r[i - 1]] - 1, 0) : 0;
                while (_s[i + ans] == _s[_sa[r[i] - 1] + ans])
                    ans++;
                _h[r[i]] = ans;
            }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z) {
        bool uniq = t[n - 1] = 1, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MAGIC(XD)
        fill_n(sa, n, 0);
        copy_n(c, z, x);
        XD;
        copy_n(c, z - 1, x + 1);
        for (int i = 0; i < n; i++)
            if (sa[i] && !t[sa[i] - 1])
                sa[x[sa[i] - 1]++] = sa[i] - 1;
        copy_n(c, z, x);
        for (int i = n - 1; i >= 0; i--)
            if (sa[i] && t[sa[i] - 1])
                sa[--x[sa[i] - 1]] = sa[i] - 1;

        fill_n(c, z, 0);
        for (int i = 0; i < n; i++) uniq &= ++c[s[i]] < 2;
        partial_sum(c, c + z, c);
        if (uniq) {
            for (int i = 0; i < n; i++) sa[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; i--)

```

```

    t[i] = (s[i] == s[i + 1] ? t[i + 1]
           : s[i] < s[i + 1]);
    MAGIC(for (int i = 1; i <= n - 1;
              i++) if (t[i] && !t[i - 1]))
        sa[--x[s[i]]] = p[q[i] = nn++] = i;
    for (int i = 0; i < n; i++)
        if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            neq = (lst < 0) ||
                !equal(s + lst,
                      s + lst + p[q[sa[i]] + 1] - sa[i],
                      s + sa[i]);
            ns[q[lst = sa[i]]] = nmzx += neq;
        }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
         nmzx + 1);
    MAGIC(for (int i = nn - 1; i >= 0; i--)
          sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
}
} sa;

```

## 5.7 Z-value [2e5c4c]

```

int z[MAXn];
void make_z(const string &s) {
    int l = 0, r = 0;
    for (int i = 1; i < SZ(s); ++i) {
        for (z[i] = max(0, min(r - i + 1, z[i - l]));
             i + z[i] < SZ(s) && s[i + z[i]] == s[z[i]];
             ++z[i])
            ;
        if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
    }
}

```

## 5.8 exSAM [0b980b]

```

struct exSAM {
    int len[N * 2], link[N * 2]; // maxlength, suflink
    int next[N * 2][CNUM], tot; // [0, tot), root = 0
    int lenSorted[N * 2]; // topo. order
    int cnt[N * 2]; // occurrence
    int newnode() {
        fill_n(next[tot], CNUM, 0);
        len[tot] = cnt[tot] = link[tot] = 0;
        return tot++;
    }
    void init() { tot = 0, newnode(), link[0] = -1; }
    int insertSAM(int last, int c) {
        int cur = next[last][c];
        len[cur] = len[last] + 1;
        int p = link[last];
        while (p != -1 && !next[p][c])
            next[p][c] = cur, p = link[p];
        if (p == -1) return link[cur] = 0, cur;
        int q = next[p][c];
        if (len[p] + 1 == len[q]) return link[cur] = q, cur;
        int clone = newnode();
        for (int i = 0; i < CNUM; ++i)
            next[clone][i] = len[next[q][i]] ? next[q][i] : 0;
        len[clone] = len[p] + 1;
        while (p != -1 && next[p][c] == q)
            next[p][c] = clone, p = link[p];
        link[link[cur] = clone] = link[q];
        link[q] = clone;
        return cur;
    }
    void insert(const string &s) {
        int cur = 0;
        for (auto ch : s) {
            int &nxt = next[cur][int(ch - 'a')];
            if (!nxt) nxt = newnode();
            cnt[cur = nxt] += 1;
        }
    }
    void build() {
        queue<int> q;
        q.push(0);
        while (!q.empty()) {
            int cur = q.front();
            q.pop();
            for (int i = 0; i < CNUM; ++i)
                if (next[cur][i])
                    q.push(insertSAM(cur, i));
        }
        vector<int> lc(tot);
    }

```

```

    for (int i = 1; i < tot; ++i) ++lc[len[i]];
    partial_sum(ALL(lc), lc.begin());
    for (int i = 1; i < tot; ++i) lenSorted[--lc[len[i]]] = i;
}
void solve() {
    for (int i = tot - 2; i >= 0; --i)
        cnt[link[lenSorted[i]]] += cnt[lenSorted[i]];
}
};

```

## 5.9 SAIS [afbad7]

```

namespace sfx {
    bool _t[N * 2];
    int SA[N * 2], H[N], RA[N];
    int _s[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2];
    // zero based, string content MUST > 0
    // SA[i]: SA[i]-th
    // suffix is the i-th lexicographically smallest suffix.
    // H[i]: longest
    // common prefix of suffix SA[i] and suffix SA[i - 1].
    void pre(int *sa, int *c, int n, int z)
    { fill_n(sa, n, 0), copy_n(c, z, x); }
    void induce
    (int *sa, int *c, int *s, bool *t, int n, int z) {
        copy_n(c, z - 1, x + 1);
        for (int i = 0; i < n; ++i)
            if (sa[i] && !t[sa[i] - 1])
                sa[x[s[sa[i] - 1]]++] = sa[i] - 1;
        copy_n(c, z, x);
        for (int i = n - 1; i >= 0; --i)
            if (sa[i] && t[sa[i] - 1])
                sa[--x[s[sa[i] - 1]]] = sa[i] - 1;
    }
    void sais(int *s, int *sa
              , int *p, int *q, bool *t, int *c, int n, int z) {
        bool uniq = t[n - 1] = true;
        int nn = 0;
        nmzx = -1, *nsa = sa + n, *ns = s + n, last = -1;
        fill_n(c, z, 0);
        for (int i = 0; i < n; ++i) uniq &= ++c[s[i]] < 2;
        partial_sum(c, c + z, c);
        if (uniq) {
            for (int i = 0; i < n; ++i) sa[--c[s[i]]] = i;
            return;
        }
        for (int i = n - 2; i >= 0; --i)
            t[i] = (
                s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
        pre(sa, c, n, z);
        for (int i = 1; i <= n - 1; ++i)
            if (t[i] && !t[i - 1])
                sa[--x[s[i]]] = p[q[i] = nn++] = i;
        induce(sa, c, s, t, n, z);
        for (int i = 0; i < n; ++i)
            if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
                bool neq = last < 0 || !equal
                    (s + sa[i], s + p[q[sa[i]] + 1], s + last);
                ns[q[last = sa[i]]] = nmzx += neq;
            }
        sais(ns,
            nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
        pre(sa, c, n, z);
        for (int i = nn - 1; i >= 0; --i)
            sa[--x[s[p[nsa[i]]]]] = p[nsa[i]];
        induce(sa, c, s, t, n, z);
    }
    void mkhei(int n) {
        for (int i = 0, j = 0; i < n; ++i) {
            if (RA[i])
                for (; _s[i + j] == _s[SA[RA[i] - 1] + j]; ++j);
            H[RA[i]] = j, j = max(0, j - 1);
        }
    }
    void build(int *s, int n) {
        copy_n(s, n, _s), _s[n] = 0;
        sais(_s, SA, _p, _q, _t, _c, n + 1, 256);
        copy_n(SA + 1, n, SA);
        for (int i = 0; i < n; ++i) RA[SA[i]] = i;
        mkhei(n);
    }
}

```

## 5.10 SAIS-C++20 [2cd2ea]

```

auto sais(const auto &s) {
    const int n = SZ(s), z = ranges::max(s) + 1;

```

```

if (n == 1) return vector{0};
vector<int> c(z); for (int x : s) ++c[x];
partial_sum(ALL(c), begin(c));
vector<int> sa(n); auto I = views::iota(0, n);
vector<bool> t(n, true);
for (int i = n - 2; i >= 0; --i)
    t[i] = (
        s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
auto is_lms = views::filter([&t](int x) {
    return x && t[x] && !t[x - 1];
});
auto induce = [&] {
    for (auto x = c; int y : sa)
        if (y-- if (!t[y]) sa[x[s[y] - 1]++] = y;
    for (auto x = c; int y : sa | views::reverse)
        if (y-- if (t[y]) sa[--x[s[y]]] = y;
};
vector<int> lms, q(n); lms.reserve(n);
for (auto x = c; int i : I | is_lms)
    q[i] = SZ(lms), lms.pb(sa[--x[s[i]]] = i);
induce(); vector<int> ns(SZ(lms));
for (int j = -1, nz = 0; int i : sa | is_lms) {
    if (j >= 0) {
        int len = min({n - i, n - j, lms[q[i] + 1] - i});
        ns[q[i]] = nz += lexicographical_compare(
            begin(s) + j, begin(s) + j + len,
            begin(s) + i, begin(s) + i + len);
    }
    j = i;
}
fill(ALL(sa), 0); auto nsa = sais(ns);
for (auto x = c; int y : nsa | views::reverse)
    y = lms[y], sa[--x[s[y]]] = y;
return induce(), sa;
}
// sa[i]: sa[i]-th suffix
// is the i-th lexicographically smallest suffix.
// hi[i]: LCP of suffix sa[i] and suffix sa[i - 1].
struct Suffix {
    int n; vector<int> sa, hi, ra;
    Suffix(
        (const auto &s, int _n) : n(_n), hi(n), ra(n) {
            vector<int> s(n + 1); // s[n] = 0;
            copy_n(_s, n, begin(s)); // _s shouldn't contain 0
            sa = sais(s); sa.erase(sa.begin());
            for (int i = 0; i < n; ++i) ra[sa[i]] = i;
            for (int i = 0, h = 0; i < n; ++i) {
                if (!ra[i]) { h = 0; continue; }
                for (int j = sa[ra[i] - 1]; max(
                    (i, j) + h < n && s[i + h] == s[j + h];) ++h;
                hi[ra[i]] = h ? h - 1 : 0;
            }
        }
};

```

## 5.11 PalTree [9bd3fb]

```

struct palindromic_tree {
    struct node {
        int next[26], fail, len;
        int cnt, num; // cnt: appear times, num: number of
        // pal. suf.
        node(int l = 0) : fail(0), len(l), cnt(0), num(0) {
            for (int i = 0; i < 26; ++i) next[i] = 0;
        }
    };
    vector<node> St;
    vector<char> s;
    int last, n;
    palindromic_tree() : St(2), last(1), n(0) {
        St[0].fail = 1, St[1].len = -1, s.pb(-1);
    }
    inline void clear() {
        St.clear(), s.clear(), last = 1, n = 0;
        St.pb(0), St.pb(-1);
        St[0].fail = 1, s.pb(-1);
    }
    inline int get_fail(int x) {
        while (s[n - St[x].len - 1] != s[n])
            x = St[x].fail;
        return x;
    }
    inline void add(int c) {
        s.push_back(c == 'a', ++n);
        int cur = get_fail(last);
        if (!St[cur].next[c]) {
            int now = SZ(St);

```

```

        St.pb(St[cur].len + 2);
        St[now].fail =
            St[get_fail(St[cur].fail)].next[c];
        St[cur].next[c] = now;
        St[now].num = St[St[now].fail].num + 1;
    }
    last = St[cur].next[c], ++St[last].cnt;
}
inline void count() { // counting cnt
    auto i = St.rbegin();
    for (; i != St.rend(); ++i) {
        St[i->fail].cnt += i->cnt;
    }
}
inline int size() { // The number of diff. pal.
    return SZ(St) - 2;
}
};

```

## 5.12 MainLorentz [2981c4]

```

vector<pair<int, int>> rep[kN]; // 0-base [l, r]
void main_lorentz(const string &s, int sft = 0) {
    const int n = s.size();
    if (n == 1) return;
    const int nu = n / 2, nv = n - nu;
    const string u = s.substr(0, nu), v = s.substr(nu),
        ru(u.rbegin(), u.rend()), rv(v.rbegin(), v.rend());
    main_lorentz(u, sft), main_lorentz(v, sft + nu);
    const auto z1 = Zalgo(ru), z2 = Zalgo(v + '#' + u),
        z3 = Zalgo(ru + '#' + rv), z4 = Zalgo(v);
    auto get_z = [](const vector<int> &z, int i) {
        return
            (0 <= i and i < (int)z.size()) ? z[i] : 0;
    };
    auto add_rep
        = [&](bool left, int c, int l, int k1, int k2) {
            const
                int L = max(1, l - k2), R = min(l - left, k1);
            if (L > R) return;
            if (left)
                rep[l].emplace_back(sft + c - R, sft + c - L);
            else
                rep[l].emplace_back
                    (sft + c - R - l + 1, sft + c - L - l + 1);
        };
    for (int cntr = 0; cntr < n; cntr++) {
        int l, k1, k2;
        if (cntr < nu) {
            l = nu - cntr;
            k1 = get_z(z1, nu - cntr);
            k2 = get_z(z2, nv + 1 + cntr);
        } else {
            l = cntr - nu + 1;
            k1 = get_z(z3, nu + 1 + nv - 1 - (cntr - nu));
            k2 = get_z(z4, (cntr - nu) + 1);
        }
        if (k1 + k2 >= l)
            add_rep(cntr < nu, cntr, l, k1, k2);
    }
} // p | in [l, r] => s[p, p + i) = s[p + i, p + 2i)

```

## 5.13 KMP [32f229]

```

int F[MAXN];
vector<int> match(string A, string B) {
    vector<int> ans;
    F[0] = -1, F[1] = 0;
    for (int i = 1, j = 0; i < SZ(B); F[++i] = ++j) {
        if (B[i] == B[j]) F[i] = F[j]; // optimize
        while (j != -1 && B[i] != B[j]) j = F[j];
    }
    for (int i = 0, j = 0; i < SZ(A); ++i) {
        while (j != -1 && A[i] != B[j]) j = F[j];
        if (++j == SZ(B)) ans.pb(i + 1 - j), j = F[j];
    }
    return ans;
}

```

## 5.14 Suffix Array [b981d5]

```

struct suffix_array {
    int box[MAXN], tp[MAXN], m;
    bool not_equ(int a, int b, int k, int n) {
        return ra[a] != ra[b] || a + k >= n ||
            b + k >= n || ra[a + k] != ra[b + k];
    }
    void radix(int *key, int *it, int *ot, int n) {

```



```

fill_n(box, m, 0);
for (int i = 0; i < n; ++i) ++box[key[i]];
partial_sum(box, box + m, box);
for (int i = n - 1; i >= 0; --i)
    ot[--box[key[it[i]]]] = it[i];
}
void make_sa(const string &s, int n) {
    int k = 1;
    for (int i = 0; i < n; ++i) ra[i] = s[i];
    do {
        iota(tp, tp + k, n - k), iota(sa + k, sa + n, 0);
        radix(ra + k, sa + k, tp + k, n - k);
        radix(ra, tp, sa, n);
        tp[sa[0]] = 0, m = 1;
        for (int i = 1; i < n; ++i) {
            m += not_equ(sa[i], sa[i - 1], k, n);
            tp[sa[i]] = m - 1;
        }
        copy_n(tp, n, ra);
        k *= 2;
    } while (k < n && m != n);
}
void make_he(const string &s, int n) {
    for (int j = 0, k = 0; j < n; ++j) {
        if (ra[j])
            for (; s[j + k] == s[sa[ra[j] - 1] + k]; ++k)
                he[ra[j]] = k, k = max(0, k - 1);
    }
    int sa[MAXN], ra[MAXN], he[MAXN];
    void build(const string &s) {
        int n = SZ(s);
        fill_n
            (sa, n, 0), fill_n(ra, n, 0), fill_n(he, n, 0);
        fill_n(box, n, 0), fill_n(tp, n, 0), m = 256;
        make_sa(s, n), make_he(s, n);
    }
};

```

## 6 Math

### 6.1 chineseRemainder [0e2467]

```

ll solve(ll x1, ll m1, ll x2, ll m2) {
    ll g = gcd(m1, m2);
    if ((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pll p = exgcd(m1, m2);
    ll lcm = m1 * m2 * g;
    ll res = p.first * (x2 - x1) * m1 + x1;
    // be careful with overflow
    return (res % lcm + lcm) % lcm;
}

```

### 6.2 PiCount [29fb4b]

```

ll PrimeCount(ll n) { // n ~ 10^13 => < 2s
    if (n <= 1) return 0;
    int v = sqrt(n), s = (v + 1) / 2, pc = 0;
    vector<int> smalls(v + 1), skip(v + 1), roughs(s);
    vector<ll> larges(s);
    for (int i = 2; i <= v; ++i) smalls[i] = (i + 1) / 2;
    for (int i = 0; i < s; ++i) {
        roughs[i] = 2 * i + 1;
        larges[i] = (n / (2 * i + 1) + 1) / 2;
    }
    for (int p = 3; p <= v; ++p) {
        if (smalls[p] > smalls[p - 1]) {
            int q = p * p;
            ++pc;
            if (1LL * q * q > n) break;
            skip[p] = 1;
            for (int i = q; i <= v; i += 2 * p) skip[i] = 1;
            int ns = 0;
            for (int k = 0; k < s; ++k) {
                int i = roughs[k];
                if (skip[i]) continue;
                ll d = 1LL * i * p;
                larges[ns] = larges[k] - (d <= v ? larges
                    [smalls[d] - pc] : smalls[n / d]) + pc;
                roughs[ns++] = i;
            }
            s = ns;
            for (int j = v / p; j >= p; --j) {
                int c =
                    smalls[j] - pc, e = min(j * p + p, v + 1);

```

```

                for (int i = j * p; i < e; ++i) smalls[i] -= c;
            }
        }
    }
    for (int k = 1; k < s; ++k) {
        const ll m = n / roughs[k];
        ll t = larges[k] - (pc + k - 1);
        for (int l = 1; l < k; ++l) {
            int p = roughs[l];
            if (1LL * p * p > m) break;
            t -= smalls[m / p] - (pc + l - 1);
        }
        larges[0] -= t;
    }
    return larges[0];
}

```

### 6.3 numbers

- Bernoulli numbers

$$B_0=1, B_1=\pm\frac{1}{2}, B_2=\frac{1}{6}, B_3=0$$

$$\sum_{j=0}^m \binom{m+1}{j} B_j = 0, \text{EGF is } B(x) = \frac{x}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

$$S_m(n) = \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k^+ n^{m+1-k}$$

- Stirling numbers of the second kind Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k), S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$$

$$x^n = \sum_{i=0}^n S(n, i) (x)_i$$

- Pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - x^n) = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right)$$

- Catalan numbers

$$C_n^{(k)} = \frac{1}{(k-1)n+1} \binom{kn}{n}$$

$$C^{(k)}(x) = 1 + x[C^{(k)}(x)]^k$$

- Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k, j$ 's s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1, j$ 's s.t.  $\pi(j) \geq j$ ,  $k, j$ 's s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

### 6.4 Estimation

$n$	2	3	4	5	6	7	8	9	20	30	40	50	100		
$p(n)$	2	3	5	7	11	15	22	30	627	5604	4e4	2e5	2e8		
$n$	100	1e3	1e6	1e9	1e12	1e15	1e18								
$d(i)$	12	32	240	1344	6720	26880	103680								
$n$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\binom{2n}{n}$	2	6	20	70	252	924	3432	12870	48620	184756	7e5	2e6	1e7	4e7	1.5e8
$n$	2	3	4	5	6	7	8	9	10	11	12	13			
$B_n$	2	5	15	52	203	877	4140	21147	115975	7e5	4e6	3e7			

### 6.5 floor sum [76c575]

```

ll floor_sum(ll n, ll m, ll a, ll b) {
    ll ans = 0;
    if (a >= m)
        ans += (n - 1) * n * (a / m) / 2, a %= m;
    if (b >= m)
        ans += n * (b / m), b %= m;
    ll y_max
        = (a * n + b) / m, x_max = (y_max * m - b);
    if (y_max == 0) return ans;
    ans += (n - (x_max + a - 1) / a) * y_max;
    ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
    return ans;
} // sum^{n-1}_0 floor((a * i + b) / m) in log(n + m + a + b)

```

### 6.6 QuadraticResidue [0b50c4]

```

int Jacobi(int a, int m) {
    int s = 1;
    for (; m > 1; ) {
        a %= m;
        if (a == 0) return 0;
        const int r = __builtin_ctz(a);
        if ((r & 1) && ((m + 2) & 4)) s = -s;
        a >>= r;
        if (a & m & 2) s = -s;
        swap(a, m);
    }
}

```



```

    return s;
}

int QuadraticResidue(int a, int p) {
    if (p == 2) return a & 1;
    const int jc = Jacobi(a, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    int b, d;
    for (; ; ) {
        b = rand() % p;
        d = (1LL * b * b + p - a) % p;
        if (Jacobi(d, p) == -1) break;
    }
    int f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (1LL + p) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = (1LL *
                g0 * f0 + 1LL * d * (1LL * g1 * f1 % p)) % p;
            g1 = (1LL * g0 * f1 + 1LL * g1 * f0) % p;
            g0 = tmp;
        }
        tmp = (1LL *
            f0 * f0 + 1LL * d * (1LL * f1 * f1 % p)) % p;
        f1 = (2LL * f0 * f1) % p;
        f0 = tmp;
    }
    return g0;
}

```

## 6.7 floor enumeration [fc55c8]

```

// enumerating x = floor(n / i), [l, r]
for (int l = 1, r; l <= n; l = r + 1) {
    int x = n / l;
    r = n / x;
}

```

## 6.8 ax+by=gcd [43bd81]

```

pll exgcd(ll a, ll b) {
    if (b == 0) return pll(1, 0);
    ll p = a / b;
    pll q = exgcd(b, a % b);
    return pll(q.Y, q.X - q.Y * p);
}

/* ax+by=res, let x be minimum non-negative
g, p = gcd(a, b), exgcd(a, b) * res / g
if p.X < 0: t = (abs(p.X) + b / g - 1) / (b / g)
else: t = -(p.X / (b / g))
p += (b / g, -a / g) * t */

```

## 6.9 cantor expansion [2d801a]

```

#define MAXN 11
int factorial[MAXN];
inline void init(){
    factorial[0]=1;
    for(int i=1;i<=MAXN;++i){
        factorial[i]=factorial[i-1]*i;
    }
}
inline int encode(const std::vector<int> &s){
    int n=s.size(),res=0;
    for(int i=0;i<n;++i){
        int t=0;
        for(int j=i+1;j<n;++j){
            if(s[j]<s[i])++t;
        }
        res+=t*factorial[n-i-1];
    }
    return res;
}
inline std::vector<int> decode(int a,int n){
    std::vector<int> res;
    std::vector<bool> vis(n,0);
    for(int i=n-1;i>=0;--i){
        int t=a/factorial[i],j;
        for(j=0;j<n;++j){
            if(!vis[j]){
                if(t==0)break;
                --t;
            }
        }
        res.push_back(j);
        vis[j]=1;
        a%=factorial[i];
    }
}

```

```

}
return res;
}

```

## 6.10 Generating function

- Ordinary Generating Function  $A(x) = \sum_{i \geq 0} a_i x^i$ 
  - $A(rx) \Rightarrow r^n a_n$
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^n a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} a_{i_1} a_{i_2} \dots a_{i_k}$
  - $x A(x)' \Rightarrow n a_n$
  - $\frac{A(x)}{1-x} \Rightarrow \sum_{i=0}^n a_i$
- Exponential Generating Function  $A(x) = \sum_{i \geq 0} \frac{a_i}{i!} x^i$ 
  - $A(x) + B(x) \Rightarrow a_n + b_n$
  - $A^{(k)}(x) \Rightarrow a_{n+k}$
  - $A(x)B(x) \Rightarrow \sum_{i=0}^n \binom{n}{i} a_i b_{n-i}$
  - $A(x)^k \Rightarrow \sum_{i_1+i_2+\dots+i_k=n} \binom{n}{i_1, i_2, \dots, i_k} a_{i_1} a_{i_2} \dots a_{i_k}$
  - $x A(x) \Rightarrow n a_n$
- Special Generating Function
  - $(1+x)^n = \sum_{i \geq 0} \binom{n}{i} x^i$
  - $\frac{1}{(1-x)^n} = \sum_{i \geq 0} \binom{i+n-1}{n-1} x^i$

## 6.11 Fraction [666134]

```

struct fraction {
    ll n, d;
    fraction(
        (const ll &n=0, const ll &d=1): n(_n), d(_d) {
            ll t = gcd(n, d);
            n /= t, d /= t;
            if (d < 0) n = -n, d = -d;
        }
    fraction operator-(const fraction &b) const {
        return fraction(n * b.d + b.n * d, d * b.d);
    }
    fraction operator+(const fraction &b) const {
        return fraction(n * b.d - b.n * d, d * b.d);
    }
    fraction operator*(const fraction &b) const {
        return fraction(n * b.n, d * b.d);
    }
    fraction operator/(const fraction &b) const {
        return fraction(n * b.d, d * b.n);
    }
    void print() {
        cout << n;
        if (d != 1) cout << "/" << d;
    }
};

```

## 6.12 Gaussian gcd [616465]

```

cpx gaussian_gcd(cpx a, cpx b) {
#define rnd
    (a, b) ((a >= 0 ? a * 2 + b : a * 2 - b) / (b * 2))
    ll c = a.real() * b.real() + a.imag() * b.imag();
    ll d = a.imag() * b.real() - a.real() * b.imag();
    ll r = b.real() * b.real() + b.imag() * b.imag();
    if (c % r == 0 && d % r == 0) return b;
    return gaussian_gcd(
        (b, a - cpx(rnd(c, r), rnd(d, r)) * b);
    }
}

```

## 6.13 Theorem

- Cramer's rule

$$\begin{aligned}
 ax+by=e \\
 cx+dy=f
 \end{aligned}
 \Rightarrow
 \begin{aligned}
 x &= \frac{ed-bf}{ad-bc} \\
 y &= \frac{af-ec}{ad-bc}
 \end{aligned}$$

- Vandermonde's Identity

$$C(n+m, k) = \sum_{i=0}^k C(n, i) C(m, k-i)$$

- Kirchhoff's Theorem

Denote  $L$  be a  $n \times n$  matrix as the Laplacian matrix of graph  $G$ , where  $L_{ii} = d(i)$ ,  $L_{ij} = -c$  where  $c$  is the number of edge  $(i, j)$  in  $G$ .

- The number of undirected spanning in  $G$  is  $|\det(\tilde{L}_{11})|$ .
- The number of directed spanning tree rooted at  $r$  in  $G$  is  $|\det(\tilde{L}_{rr})|$ .

- Tutte's Matrix

Let  $D$  be a  $n \times n$  matrix, where  $d_{ij} = x_{ij}$  ( $x_{ij}$  is chosen uniformly at random) if  $i < j$  and  $(i, j) \in E$ , otherwise  $d_{ij} = -d_{ji}$ .  $\frac{\text{rank}(D)}{2}$  is the maximum matching on  $G$ .

- Cayley's Formula

- Given a degree sequence  $d_1, d_2, \dots, d_n$  for each *labeled* vertices, there are  $\frac{(n-2)!}{(d_1-1)!(d_2-1)!\dots(d_n-1)!}$  spanning trees.
- Let  $T_{n,k}$  be the number of *labeled* forests on  $n$  vertices with  $k$  components, such that vertex  $1, 2, \dots, k$  belong to different components. Then  $T_{n,k} = kn^{n-k-1}$ .

#### Erdős–Gallai theorem

A sequence of nonnegative integers  $d_1 \geq \dots \geq d_n$  can be represented as the degree sequence of a finite simple graph on  $n$  vertices if and only if

$$d_1 + \dots + d_n \text{ is even and } \sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k) \text{ holds for every } 1 \leq k \leq n.$$

#### Gale–Ryser theorem

A pair of sequences of nonnegative integers  $a_1 \geq \dots \geq a_n$  and  $b_1, \dots, b_n$

is bigraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and  $\sum_{i=1}^k a_i \leq \sum_{i=1}^n \min(b_i, k)$  holds for every  $1 \leq k \leq n$ .

#### Fulkerson–Chen–Anstee theorem

A sequence  $(a_1, b_1), \dots, (a_n, b_n)$  of nonnegative integer pairs

with  $a_1 \geq \dots \geq a_n$  is digraphic if and only if  $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i$  and

$$\sum_{i=1}^k a_i \leq \sum_{i=1}^k \min(b_i, k-1) + \sum_{i=k+1}^n \min(b_i, k) \text{ holds for every } 1 \leq k \leq n.$$

#### Pick's theorem

For simple polygon, when points are all integer, we have  $A = \#\{\text{lattice points in the interior}\} + \frac{\#\{\text{lattice points on the boundary}\}}{2} - 1$ .

#### Möbius inversion formula

$$\begin{aligned} - f(n) &= \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) \\ - f(n) &= \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d) \end{aligned}$$

#### Spherical cap

- A portion of a sphere cut off by a plane.
- $r$ : sphere radius,  $a$ : radius of the base of the cap,  $h$ : height of the cap,  $\theta$ :  $\arcsin(a/r)$ .
- Volume  $= \pi h^2(3r - h)/3 = \pi h(3a^2 + h^2)/6 = \pi r^3(2 + \cos \theta)(1 - \cos \theta)^2/3$ .
- Area  $= 2\pi r h = \pi(a^2 + h^2) = 2\pi r^2(1 - \cos \theta)$ .

#### Lagrange multiplier

- Optimize  $f(x_1, \dots, x_n)$  when  $k$  constraints  $g_i(x_1, \dots, x_n) = 0$ .
- Lagrangian function  $\mathcal{L}(x_1, \dots, x_n, \lambda_1, \dots, \lambda_k) = f(x_1, \dots, x_n) - \sum_{i=1}^k \lambda_i g_i(x_1, \dots, x_n)$ .
- The solution corresponding to the original constrained optimization is always a saddle point of the Lagrangian function.

#### Nearest points of two skew lines

- Line 1:  $v_1 = p_1 + t_1 d_1$
- Line 2:  $v_2 = p_2 + t_2 d_2$
- $n = d_1 \times d_2$
- $n_1 = d_1 \times n$
- $n_2 = d_2 \times n$
- $c_1 = p_1 + \frac{(p_2 - p_1) \cdot n_2}{d_1 \cdot n_2} d_1$
- $c_2 = p_2 + \frac{(p_1 - p_2) \cdot n_1}{d_2 \cdot n_1} d_2$

#### Derivatives/Integrals

Integration by parts:  $\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$

$$\left| \begin{aligned} \frac{d}{dx} \sin^{-1} x &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx} \cos^{-1} x &= -\frac{1}{\sqrt{1-x^2}} & \frac{d}{dx} \tan^{-1} x &= \frac{1}{1+x^2} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \int \tan ax &= -\frac{\ln|\cos ax|}{a} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1) \\ \int \sqrt{a^2 + x^2} &= \frac{1}{2} \left( x \sqrt{a^2 + x^2} + a^2 \operatorname{asinh}(x/a) \right) \end{aligned} \right|$$

#### Spherical Coordinate

$$(x, y, z) = (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta)$$

$$(r, \theta, \phi) = (\sqrt{x^2 + y^2 + z^2}, \arccos(z/\sqrt{x^2 + y^2 + z^2}), \operatorname{atan2}(y, x))$$

#### Rotation Matrix

$$M(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}, R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & -\sin \theta_x \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

## 6.14 Determinant [a4d696]

```
struct Matrix {
    int n, m;
    ll M[MAXN][MAXN];
    int row_swap(int i, int j) {
        if (i == j) return 0;
        for (int k = 0; k < m; ++k)
            swap(M[i][k], M[j][k]);
        return 1;
    }
    ll det() { // return the number of swaps
        int rt = 0;
        for (int i = 0; i < n; ++i) {
            int piv = i;
            while (piv < n && !M[piv][i]) ++piv;
            if (piv == n) continue;
            rt += row_swap(i, piv);
            for (int j = i + 1; j < n; ++j) {
                while (M[j][i]) {
                    int tmp = P - M[j][i] / M[i][i];
                    for (int k = i; k < m; ++k)
                        M[j][k] = (M[j][k] * tmp + M[i][k]) % P;
                    rt += row_swap(i, j);
                }
            }
            rt = (rt & 1) ? P - 1 : 1;
            for (int i = 0; i < n; ++i)
                rt = rt * M[i][i] % P;
            return rt;
        }
        // round(rt) if using double to cal. int. det
    }
};
```

## 6.15 ModMin [05065e]

```
// min{k | l <= ((ak) mod m) <= r}, no solution -> -1
ll mod_min(ll a, ll m, ll l, ll r) {
    if (a == 0) return l ? -1 : 0;
    if (ll k = (l + a - 1) / a; k * a <= r)
        return k;
    ll b = m / a, c = m % a;
    if (ll y = mod_min(c, a, a - r % a, a - l % a))
        return (l + y * c + a - 1) / a + y * b;
    return -1;
}
```

## 6.16 Primes [2464ae]

```
/* 12721 13331 14341 75577 123457 222557
   556679 999983 1097774749 1076767633 100102021
   999997771 1001010013 1000512343 987654361 999991231
   999888733 98789101 987777733 999991921 1010101333
   1010102101 1000000000039 100000000000037
   2305843009213693951 4611686018427387847
   9223372036854775783 18446744073709551557 */
```

## 6.17 Pollard Rho [a5802e]

```
map<ll, int> cnt;
void PollardRho(ll n) {
    if (n == 1) return;
    if (prime(n)) return ++cnt[n], void();
    if (n % 2 == 0) return PollardRho(n / 2), ++cnt[2], void();
    ll x = 2, y = 2, d = 1, p = 1;
    #define f(x, n, p) ((mul(x, x, n) + p) % n)
    while (true) {
        if (d != n && d != 1) {
            PollardRho(n / d);
            PollardRho(d);
            return;
        }
        if (d == n) ++p;
        x = f(x, n, p), y = f(f(y, n, p), n, p);
        d = gcd(abs(x - y), n);
    }
}
```

## 6.18 Simultaneous Equations [b8b03f]

```
struct matrix { //m variables, n equations
    int n, m;
    fraction M[MAXN][MAXN + 1], sol[MAXN];
    int solve() { // -1: inconsistent, >= 0: rank
        for (int i = 0; i < n; ++i) {
            int piv = 0;
```

```

while (piv < m && !M[i][piv].n) ++piv;
if (piv == m) continue;
for (int j = 0; j < n; ++j) {
    if (i == j) continue;
    fraction tmp = -M[j][piv] / M[i][piv];
    for (int k = 0; k <= m; ++k) M[j][k] = tmp * M[i][k] + M[j][k];
}
int rank = 0;
for (int i = 0; i < n; ++i) {
    int piv = 0;
    while (piv < m && !M[i][piv].n) ++piv;
    if (piv == m && M[i][m].n) return -1;
    else if (piv < m) ++rank, sol[piv] = M[i][m] / M[i][piv];
}
return rank;
};

```

## 6.19 Big number [1c17ab]

```

template<typename T>
inline string to_string(const T& x){
    stringstream ss;
    return ss<<x,ss.str();
}
struct bigN:vector<ll>{
    const static int base=1000000000,width=log10(base);
    bool negative;
    bigN(const_iterator
        a,const_iterator b):vector<ll>(a,b){}
    bigN(string s){
        if(s.empty())return;
        if(s[0]=='-')negative=1,s=s.substr(1);
        else negative=0;
        for(int i=int(s.size())-1;i>=0;i-=width){
            ll t=0;
            for(int j=max(0,i-width+1);j<=i;++j)
                t=t*10+s[j]-'0';
            push_back(t);
        }
        trim();
    }
    template<typename T>
    bigN(const T &x):bigN(to_string(x)){}
    bigN():negative(0){}
    void trim(){
        while(size()&&!back())pop_back();
        if(empty())negative=0;
    }
    void carry(int _base=base){
        for(size_t i=0;i<size();++i){
            if(at(i)>=0&&at(i)<_base)continue;
            if(i+1u==size())push_back(0);
            int r=at(i)%_base;
            if(r<0)r+=_base;
            at(i+1)+=(at(i)-r)/_base,at(i)=r;
        }
    }
    int abscmp(const bigN &b)const{
        if(size()>b.size())return 1;
        if(size()<b.size())return -1;
        for(int i=int(size())-1;i>=0;--i){
            if(at(i)>b[i])return 1;
            if(at(i)<b[i])return -1;
        }
        return 0;
    }
    int cmp(const bigN &b)const{
        if(negative!=b.negative)return negative?-1:1;
        return negative?-abscmp(b):abscmp(b);
    }
    bool operator<(const bigN&b)const{return cmp(b)<0;}
    bool operator>(const bigN&b)const{return cmp(b)>0;}
    bool operator<=(const bigN&b)const{return cmp(b)<=0;}
    bool operator>=(const bigN&b)const{return cmp(b)>=0;}
    bool operator==(const bigN&b)const{return !cmp(b);}
    bool operator!=(const bigN&b)const{return cmp(b)!=0;}
    bigN abs()const{
        bigN res=*this;
        return res.negative=0, res;
    }
    bigN operator-()const{
        bigN res=*this;
        return res.negative=!negative,res.trim(),res;
    }
};

```

```

}
bigN operator+(const bigN &b)const{
    if(negative)return -(-(*this)+(-b));
    if(b.negative)return *this-(-b);
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]+=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator-(const bigN &b)const{
    if(negative)return -(-(*this)-(-b));
    if(b.negative)return *this+(-b);
    if(abscmp(b)<0)return -(b-(*this));
    bigN res=*this;
    if(b.size()>size())res.resize(b.size());
    for(size_t i=0;i<b.size();++i)res[i]-=b[i];
    return res.carry(),res.trim(),res;
}
bigN operator*(const bigN &b)const{
    bigN res;
    res.negative=negative!=b.negative;
    res.resize(size()+b.size());
    for(size_t i=0;i<size();++i)
        for(size_t j=0;j<b.size();++j)
            if((res[i+j]+at(i)*b[j])>=base){
                res[i+j+1]+=res[i+j]/base;
                res[i+j]%=base;
            }
    return res.trim(),res;
}
bigN operator/(const bigN &b)const{
    int norm=base/(b.back()+1);
    bigN x=abs()*norm;
    bigN y=b.abs()*norm;
    bigN q,r;
    q.resize(x.size());
    for(int i=int(x.size())-1;i>=0;--i){
        r=r*base+x[i];
        int s1=r.size()<=y.size()?0:r[y.size()];
        int s2=r.size()<y.size()?0:r[y.size()-1];
        int d=(ll(base)*s1+s2)/y.back();
        r=r-y*d;
        while(r.negative)r=r+y,--d;
        q[i]=d;
    }
    q.negative=negative!=b.negative;
    return q.trim(),q;
}
bigN operator%(const bigN &b)const{
    return *this-(*this/b)*b;
}
friend istream& operator>>(istream &ss,bigN &b){
    string s;
    return ss>>s, b=s, ss;
}
friend
    ostream& operator<<(ostream &ss,const bigN &b){
        if(b.negative)ss<<"-";
        ss<<(b.empty()?0:b.back());
        for(int i=int(b.size())-2;i>=0;--i)
            ss<<setw(width)<<setfill('0')<<b[i];
        return ss;
    }
template<typename T>
operator T(){
    stringstream ss;
    ss<<*this;
    T res;
    return ss>>res,res;
}
};

```

## 6.20 Euclidean

- $m = \lfloor \frac{an+b}{c} \rfloor$
- Time complexity:  $O(\log n)$

$$f(a,b,c,n) = \sum_{i=0}^n \left\lfloor \frac{ai+b}{c} \right\rfloor$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)}{2} + \left\lfloor \frac{b}{c} \right\rfloor \cdot (n+1) \\ \quad + f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm - f(c, c-b-1, a, m-1), & \text{otherwise} \end{cases}$$

$$g(a,b,c,n) = \sum_{i=0}^n i \left\lfloor \frac{ai+b}{c} \right\rfloor$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor \cdot \frac{n(n+1)}{2} \\ + g(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ \frac{1}{2} \cdot (n(n+1)m - f(c, c-b-1, a, m-1) \\ - h(c, c-b-1, a, m-1)), & \text{otherwise} \end{cases}$$

$$h(a,b,c,n) = \sum_{i=0}^n \left\lfloor \frac{ai+b}{c} \right\rfloor^2$$

$$= \begin{cases} \left\lfloor \frac{a}{c} \right\rfloor^2 \cdot \frac{n(n+1)(2n+1)}{6} + \left\lfloor \frac{b}{c} \right\rfloor^2 \cdot (n+1) \\ + \left\lfloor \frac{a}{c} \right\rfloor \cdot \left\lfloor \frac{b}{c} \right\rfloor \cdot n(n+1) \\ + h(a \bmod c, b \bmod c, c, n) \\ + 2 \left\lfloor \frac{a}{c} \right\rfloor \cdot g(a \bmod c, b \bmod c, c, n) \\ + 2 \left\lfloor \frac{b}{c} \right\rfloor \cdot f(a \bmod c, b \bmod c, c, n), & a \geq c \vee b \geq c \\ 0, & n < 0 \vee a = 0 \\ nm(m+1) - 2g(c, c-b-1, a, m-1) \\ - 2f(c, c-b-1, a, m-1) - f(a, b, c, n), & \text{otherwise} \end{cases}$$

## 6.21 Miller Rabin [969881]

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633 4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383 6 : primes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool Miller_Rabin(ll a, ll n) {
    if ((a = a % n) == 0) return 1;
    if (n % 2 == 0) return n == 2;
    ll tmp = (n - 1) / ((n - 1) & (1 - n));
    ll t = __lg(((n - 1) & (1 - n))), x = 1;
    for (; tmp; tmp >>= 1, a = mul(a, a, n))
        if (tmp & 1) x = mul(x, a, n);
    if (x == 1 || x == n - 1) return 1;
    while (--t)
        if ((x = mul(x, x, n)) == n - 1) return 1;
    return 0;
}
```

## 6.22 Berlekamp-Massey [cdb091]

```
template <typename T>
vector<T> BerlekampMassey(const vector<T> &output) {
    vector<T> d(SZ(output) + 1), me, he;
    for (int f = 0, i = 1; i <= SZ(output); ++i) {
        for (int j = 0; j < SZ(me); ++j)
            d[i] += output[i - j - 2] * me[j];
        if ((d[i] - output[i - 1]) == 0) continue;
        if (me.empty()) {
            me.resize(f = i);
            continue;
        }
        vector<T> o(i - f - 1);
        T k = -d[i] / d[f]; o.pb(-k);
        for (T x : he) o.pb(x * k);
        o.resize(max(SZ(o), SZ(me)));
        for (int j = 0; j < SZ(me); ++j) o[j] += me[j];
        if (i - f + SZ(he) >= SZ(me)) he = me, f = i;
        me = o;
    }
    return me;
}
```

## 6.23 floor ceil [f84849]

```
int floor(int a, int b)
{ return a / b - (a % b && (a < 0) ^ (b < 0)); }
int ceil(int a, int b)
{ return a / b + (a % b && (a < 0) ^ (b > 0)); }
```

## 6.24 fac no p [86ad89]

```
// O(p^k + log^2 n), pk = p^k
ll prod[MAXP];
ll fac_no_p(ll n, ll p, ll pk) {
    prod[0] = 1;
    for (int i = 1; i <= pk; ++i)
        if (i % p) prod[i] = prod[i - 1] * i % pk;
        else prod[i] = prod[i - 1];
    ll rt = 1;
    for (; n; n /= p) {
        rt = rt * mpow(prod[pk], n / pk, pk) % pk;
        rt = rt * prod[n % pk] % pk;
    }
    return rt;
} // (n! without factor p) % p^k
```

## 6.25 DiscreteLog [21f791]

```
int DiscreteLog(int s, int x, int y, int m) {
    constexpr int kStep = 32000;
    unordered_map<int, int> p;
    int b = 1;
    for (int i = 0; i < kStep; ++i) {
        p[y] = i;
        y = 1LL * y * x % m;
        b = 1LL * b * x % m;
    }
    for (int i = 0; i < m + 10; i += kStep) {
        s = 1LL * s * b % m;
        if (p.find(s) != p.end()) return i + kStep - p[s];
    }
    return -1;
}

int DiscreteLog(int x, int y, int m) {
    if (m == 1) return 0;
    int s = 1;
    for (int i = 0; i < 100; ++i) {
        if (s == y) return i;
        s = 1LL * s * x % m;
    }
    if (s == y) return 100;
    int p = 100 + DiscreteLog(s, x, y, m);
    if (fpow(x, p, m) != y) return -1;
    return p;
}
```

## 6.26 SimplexConstruction

Primal	Dual
Maximize $c^T x$ s.t. $Ax \leq b, x \geq 0$	Minimize $b^T y$ s.t. $A^T y \geq c, y \geq 0$
Maximize $c^T x$ s.t. $Ax \leq b$	Minimize $b^T y$ s.t. $A^T y = c, y \geq 0$
Maximize $c^T x$ s.t. $Ax = b, x \geq 0$	Minimize $b^T y$ s.t. $A^T y \geq c$

$\bar{x}$  and  $\bar{y}$  are optimal if and only if for all  $i \in [1, n]$ , either  $\bar{x}_i = 0$  or  $\sum_{j=1}^m A_{ji} \bar{y}_j = c_i$  holds and for all  $i \in [1, m]$  either  $\bar{y}_i = 0$  or  $\sum_{j=1}^n A_{ij} \bar{x}_j = b_j$  holds.

- In case of minimization, let  $c'_i = -c_i$
- $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j \rightarrow \sum_{1 \leq i \leq n} -A_{ji} x_i \leq -b_j$
- $\sum_{1 \leq i \leq n} A_{ji} x_i = b_j$ 
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \leq b_j$
  - $\sum_{1 \leq i \leq n} A_{ji} x_i \geq b_j$
- If  $x_i$  has no lower bound, replace  $x_i$  with  $x_i - x'_i$

## 6.27 Simplex Algorithm [ad99b3]

```
const int MAXN = 11000, MAXM = 405;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN];
double d[MAXN][MAXM], x[MAXN];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax <= b, x >= 0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(int n, int m) {
    ++m;
    fill_n(d[n], m + 1, 0);
    fill_n(d[n + 1], m + 1, 0);
    iota(ix, ix + n + m, 0);
    int r = n, s = m - 1;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    copy_n(c, m - 1, d[n]);
    d[n + 1][m - 1] = -1;
    for (double dd; ) {
        if (r < n) {
            swap(ix[s], ix[r + m]);
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i) if (i != r) {
                for (int j = 0; j <= m; ++j) if (j != s)
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
```

```

    if (d[n + 1][j] > eps ||
        (d[n + 1][j] > -eps && d[n][j] > eps))
        s = j;
}
if (s < 0) break;
for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
    if (r < 0 ||
        (dd = d[r][m]
            / d[r][s] - d[i][m] / d[i][s]) < -eps ||
        (dd < eps && ix[r + m] > ix[i + m]))
        r = i;
}
if (r < 0) return -1; // not bounded
}
if (d[n + 1][m] < -eps) return -1; // not executable
double ans = 0;
fill_n(x, m, 0);
for (int i = m; i <
    n + m; ++i) { // the missing enumerated x[i] = 0
    if (ix[i] < m - 1) {
        ans += d[i - m][m] * c[ix[i]];
        x[ix[i]] = d[i - m][m];
    }
}
return ans;
}
}

```

## 6.28 SchreierSims [b17b78]

```

namespace schreier {
int n;
vector<vector<vector<int>>> bkets, binv;
vector<vector<int>>> lk;
vector<int> operator
*(const vector<int> &a, const vector<int> &b) {
    vector<int> res(SZ(a));
    for (int i = 0; i < SZ(a); ++i) res[i] = b[a[i]];
    return res;
}
vector<int> inv(const vector<int> &a) {
    vector<int> res(SZ(a));
    for (int i = 0; i < SZ(a); ++i) res[a[i]] = i;
    return res;
}
int filter(const vector<int> &g, bool add = true) {
    n = SZ(bkets);
    vector<int> p = g;
    for (int i = 0; i < n; ++i) {
        assert(p[i] >= 0 && p[i] < SZ(lk[i]));
        if (lk[i][p[i]] == -1) {
            if (add) {
                bkets[i].pb(p);
                binv[i].pb(inv(p));
                lk[i][p[i]] = SZ(bkets[i]) - 1;
            }
            return i;
        }
    }
    p = p * binv[i][lk[i][p[i]]];
}
return -1;
}
bool inside(const
    vector<int> &g) { return filter(g, false) == -1; }
void solve(const vector<vector<int>>> &gen, int _n) {
    n = _n;
    bkets.clear(), bkets.resize(n);
    binv.clear(), binv.resize(n);
    lk.clear(), lk.resize(n);
    vector<int> iden(n);
    iota(iden.begin(), iden.end(), 0);
    for (int i = 0; i < n; ++i) {
        lk[i].resize(n, -1);
        bkets[i].pb(iden);
        binv[i].pb(iden);
        lk[i][i] = 0;
    }
    for (int i = 0; i < SZ(gen); ++i) filter(gen[i]);
    queue<pair<pii, pii>> upd;
    for (int i = 0; i < n; ++i)
        for (int j = i; j < n; ++j)
            for (int k = 0; k < SZ(bkets[i]); ++k)
                for (int l = 0; l < SZ(bkets[j]); ++l)
                    upd.emplace(pii(i, k), pii(j, l));
    while (!upd.empty()) {
        auto a = upd.front().X;
        auto b = upd.front().Y;
        upd.pop();

```

```

int res = filter(bkets[a.X][a.Y] * bkets[b.X][b.Y]);
if (res == -1) continue;
pii pr = pii(res, SZ(bkets[res]) - 1);
for (int i = 0; i < n; ++i)
    for (int j = 0; j < SZ(bkets[i]); ++j) {
        if (i <= res) upd.emplace(pii(i, j), pr);
        if (res <= i) upd.emplace(pr, pii(i, j));
    }
}
ll size() {
    ll res = 1;
    for (int i = 0; i < n; ++i) res = res * SZ(bkets[i]);
    return res;
}
}

```

## 7 Polynomial

### 7.1 Polynomial Operation [dcba1b]

```

#define
    fi(s, n) for (int i = (int)(s); i < (int)(n); ++i)
template<int MAXN, ll P, ll RT> // MAXN = 2^k
struct Poly : vector<ll> { // coefficients in [0, P)
    using vector<ll>::vector;
    static NTT<MAXN, P, RT> ntt;
    int n() const { return (int)size(); } // n() >= 1
    Poly(const Poly &p, int m) : vector<ll>(m) {
        copy_n(p.data(), min(p.n(), m), data());
    }
    Poly& irev() {
        return reverse(data(), data() + n(), *this);
    }
    Poly& isz(int m) { return resize(m), *this; }
    Poly& iadd(const Poly &rhs) { // n() == rhs.n()
        fi(0, n()) if
            (((*this)[i] += rhs[i]) >= P) (*this)[i] -= P;
        return *this;
    }
    Poly& imul(ll k) {
        fi(0, n()) (*this)[i] = (*this)[i] * k % P;
        return *this;
    }
    Poly Mul(const Poly &rhs) const {
        int m = 1;
        while (m < n() + rhs.n() - 1) m <= 1;
        Poly X(*this, m), Y(rhs, m);
        ntt(X.data(), m), ntt(Y.data(), m);
        fi(0, m) X[i] = X[i] * Y[i] % P;
        ntt(X.data(), m, true);
        return X.isz(n() + rhs.n() - 1);
    }
    Poly Inv() const { // (*this)[0] != 0, 1e5/95ms
        if (n() == 1) return {ntt.minv((*this)[0])};
        int m = 1;
        while (m < n() * 2) m <= 1;
        Poly Xi = Poly(*this, (n() + 1) / 2).Inv().isz(m);
        Poly Y(*this, m);
        ntt(Xi.data(), m), ntt(Y.data(), m);
        fi(0, m) {
            Xi[i] *= (2 - Xi[i] * Y[i]) % P;
            if ((Xi[i] % = P) < 0) Xi[i] += P;
        }
        ntt(Xi.data(), m, true);
        return Xi.isz(n());
    }
    Poly Sqrt()
        const { // Jacobi((*this)[0], P) = 1, 1e5/235ms
        if (n()
            == 1) return {QuadraticResidue((*this)[0], P)};
        Poly
            X = Poly(*this, (n() + 1) / 2).Sqrt().isz(n());
        return
            X.iadd(Mul(X.Inv()).isz(n())).imul(P / 2 + 1);
    }
    pair<Poly, Poly> DivMod
        (const Poly &rhs) const { // (rhs.)back() != 0
        if (n() < rhs.n()) return {0, *this};
        const int m = n() - rhs.n() + 1;
        Poly X(rhs); X.irev().isz(m);
        Poly Y(*this); Y.irev().isz(m);
        Poly Q = Y.Mul(X.Inv()).isz(m).irev();
        X = rhs.Mul(Q), Y = *this;
        fi(0, n()) if ((Y[i] -= X[i]) < 0) Y[i] += P;
        return {Q, Y.isz(max(1, rhs.n() - 1))};
    }
    Poly Dx() const {
        Poly ret(n() - 1);

```



```

    fi(0,
        ret.n()) ret[i] = (i + 1) * (*this)[i + 1] % P;
    return ret.isz(max(1, ret.n()));
}
Poly Sx() const {
    Poly ret(n() + 1);
    fi(0, n())
        ret[i + 1] = ntt.minv(i + 1) * (*this)[i] % P;
    return ret;
}
Poly _tmul(int nn, const Poly &rhs) const {
    Poly Y = Mul(rhs).isz(n() + nn - 1);
    return Poly(Y.data() + n() - 1, Y.data() + Y.n());
}
vector<ll> _eval(const
    vector<ll> &x, const vector<Poly> &up) const {
    const int m = (int)x.size();
    if (!m) return {};
    vector<Poly> down(m * 2);
    // down[1] = DivMod(up[1]).second;
    // fi(2, m *
    // 2) down[i] = down[i / 2].DivMod(up[i]).second;
    down[1] = Poly(up[1])
        .irev().isz(n()).Inv().irev()._tmul(m, *this);
    fi(2, m * 2) down[i]
        = up[i ^ 1]._tmul(up[i].n() - 1, down[i / 2]);
    vector<ll> y(m);
    fi(0, m) y[i] = down[m + i][0];
    return y;
}
static vector<Poly> _tree1(const vector<ll> &x) {
    const int m = (int)x.size();
    vector<Poly> up(m * 2);
    fi(0, m) up[m + i] = {(x[i] ? P - x[i] : 0), 1};
    for (int i = m - 1; i
        > 0; --i) up[i] = up[i * 2].Mul(up[i * 2 + 1]);
    return up;
}
vector
    <ll> Eval(const vector<ll> &x) const { // 1e5, 1s
    auto up = _tree1(x); return _eval(x, up);
}
static Poly Interpolate(const vector
    <ll> &x, const vector<ll> &y) { // 1e5, 1.4s
    const int m = (int)x.size();
    vector<Poly> up = _tree1(x), down(m * 2);
    vector<ll> z = up[1].Dx()._eval(x, up);
    fi(0, m) z[i] = y[i] * ntt.minv(z[i]) % P;
    fi(0, m) down[m + i] = {z[i]};
    for (int i = m -
        1; i > 0; --i) down[i] = down[i * 2].Mul(up[i
        * 2 + 1]).iadd(down[i * 2 + 1].Mul(up[i * 2]));
    return down[1];
}
Poly Ln() const { // (*this)[0] == 1, 1e5/170ms
    return Dx().Mul(Inv()).Sx().isz(n());
}
Poly Exp() const { // (*this)[0] == 0, 1e5/360ms
    if (n() == 1) return {1};
    Poly X = Poly(*this, (n() + 1) / 2).Exp().isz(n());
    Poly Y = X.Ln(); Y[0] = P - 1;
    fi(0, n())
        if ((Y[i] = (*this)[i] - Y[i]) < 0) Y[i] += P;
    return X.Mul(Y).isz(n());
}
// M := P(P - 1). If k >= M, k := k % M + M.
Poly Pow(ll k) const {
    int nz = 0;
    while (nz < n() && !(*this)[nz]) ++nz;
    if (nz * min(k, (ll)n()) >= n()) return Poly(n());
    if (!k) return Poly(Poly{1}, n());
    Poly X(data() + nz, data() + nz + n() - nz * k);
    const ll c = ntt.mpow(X[0], k % (P - 1));
    return X.Ln().imul
        (k % P).Exp().imul(c).irev().isz(n()).irev();
}
static ll
    LinearRecursion(const vector<ll> &a, const vector
    <ll> &coef, ll n) { // a_n = \sum c_{-j} a_{(n-j)}
    const int k = (int)a.size();
    assert((int)coef.size() == k + 1);
    Poly C(k + 1), W(Poly{1}, k), M = {0, 1};
    fi(1, k + 1) C[k - i] = coef[i] ? P - coef[i] : 0;
    C[k] = 1;
    while (n) {
        if (n % 2) W = W.Mul(M).DivMod(C).second;

```

```

        n /= 2, M = M.Mul(M).DivMod(C).second;
    }
    ll ret = 0;
    fi(0, k) ret = (ret + W[i] * a[i]) % P;
    return ret;
}
};
#undef fi
using Poly_t = Poly<131072 * 2, 998244353, 3>;
template<> decltype(Poly_t::ntt) Poly_t::ntt = {};

```

## 7.2 Fast Walsh Transform [820c20]

```

/* x: a[j], y: a[j + (L >> 1)]
or: (y += x * op), and: (x += y * op)
xor: (x, y = (x + y) * op, (x - y) * op)
invop: or, and, xor = -1, -1, 1/2 */
void fwt(int *a, int n, int op) { //or
    for (int L = 2; L <= n; L <= 1)
        for (int i = 0; i < n; i += L)
            for (int j = i; j < i + (L >> 1); ++j)
                a[j + (L >> 1)] += a[j] * op;
}
const int N = 21;
int f[
    N][1 << N], g[N][1 << N], h[N][1 << N], ct[1 << N];
void
    subset_convolution(int *a, int *b, int *c, int L) {
    // c_k = \sum_{i+j=k, i&j=0} a_i * b_j
    int n = 1 << L;
    for (int i = 1; i < n; ++i)
        ct[i] = ct[i & (i - 1)] + 1;
    for (int i = 0; i < n; ++i)
        f[ct[i]][i] = a[i], g[ct[i]][i] = b[i];
    for (int i = 0; i <= L; ++i)
        fwt(f[i], n, 1), fwt(g[i], n, 1);
    for (int i = 0; i <= L; ++i)
        for (int j = 0; j <= i; ++j)
            for (int x = 0; x < n; ++x)
                h[i][x] += f[j][x] * g[i - j][x];
    for (int i = 0; i <= L; ++i)
        fwt(h[i], n, -1);
    for (int i = 0; i < n; ++i)
        c[i] = h[ct[i]][i];
}

```

## 7.3 Number Theory Transform [9a0ea6]

```

//(2^16)+1, 65537, 3
//7*17*(2^23)+1, 998244353, 3
//1255*(2^20)+1, 1315962881, 3
//51*(2^25)+1, 1711276033, 29
template<int MAXN, ll P, ll RT> //MAXN must be 2^k
struct NTT {
    ll w[MAXN];
    ll mpow(ll a, ll n);
    ll minv(ll a) { return mpow(a, P - 2); }
    NTT() {
        ll dw = mpow(RT, (P - 1) / MAXN);
        w[0] = 1;
        for (int
            i = 1; i < MAXN; ++i) w[i] = w[i - 1] * dw % P;
    }
    void bitrev(ll *a, int n) {
        int i = 0;
        for (int j = 1; j < n - 1; ++j) {
            for (int k = n >> 1; (i ^ k) < k; k >>= 1);
            if (j < i) swap(a[i], a[j]);
        }
    }
    void operator()(
        ll *a, int n, bool inv = false) { //0 <= a[i] < P
        bitrev(a, n);
        for (int L = 2; L <= n; L <= 1) {
            int dx = MAXN / L, dl = L >> 1;
            for (int i = 0; i < n; i += L) {
                for (int
                    j = i, x = 0; j < i + dl; ++j, x += dx) {
                    ll tmp = a[j + dl] * w[x] % P;
                    if ((a[j
                        + dl] = a[j] - tmp) < 0) a[j + dl] += P;
                    if ((a[j] += tmp) >= P) a[j] -= P;
                }
            }
        }
        if (inv) {
            reverse(a + 1, a + n);

```

```

    ll invn = minv(n);
    for (int i = 0; i < n; ++i) a[i] = a[i] * invn % P;
}
};

```

## 7.4 Value Poly [6438ba]

```

struct Poly {
    mint base; // f(x) = poly[x - base]
    vector<mint> poly;
    Poly(mint b = 0, mint x = 0): base(b), poly(1, x) {}
    mint get_val(const mint &x) {
        if (x >= base && x < base + SZ(poly))
            return poly[x - base];
        mint rt = 0;
        vector<mint> lmul(SZ(poly), 1), rmul(SZ(poly), 1);
        for (int i = 1; i < SZ(poly); ++i)
            lmul[i] = lmul[i - 1] * (x - (base + i - 1));
        for (int i = SZ(poly) - 2; i >= 0; --i)
            rmul[i] = rmul[i + 1] * (x - (base + i + 1));
        for (int i = 0; i < SZ(poly); ++i)
            rt += poly[i] * ifac[i] * inegfac
                [SZ(poly) - 1 - i] * lmul[i] * rmul[i];
        return rt;
    }
    void raise() { // g(x) = sigma{base:x} f(x)
        if (SZ(poly) == 1 && poly[0] == 0)
            return;
        mint nw = get_val(base + SZ(poly));
        poly.pb(nw);
        for (int i = 1; i < SZ(poly); ++i)
            poly[i] += poly[i - 1];
    }
};

```

## 7.5 NTT.2 [6997db]

```

#include <bits/stdc++.h>
using namespace std;
using ll = long long;
constexpr int MAXN = 1 << 20;
template<int MOD, int RT>
struct Zp {
    #define OP(op) static int op(int x, int y)
    OP(add) { return (x += y) >= MOD ? x - MOD : x; }
    OP(sub) { return (x -= y) < 0 ? x + MOD : x; }
    OP(mul) { return int(ll(x) * y % MOD); }
    static int mpow(int a, int n) {
        int r = 1;
        while (n) {
            if (n % 2) r = mul(r, a);
            n /= 2, a = mul(a, a);
        }
        return r;
    }
    static int minv(int a) { return mpow(a, MOD - 2); }
    struct NTT; struct Poly;
    static NTT ntt;
};
template<int MOD, int RT>
struct Zp<MOD, RT>::NTT {
    int w[MAXN];
    NTT() {
        int s =
            MAXN / 2, dw = mpow(RT, (MOD - 1) / MAXN);
        for (; s; s >>= 1, dw = mul(dw, dw)) {
            w[s] = 1;
            for (int j = 1; j < s; ++j)
                w[s + j] = mul(w[s + j - 1], dw);
        }
    }
    void apply
        (int *a, int n, bool inv = 0) { // 0 <= a_i < P
        for (int i = 0, j = 1; j < n - 1; ++j) {
            for (
                int k = n >> 1; (i ^ k) < k; k >>= 1;
                if (j < i) swap(a[i], a[j]));
        }
        for (int s = 1; s < n; s <= 1) {
            for (int i = 0; i < n; i += s * 2) {
                for (int j = 0; j < s; ++j) {
                    int tmp
                        = mul(a[i + s + j], w[s + j]);
                    a[i + s + j] = sub(a[i + j], tmp);
                    a[i + j] = add(a[i + j], tmp);
                }
            }
        }
    }
};

```

```

    }
}
}
if (!inv) return;
int iv = minv(n); reverse(a + 1, a + n);
for (int i = 0; i < n; ++i) a[i] = mul(a[i], iv);
}
};
template<int MOD, int RT>
typename Zp<MOD, RT>::NTT Zp<MOD, RT>::ntt;
using ctx1 = Zp<998244353, 3>;
int a[MAXN];
int main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);
    for (int i = 0; i < 10; ++i) {
        a[i] = rand() % 100;
        cout << a[i] << " \n"[i == 9];
    }
    ctx1::ntt.apply(a, MAXN);
    for (int i = 0; i < 10; ++i) {
        cout << a[i] << " \n"[i == 9];
    }
    ctx1::ntt.apply(a, MAXN, 1);
    for (int i = 0; i < 10; ++i) {
        cout << a[i] << " \n"[i == 9];
    }
    return 0;
}

```

## 7.6 Newton

Given  $F(x)$  where

$$F(x) = \sum_{i=0}^{\infty} \alpha_i (x - \beta)^i$$

for  $\beta$  being some constant. Polynomial  $P$  such that  $F(P) = 0$  can be found iteratively. Denote by  $Q_k$  the polynomial such that  $F(Q_k) = 0 \pmod{x^{2^k}}$ , then

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)} \pmod{x^{2^{k+1}}}$$

## 7.7 Fast Fourier Transform [e5f7dc]

```

template<int MAXN>
struct FFT {
    using val_t = complex<double>;
    const double PI = acos(-1);
    val_t w[MAXN];
    FFT() {
        for (int i = 0; i < MAXN; ++i) {
            double arg = 2 * PI * i / MAXN;
            w[i] = val_t(cos(arg), sin(arg));
        }
    }
    void bitrev(val_t *a, int n); // see NTT
    void trans
        (val_t *a, int n, bool inv = false); // see NTT;
        // remember to replace LL with val_t
};

```

# 8 Geometry

## 8.1 PolyUnion [bf776d]

```

double rat(pll a, pll b) {
    return sign
        (b.X ? (double)a.X / b.X : (double)a.Y / b.Y);
} // all poly. should be ccw
double polyUnion(vector<vector<pll>> &poly) {
    double res = 0;
    for (auto &p : poly)
        for (int a = 0; a < SZ(p); ++a) {
            pll A = p[a], B = p[(a + 1) % SZ(p)];
            vector
                <pair<double, int>> segs = {{0, 0}, {1, 0}};
            for (auto &q : poly) {
                if (&p == &q) continue;
                for (int b = 0; b < SZ(q); ++b) {
                    pll C = q[b], D = q[(b + 1) % SZ(q)];
                    int sc = ori(A, B, C), sd = ori(A, B, D);
                    if (sc != sd && min(sc, sd) < 0) {
                        double sa = cross(D
                            - C, A - C), sb = cross(D - C, B - C);
                    }
                }
            }
        }
    }
}

```



```

        segs.emplace_back
            (sa / (sa - sb), sign(sc - sd));
    }
    if (!sc && !sd &&
        &q < &p && sign(dot(B - A, D - C)) > 0) {
        segs.emplace_back(rat(C - A, B - A), 1);
        segs.emplace_back(rat(D - A, B - A), -1);
    }
}
sort(ALL(segs));
for (auto &s : segs) s.X = clamp(s.X, 0.0, 1.0);
double sum = 0;
int cnt = segs[0].second;
for (int j = 1; j < SZ(segs); ++j) {
    if (!cnt) sum += segs[j].X - segs[j - 1].X;
    cnt += segs[j].Y;
}
res += cross(A, B) * sum;
}
return res / 2;
}
}

```

## 8.2 external\_bisector [f088cc]

```

pdd external_bisector(pdd p1, pdd p2, pdd p3) { //213
    pdd l1 = p2 - p1, l2 = p3 - p1;
    L2 = L2 * abs(l1) / abs(l2);
    return l1 + l2;
}

```

## 8.3 Convexhull3D [fc330d]

```

struct convex_hull_3D {
    struct Face {
        int a, b, c;
        Face(int ta, int tb, int tc): a(ta), b(tb), c(tc) {}
    }; // return the faces with pt indexes
    vector<Face> res;
    vector<Point> P;
    convex_hull_3D(const vector<Point> &_P): res(), P(_P) {
        // all points coplanar case will WA, O(n^2)
        int n = SZ(P);
        if (n <= 2) return; // be careful about edge case
        // ensure first 4 points are not coplanar
        swap(P[1], *find_if(ALL(P), [&](auto p) { return
            sign(abs2(P[0] - p)) != 0; }));
        swap(P[2], *find_if(ALL(P), [&](auto p) { return
            sign(abs2(cross3(p, P[0], P[1]))) != 0; }));
        swap(P[3], *find_if(ALL(P), [&](auto p) { return
            sign(volume(P[0], P[1], P[2], p)) != 0; }));
        vector<vector<int>> flag(n, vector<int>(n));
        res.emplace_back(0, 1, 2); res.emplace_back(2, 1, 0);
        for (int i = 3; i < n; ++i) {
            vector<Face> next;
            for (auto f : res) {
                int d = sign(volume(P[f.a], P[f.b], P[f.c], P[i]));
                if (d <= 0) next.pb(f);
                int ff = (d > 0) - (d < 0);
                flag[f.a][f.b] = flag[f.b][f.c] = flag[f.c][f.a] = ff;
            }
            for (auto f : res) {
                auto F = [&](int x, int y) {
                    if (flag[x][y] > 0 && flag[y][x] <= 0)
                        next.emplace_back(x, y, i);
                };
                F(f.a, f.b); F(f.b, f.c); F(f.c, f.a);
            }
            res = next;
        }
    }
    bool same(Face s, Face t) {
        if (sign(volume(P[s.a], P[s.b], P[s.c], P[t.a])) != 0) return 0;
        if (sign(volume(P[s.a], P[s.b], P[s.c], P[t.b])) != 0) return 0;
        if (sign(volume(P[s.a], P[s.b], P[s.c], P[t.c])) != 0) return 0;
        return 1;
    }
    int polygon_face_num() {
        int ans = 0;
        for (int i = 0; i < SZ(res); ++i)
            ans += none_of(res.begin(), res.begin()
                + i, [&](Face g) { return same(res[i], g); });
    }
}

```

```

return ans;
}
double get_volume() {
    double ans = 0;
    for (auto f : res)
        ans += volume(Point(0, 0, 0), P[f.a], P[f.b], P[f.c]);
    return fabs(ans / 6);
}
double get_dis(Point p, Face f) {
    Point p1 = P[f.a], p2 = P[f.b], p3 = P[f.c];
    double a = (p2.y - p1.y) * (p3.z - p1.z) - (p2.z - p1.z) * (p3.y - p1.y);
    double b = (p2.z - p1.z) * (p3.x - p1.x) - (p2.x - p1.x) * (p3.z - p1.z);
    double c = (p2.x - p1.x) * (p3.y - p1.y) - (p2.y - p1.y) * (p3.x - p1.x);
    double d = 0 - (a * p1.x + b * p1.y + c * p1.z);
    return fabs(a * p.x + b * p.y + c * p.z + d) / sqrt(a * a + b * b + c * c);
}
};
// n^2 delaunay: facets with negative z normal of
// convexhull of (x, y, x^2 + y^2), use a pseudo-point
// (0, 0, inf) to avoid degenerate case

```

## 8.4 Triangulation Voronoi [433667]

```

// all coord. is even
// you may want to call halfPlaneInter after then
vector<vector<Line>> vec;
void build_voronoi_line(int n, pll *arr) {
    tool.init(n, arr); // Delaunay
    vec.clear(), vec.resize(n);
    for (int i = 0; i < n; ++i)
        for (auto e : tool.head[i]) {
            int u = tool.oidx[i], v = tool.oidx[e.id];
            pll m = (arr[v]
                + arr[u]) / 2LL, d = perp(arr[v] - arr[u]);
            vec[u].pb(Line(m, m + d));
        }
}

```

## 8.5 Default code int [111a95]

```

typedef pair<double, double> pdd;
typedef pair<pll, pll> Line;
pll operator+(pll a, pll b) {
    return pll(a.X + b.X, a.Y + b.Y);
}
pll operator-(pll a, pll b) {
    return pll(a.X - b.X, a.Y - b.Y);
}
pll operator*(pll a, pll b) {
    return pll(a.X * b.X, a.Y * b.Y);
}
pll operator/(pll a, pll b) {
    return pll(a.X / b.X, a.Y / b.Y);
}
pdd operator/(pll a, double b) {
    return pdd(a.X / b, a.Y / b);
}
ll dot(pll a, pll b) {
    return a.X * b.X + a.Y * b.Y;
}
ll cross(pll a, pll b) {
    return a.X * b.Y - a.Y * b.X;
}
ll abs2(pll a) {
    return dot(a, a);
}
int sign(ll a) {
    return a == 0 ? 0 : a > 0 ? 1 : -1;
}
int ori(pll a, pll b, pll c) {
    return sign(cross(b - a, c - a));
}
bool collinearity(pll p1, pll p2, pll p3) {
    return sign(cross(p1 - p3, p2 - p3)) == 0;
}
bool btw(pll p1, pll p2, pll p3) {
    if (!collinearity(p1, p2, p3)) return 0;
    return sign(dot(p1 - p3, p2 - p3)) <= 0;
}
bool seg_intersect(pll p1, pll p2, pll p3, pll p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if (a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(pll p1, pll p2, pll p3, pll p4) {
    ll a123 = cross(p2 - p1, p3 - p1);
    ll a124 = cross(p2 - p1, p4 - p1);
    return (p4 * a123
        - p3 * a124) / double(a123 - a124); // C^3 / C^2
}

```

```

}
pll perp(pll p1)
{ return pll(-p1.Y, p1.X); }

```

## 8.6 Polar Angle Sort [2804b5]

```

int cmp(pll a, pll b, bool same = true) {
#define is_neg(k) (
    sign(k.Y) < 0 || (sign(k.Y) == 0 && sign(k.X) < 0))
    int A = is_neg(a), B = is_neg(b);
    if (A != B)
        return A < B;
    if (sign(cross(a, b)) == 0)
        return same ? abs2(a) < abs2(b) : -1;
    return sign(cross(a, b)) > 0;
}

```

## 8.7 Default code [3efc61]

```

typedef pair<double, double> pdd;
typedef pair<pdd, pdd> Line;
struct Cir{ pdd O; double R; };
const double eps = 1e-8;
pdd operator+(pdd a, pdd b)
{ return pdd(a.X + b.X, a.Y + b.Y); }
pdd operator-(pdd a, pdd b)
{ return pdd(a.X - b.X, a.Y - b.Y); }
pdd operator*(pdd a, double b)
{ return pdd(a.X * b, a.Y * b); }
pdd operator/(pdd a, double b)
{ return pdd(a.X / b, a.Y / b); }
double dot(pdd a, pdd b)
{ return a.X * b.X + a.Y * b.Y; }
double cross(pdd a, pdd b)
{ return a.X * b.Y - a.Y * b.X; }
double abs2(pdd a)
{ return dot(a, a); }
double abs(pdd a)
{ return sqrt(dot(a, a)); }
int sign(double a)
{ return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(pdd a, pdd b, pdd c)
{ return sign(cross(b - a, c - a)); }
bool collinearity(pdd p1, pdd p2, pdd p3)
{ return sign(cross(p1 - p3, p2 - p3)) == 0; }
bool btw(pdd p1, pdd p2, pdd p3) {
    if (!collinearity(p1, p2, p3)) return 0;
    return sign(dot(p1 - p3, p2 - p3)) <= 0;
}
bool seg_intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
    int a123 = ori(p1, p2, p3);
    int a124 = ori(p1, p2, p4);
    int a341 = ori(p3, p4, p1);
    int a342 = ori(p3, p4, p2);
    if (a123 == 0 && a124 == 0)
        return btw(p1, p2, p3) || btw(p1, p2, p4) ||
            btw(p3, p4, p1) || btw(p3, p4, p2);
    return a123 * a124 <= 0 && a341 * a342 <= 0;
}
pdd intersect(pdd p1, pdd p2, pdd p3, pdd p4) {
    double a123 = cross(p2 - p1, p3 - p1);
    double a124 = cross(p2 - p1, p4 - p1);
    return p4
        * a123 - p3 * a124 / (a123 - a124); // C^3 / C^2
}
pdd perp(pdd p1)
{ return pdd(-p1.Y, p1.X); }
pdd projection(pdd p1, pdd p2, pdd p3)
{ return p1 + (
    p2 - p1) * dot(p3 - p1, p2 - p1) / abs2(p2 - p1); }
pdd reflection(pdd p1, pdd p2, pdd p3)
{ return p3 + perp(p2 - p1
    ) * cross(p3 - p1, p2 - p1) / abs2(p2 - p1) * 2; }
pdd linearTransformation
    (pdd p0, pdd p1, pdd q0, pdd q1, pdd r) {
    pdd dp = p1 - p0
        , dq = q1 - q0, num(cross(dp, dq), dot(dp, dq));
    return q0 + pdd(
        cross(r - p0, num), dot(r - p0, num)) / abs2(dp);
} // from line p0--p1 to q0--q1, apply to r

```

## 8.8 PointInConvex Slow [dd78ba]

```

bool PointInConvex(const vector<pll> &C, pdd p) {
    if (SZ(C) == 0) return false;
    if (SZ(C) == 1) return abs(C[0] - p) < eps;
    if (SZ(C) == 2) return btw(C[0], C[1], p);
}

```

```

for (int i = 0; i < SZ(C); ++i) {
    const int j = i + 1 == SZ(C) ? 0 : i + 1;
    if (cross(C[j] - C[i], p - C[i]) < -eps)
        return false;
}
return true;
}

```

## 8.9 Intersection of polygon and circle [cbe8f5]

```

// Divides into multiple triangle, and sum up
const double PI=acos(-1);
double _area(pdd pa, pdd pb, double r){
    if(abs(pa)<abs(pb)) swap(pa, pb);
    if(abs(pb)<eps) return 0;
    double S, h, theta;
    double a=abs(pb),b=abs(pa),c=abs(pb-pa);
    double cosB = dot(pb,pb-pa) / a / c, B = acos(cosB);
    double cosC = dot(pa,pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if (h < r && B
            < PI/2) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    }
    else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else S = .5*sin(C)*a*b;
    return S;
}
double area_poly_circle(const
    vector<pdd> poly,const pdd &O,const double r){
    double S=0;
    for(int i=0;i<SZ(poly);++i)
        S+=_area(poly[i]-O,poly[(i+1)%SZ(poly)
            ]-O,r)*ori(O,poly[i],poly[(i+1)%SZ(poly)]);
    return fabs(S);
}

```

## 8.10 Tangent line of two circles [5ad86c]

```

vector<Line>
    > go( const Cir& c1 , const Cir& c2 , int sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = abs2(c1.O - c2.O);
    if (sign(d_sq) == 0) return ret;
    double d = sqrt(d_sq);
    pdd v = (c2.O - c1.O) / d;
    double c = (c1.R - sign1 * c2.R) / d;
    if (c * c > 1) return ret;
    double h = sqrt(max(0.0, 1.0 - c * c));
    for (int sign2 = 1; sign2 >= -1; sign2 -= 2) {
        pdd n = pdd(v.X * c - sign2 * h * v.Y,
            v.Y * c + sign2 * h * v.X);
        pdd p1 = c1.O + n * c1.R;
        pdd p2 = c2.O + n * (c2.R * sign1);
        if (sign(p1.X - p2.X) == 0 and
            sign(p1.Y - p2.Y) == 0)
            p2 = p1 + perp(c2.O - c1.O);
        ret.pb(Line(p1, p2));
    }
    return ret;
}

```

## 8.11 CircleCover [1d09aa]

```

const int N = 1021;
struct CircleCover {
    int C;
    Cir c[N];
    bool g[N][N], overlap[N][N];
    // Area[i] : area covered by at least i circles
    double Area[ N ];
    void init(int _C){ C = _C; }
    struct Teve {
        pdd p; double ang; int add;
        Teve() {}
        Teve(pdd _a
            , double _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a)const {
            return ang < a.ang; }
    }eve[N * 2];
    // strict: x = 0, otherwise x = -1
    bool disjunct(Cir &a, Cir &b, int x)
}

```

```

{return sign(abs(a.0 - b.0) - a.R - b.R) > x;}
bool contain(Cir &a, Cir &b, int x)
{return sign(a.R - b.R - abs(a.0 - b.0)) > x;}
bool contain(int i, int j) {
    /* c[j] is non-strictly in c[i]. */
    return (sign
        (c[i].R - c[j].R) > 0 || (sign(c[i].R - c[j].
            R) == 0 && i < j)) && contain(c[i], c[j], -1);
}
void solve(){
    fill_n(Area, C + 2, 0);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            overlap[i][j] = contain(i, j);
    for(int i = 0; i < C; ++i)
        for(int j = 0; j < C; ++j)
            g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                disjunct(c[i], c[j], -1));
    for(int i = 0; i < C; ++i){
        int E = 0, cnt = 1;
        for(int j = 0; j < C; ++j)
            if(j != i && overlap[j][i])
                ++cnt;
        for(int j = 0; j < C; ++j)
            if(i != j && g[i][j]) {
                pdd aa, bb;
                CCinter(c[i], c[j], aa, bb);
                double A =
                    atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
                double B =
                    atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
                eve[E++] = Teve
                    (bb, B, 1), eve[E++] = Teve(aa, A, -1);
                if(B > A) ++cnt;
            }
        if(E == 0) Area[cnt] += pi * c[i].R * c[i].R;
        else{
            sort(eve, eve + E);
            eve[E] = eve[0];
            for(int j = 0; j < E; ++j){
                cnt += eve[j].add;
                Area[cnt]
                    += cross(eve[j].p, eve[j + 1].p) * .5;
                double theta = eve[j + 1].ang - eve[j].ang;
                if (theta < 0) theta += 2. * pi;
                Area[cnt] += (theta
                    - sin(theta)) * c[i].R * c[i].R * .5;
            }
        }
    }
}
};

```

## 8.12 Heart [4698ba]

```

pdd circenter
    (pdd p0, pdd p1, pdd p2) { // radius = abs(center)
    p1 = p1 - p0, p2 = p2 - p0;
    double x1 = p1.X, y1 = p1.Y, x2 = p2.X, y2 = p2.Y;
    double m = 2. * (x1 * y2 - y1 * x2);
    center.X = (x1 * x1
        * y2 - x2 * x2 * y1 + y1 * y2 * (y1 - y2)) / m;
    center.Y = (x1 * x2
        * (x2 - x1) - y1 * y1 * x2 + x1 * y2 * y2) / m;
    return center + p0;
}
pdd incenter
    (pdd p1, pdd p2, pdd p3) { // radius = area / s * 2
    double a =
        abs(p2 - p3), b = abs(p1 - p3), c = abs(p1 - p2);
    double s = a + b + c;
    return (a * p1 + b * p2 + c * p3) / s;
}
pdd masscenter(pdd p1, pdd p2, pdd p3)
{ return (p1 + p2 + p3) / 3; }
pdd orthcenter(pdd p1, pdd p2, pdd p3)
{ return masscenter
    (p1, p2, p3) * 3 - circenter(p1, p2, p3) * 2; }

```

## 8.13 PointSegDist [See686]

```

double PointSegDist(pdd q0, pdd q1, pdd p) {
    if (sign(abs(q0 - q1)) == 0) return abs(q0 - p);
    if (sign(dot(q1 - q0,
        p - q0)) >= 0 && sign(dot(q0 - q1, p - q1)) >= 0)
        return fabs(cross(q1 - q0, p - q0) / abs(q0 - q1));
    return min(abs(p - q0), abs(p - q1));
}

```

## 8.14 Minkowski Sum [95f4a0]

```

vector<pll> Minkowski
    (vector<pll> A, vector<pll> B) { // |A|,|B|>=3
    hull(A), hull(B);
    vector<pll> C(1, A[0] + B[0]), s1, s2;
    for (int i = 0; i < SZ(A); ++i)
        s1.pb(A[(i + 1) % SZ(A)] - A[i]);
    for (int i = 0; i < SZ(B); ++i)
        s2.pb(B[(i + 1) % SZ(B)] - B[i]);
    for (int i = 0, j = 0; i < SZ(A) || j < SZ(B);)
        if (j >= SZ
            (B) || (i < SZ(A) && cross(s1[i], s2[j]) >= 0))
            C.pb(B[j % SZ(B)] + A[i++]);
        else
            C.pb(A[i % SZ(A)] + B[j++]);
    return hull(C), C;
}

```

## 8.15 TangentPointToHull [5668cc]

```

/* The point should be strictly out of hull
    return arbitrary point on the tangent line */
pii get_tangent(vector<pll> &C, pll p) {
    auto gao = [&](int s) {
        return cyc_tsearch(SZ(C), [&](int x, int y)
            { return ori(p, C[x], C[y]) == s; });
    };
    return pii(gao(1), gao(-1));
} // return (a, b), ori(p, C[a], C[b]) >= 0

```

## 8.16 Intersection of two circles [b062ba]

```

bool CCinter(Cir &a, Cir &b, pdd &p1, pdd &p2) {
    pdd o1 = a.O, o2 = b.O;
    double r1 =
        a.R, r2 = b.R, d2 = abs2(o1 - o2), d = sqrt(d2);
    if(d < max
        (r1, r2) - min(r1, r2) || d > r1 + r2) return 0;
    pdd u = (o1 + o2) * 0.5
        + (o1 - o2) * ((r2 * r2 - r1 * r1) / (2 * d2));
    double A = sqrt((r1 + r2 + d) *
        (r1 - r2 + d) * (r1 + r2 - d) * (-r1 + r2 + d));
    pdd v
        = pdd(o1.Y - o2.Y, -o1.X + o2.X) * A / (2 * d2);
    p1 = u + v, p2 = u - v;
    return 1;
}

```

## 8.17 PointInConvex [9136f4]

```

bool PointInConvex
    (const vector<pll> &C, pll p, bool strict = true) {
    int a = 1, b = SZ(C) - 1, r = !strict;
    if (SZ(C) == 0) return false;
    if (SZ(C) < 3) return r && btw(C[0], C.back(), p);
    if (ori(C[0], C[a], C[b]) > 0) swap(a, b);
    if (ori
        (C[0], C[a], p) >= r || ori(C[0], C[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (ori(C[0], C[c], p) > 0 ? b : a) = c;
    }
    return ori(C[a], C[b], p) < r;
}

```

## 8.18 Intersection of line and circle [894afd]

```

vector<pdd> circleLine(pdd c, double r, pdd a, pdd b) {
    pdd p
        = a + (b - a) * dot(c - a, b - a) / abs2(b - a);
    double s = cross
        (b - a, c - a), h2 = r * r - s * s / abs2(b - a);
    if (h2 < 0) return {};
    if (h2 == 0) return {p};
    pdd h = (b - a) / abs(b - a) * sqrt(h2);
    return {p - h, p + h};
}

```

## 8.19 Trapezoidalization [4e01c8]

```

template<class T>
struct SweepLine {
    struct cmp {
        cmp(const SweepLine &swp): swp(_swp) {}
        bool operator()(int a, int b) const {
            if (abs(swp.get_y(a) - swp.get_y(b)) <= swp.eps)

```

```

    return swp.slope_cmp(a, b);
    return swp.get_y(a) + swp.eps < swp.get_y(b);
}
const SweepLine &swp;
} _cmp;
T curTime, eps, curQ;
vector<Line> base;
multiset<int, cmp> sweep;
multiset<pair<T, int>> event;
vector<typename multiset<int, cmp>::iterator> its;
vector
<typename multiset<pair<T, int>>::iterator> eits;
bool slope_cmp(int a, int b) const {
    assert(a != -1);
    if (b == -1) return 0;
    return sign(cross(base
        [a].Y - base[a].X, base[b].Y - base[b].X)) < 0;
}
T get_y(int idx) const {
    if (idx == -1) return curQ;
    Line l = base[idx];
    if (l.X.X == l.Y.X) return l.Y.Y;
    return ((curTime - l.X.X) * l.Y.Y
        + (l.Y.X - curTime) * l.X.Y) / (l.Y.X - l.X.X);
}
void insert(int idx) {
    its[idx] = sweep.insert(idx);
    if (its[idx] != sweep.begin())
        update_event(*prev(its[idx]));
    update_event(idx);
    event.emplace(base[idx].Y.X, idx + 2 * SZ(base));
}
void erase(int idx) {
    assert(eits[idx] == event.end());
    auto p = sweep.erase(its[idx]);
    its[idx] = sweep.end();
    if (p != sweep.begin())
        update_event(*prev(p));
}
void update_event(int idx) {
    if (eits[idx] != event.end())
        event.erase(eits[idx]);
    eits[idx] = event.end();
    auto nxt = next(its[idx]);
    if (nxt ==
        sweep.end() || !slope_cmp(idx, *nxt)) return;
    auto t = intersect(base[idx].
        X, base[idx].Y, base[*nxt].X, base[*nxt].Y).X;
    if (t + eps < curTime || t
        >= min(base[idx].Y.X, base[*nxt].Y.X)) return;
    eits[idx] = event.emplace(t, idx + SZ(base));
}
void swp(int idx) {
    assert(eits[idx] != event.end());
    eits[idx] = event.end();
    int nxt = *next(its[idx]);
    swap((int&)*its[idx], (int&)*its[nxt]);
    swap(its[idx], its[nxt]);
    if (its[nxt] != sweep.begin())
        update_event(*prev(its[nxt]));
    update_event(idx);
}
// only expected to call the functions below
SweepLine(T t, T e, vector
    <Line> vec): _cmp(*this), curTime(t), eps(e)
    , curQ(), base(vec), sweep(_cmp), event(), its(SZ
    (vec), sweep.end()), eits(SZ(vec), event.end()) {
    for (int i = 0; i < SZ(base); ++i) {
        auto &[p, q] = base[i];
        if (p > q) swap(p, q);
        if (p.X <= curTime && curTime <= q.X)
            insert(i);
        else if (curTime < p.X)
            event.emplace(p.X, i);
    }
}
void setTime(T t, bool ers = false) {
    assert(t >= curTime);
    while (!event.empty() && event.begin()->X <= t) {
        auto [et, idx] = *event.begin();
        int s = idx / SZ(base);
        idx %= SZ(base);
        if (abs(et - t) <= eps && s == 2 && !ers) break;
        curTime = et;
        event.erase(event.begin());
        if (s == 2) erase(idx);
    }
}

```

```

    else if (s == 1) swp(idx);
    else insert(idx);
}
curTime = t;
}
T nextEvent() {
    if (event.empty()) return INF;
    return event.begin()->X;
}
int lower_bound(T y) {
    curQ = y;
    auto p = sweep.lower_bound(-1);
    if (p == sweep.end()) return -1;
    return *p;
}
};

```

## 8.20 point in circle [9ae6d9]

```

// return q'
s relation with circumcircle of tri(p[0],p[1],p[2])
bool in_cc(const array<pll, 3> &p, pll q) {
    __int128 det = 0;
    for (int i = 0; i < 3; ++i)
        det += __int128(abs2(p[i]) - abs2(q)) *
            cross(p[(i + 1) % 3] - q, p[(i + 2) % 3] - q);
    return det > 0; // in: >0, on: =0, out: <0
}

```

## 8.21 PolyCut [417264]

```

vector<pdd> cut(vector<pdd> poly, pdd s, pdd e) {
    vector<pdd> res;
    for (int i = 0; i < SZ(poly); ++i) {
        pdd cur
            = poly[i], prv = i ? poly[i - 1] : poly.back();
        bool side = ori(s, e, cur) < 0;
        if (side != (ori(s, e, prv) < 0))
            res.pb(intersect(s, e, cur, prv));
        if (side)
            res.pb(cur);
    }
    return res;
}

```

## 8.22 minDistOfTwoConvex [d62c1f]

```

double ConvexHullDist(vector<pdd> A, vector<pdd> B) {
    for (auto &p : B) p = {-p.X, -p.Y};
    auto C = Minkowski(A, B); // assert SZ(C) > 0
    if (PointInConvex(C, pdd(0, 0))) return 0;
    double
        ans = PointSegDist(C.back(), C[0], pdd(0, 0));
    for (int i = 0; i + 1 < SZ(C); ++i) {
        ans = min(ans
            , PointSegDist(C[i], C[i + 1], pdd(0, 0)));
    }
    return ans;
}

```

## 8.23 DelaunayTriangulation [1d8107]

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
Voronoi diagram: for each triangle in triangulation,
the bisector of all its edges will split the region.
nearest point will belong to the triangle containing it
*/
const
    ll inf = MAXC * MAXC * 100; // lower_bound unknown
struct Tri;
struct Edge {
    Tri* tri; int side;
    Edge(): tri(0), side(0){}
    Edge(Tri* _tri, int _side): tri(_tri), side(_side){}
};
struct Tri {
    pll p[3];
    Edge edge[3];
    Tri* chd[3];
    Tri() {}
}

```

```

Tri(const pll& p0, const pll& p1, const pll& p2) {
    p[0] = p0; p[1] = p1; p[2] = p2;
    chd[0] = chd[1] = chd[2] = 0;
}
bool has_chd() const { return chd[0] != 0; }
int num_chd() const {
    return !!chd[0] + !!chd[1] + !!chd[2];
}
bool contains(pll const& q) const {
    for (int i = 0; i < 3; ++i)
        if (ori(p[i], p[(i + 1) % 3], q) < 0)
            return 0;
    return 1;
}
} pool[N * 10], *tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {
        the_root
            = // Tri should at least contain all points
            new(tris++) Tri(pll(-inf, -inf),
                pll(inf + inf, -inf), pll(-inf, inf + inf));
    }
    Tri* find(pll p) { return find(the_root, p); }
    void add_point(const
        pll &p) { add_point(find(the_root, p), p); }
    Tri* the_root;
    static Tri* find(Tri* root, const pll &p) {
        while (1) {
            if (!root->has_chd())
                return root;
            for (int i = 0; i < 3 && root->chd[i]; ++i)
                if (root->chd[i]->contains(p)) {
                    root = root->chd[i];
                    break;
                }
        }
        assert(0); // "point not found"
    }
    void add_point(Tri* root, pll const& p) {
        Tri* t[3];
        /* split it into three triangles */
        for (int i = 0; i < 3; ++i)
            t[i] = new(tris
                ++) Tri(root->p[i], root->p[(i + 1) % 3], p);
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 0), Edge(t[(i + 1) % 3], 1));
        for (int i = 0; i < 3; ++i)
            edge(Edge(t[i], 2), root->edge[(i + 2) % 3]);
        for (int i = 0; i < 3; ++i)
            root->chd[i] = t[i];
        for (int i = 0; i < 3; ++i)
            flip(t[i], 2);
    }
    void flip(Tri* tri, int pi) {
        Tri* trj = tri->edge[pi].tri;
        int pj = tri->edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri->p
            [0], tri->p[1], tri->p[2], trj->p[pj])) return;
        /* flip edge between tri, trj */
        Tri* trk = new(tris++) Tri
            (tri->p[(pi + 1) % 3], trj->p[pj], tri->p[pi]);
        Tri* trl = new(tris++) Tri
            (trj->p[(pj + 1) % 3], tri->p[pi], trj->p[pj]);
        edge(Edge(trk, 0), Edge(trl, 0));
        edge(Edge(trk, 1), tri->edge[(pi + 2) % 3]);
        edge(Edge(trk, 2), trj->edge[(pj + 1) % 3]);
        edge(Edge(trl, 1), trj->edge[(pj + 2) % 3]);
        edge(Edge(trl, 2), tri->edge[(pi + 1) % 3]);
        tri->chd
            [0] = trk; tri->chd[1] = trl; tri->chd[2] = 0;
        trj->chd
            [0] = trk; trj->chd[1] = trl; trj->chd[2] = 0;
        flip(trk, 1); flip(trk, 2);
        flip(trl, 1); flip(trl, 2);
    }
};
vector<Tri*> triang; // vector of all triangle
set<Tri*> vst;
void go(Tri* now) { // store all tri into triang
    if (vst.find(now) != vst.end())
        return;

```

```

    vst.insert(now);
    if (!now->has_chd())
        return triang.pb(now);
    for (int i = 0; i < now->num_chd(); ++i)
        go(now->chd[i]);
}
void build(int n, pll* ps) { // build triangulation
    tris = pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps + n);
    Trig tri; // the triangulation structure
    for (int i = 0; i < n; ++i)
        tri.add_point(ps[i]);
    go(tri.the_root);
}

```

## 8.24 rotatingSweepLine [374fec]

```

void rotatingSweepLine(vector<pii> &ps) {
    int n = SZ(ps), m = 0;
    vector<int> id(n), pos(n);
    vector<pii> line(n * (n - 1));
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            if (i != j) line[m++] = pii(i, j);
    sort(ALL(line), [&](pii a, pii b) {
        return cmp(ps[a.Y] - ps[a.X], ps[b.Y] - ps[b.X]);
    }); // cmp(): polar angle compare
    iota(ALL(id), 0);
    sort(ALL(id), [&](int a, int b) {
        if (ps[a].Y != ps[b].Y) return ps[a].Y < ps[b].Y;
        return ps[a] < ps[b];
    }); // initial order, since (1, 0) is the smallest
    for (int i = 0; i < n; ++i) pos[id[i]] = i;
    for (int i = 0; i < m; ++i) {
        auto l = line[i];
        // do something
        tie(pos[l.X], pos[l.Y], id[pos[l.X]], id[pos[l.Y]
            ]) = make_tuple(pos[l.Y], pos[l.X], l.Y, l.X);
    }
}

```

## 8.25 Intersection of line and convex [e14a5c]

```

int TangentDir(vector<pll> &C, pll dir) {
    return cyc_tsearch(SZ(C), [&](int a, int b) {
        return cross(dir, C[a]) > cross(dir, C[b]);
    });
}
#define cmpl(i) sign(cross(C[i] - a, b - a))
pii lineHull(pll a, pll b, vector<pll> &C) {
    int A = TangentDir(C, a - b);
    int B = TangentDir(C, b - a);
    int n = SZ(C);
    if (cmpl(A) < 0 || cmpl(B) > 0)
        return pii(-1, -1); // no collision
    auto gao = [&](int l, int r) {
        for (int t = l; (l + 1) % n != r; ) {
            int m = ((l + r + (l < r ? 0 : n)) / 2) % n;
            (cmpl(m) == cmpl(t) ? l : r) = m;
        }
        return (l + !cmpl(r)) % n;
    };
    pii res = pii(gao(B, A), gao(A, B)); // (i, j)
    if (res.X == res.Y) // touching the corner i
        return pii(res.X, -1);
    if (!
        cmpl(res.X) && !cmpl(res.Y)) // along side i, i+1
        switch ((res.X - res.Y + n + 1) % n) {
            case 0: return pii(res.X, res.X);
            case 2: return pii(res.Y, res.Y);
        }
    /* crossing sides (i, i+1) and (j, j+1)
    crossing corner i is treated as side (i, i+1)
    returned
    in the same order as the line hits the convex */
    return res;
} // convex cut: (r, l)

```

## 8.26 3Dpoint [374a83]

```

struct Point {
    double x, y, z;
    Point(double _x = 0, double
        _y = 0, double _z = 0): x(_x), y(_y), z(_z){}
    Point(pdd p) { x = p.X, y = p.Y, z = abs2(p); }
};
Point operator-(Point p1, Point p2)

```



```

{ return
    Point(p1.x - p2.x, p1.y - p2.y, p1.z - p2.z); }
Point operator+(Point p1, Point p2)
{ return
    Point(p1.x + p2.x, p1.y + p2.y, p1.z + p2.z); }
Point operator*(Point p1, double v)
{ return Point(p1.x * v, p1.y * v, p1.z * v); }
Point operator/(Point p1, double v)
{ return Point(p1.x / v, p1.y / v, p1.z / v); }
Point cross(Point p1, Point p2)
{ return Point(p1.y * p2.z - p1.z * p2.y, p1.z * p2.x - p1.x * p2.z, p1.x * p2.y - p1.y * p2.x); }
double dot(Point p1, Point p2)
{ return p1.x * p2.x + p1.y * p2.y + p1.z * p2.z; }
double abs(Point a)
{ return sqrt(dot(a, a)); }
Point cross3(Point a, Point b, Point c)
{ return cross(b - a, c - a); }
double area(Point a, Point b, Point c)
{ return abs(cross3(a, b, c)); }
double volume(Point a, Point b, Point c, Point d)
{ return dot(cross3(a, b, c), d - a); }
//Azimuthal
    angle (longitude) to x-axis in interval [-pi, pi]
double phi(Point p) { return atan2(p.y, p.x); }
//Zenith
    angle (latitude) to the z-axis in interval [0, pi]
double theta(Point p)
{ return atan2(sqrt(p.x * p.x + p.y * p.y), p.z); }
Point masscenter(Point a, Point b, Point c, Point d)
{ return (a + b + c + d) / 4; }
pdd proj(Point a, Point b, Point c, Point u) {
// proj. u to the plane of a, b, and c
    Point e1 = b - a;
    Point e2 = c - a;
    e1 = e1 / abs(e1);
    e2 = e2 - e1 * dot(e2, e1);
    e2 = e2 / abs(e2);
    Point p = u - a;
    return pdd(dot(p, e1), dot(p, e2));
}
Point
    rotate_around(Point p, double angle, Point axis) {
    double s = sin(angle), c = cos(angle);
    Point u = axis / abs(axis);
    return u
        * dot(u, p) * (1 - c) + p * c + cross(u, p) * s;
}

```

## 8.27 HPIGeneralLine [e36115]

```

using i128 = __int128;
struct LN {
    ll a, b, c; // ax + by + c <= 0
    pll dir() const { return pll(a, b); }
    LN(ll ta, ll tb, ll tc) : a(ta), b(tb), c(tc) {}
    LN(pll S,
        pll T) : a((T-S).Y), b(-(T-S).X), c(cross(T,S)) {}
};
pdd intersect(LN A, LN B) {
    double c = cross(A.dir(), B.dir());
    i128 a = i128(A.c) * B.a - i128(B.c) * A.a;
    i128 b = i128(A.c) * B.b - i128(B.c) * A.b;
    return pdd(-b / c, a / c);
}
bool cov(LN l, LN A, LN B) {
    i128 c = cross(A.dir(), B.dir());
    i128 a = i128(A.c) * B.a - i128(B.c) * A.a;
    i128 b = i128(A.c) * B.b - i128(B.c) * A.b;
    return
        sign(a * l.b - b * l.a + c * l.c) * sign(c) >= 0;
}
bool operator<(LN a, LN b) {
    if (int c =
        cmp(a.dir(), b.dir(), false); c != -1) return c;
    return i128(abs(b.a) + abs
        (b.b)) * a.c > i128(abs(a.a) + abs(a.b)) * b.c;
}
}

```

## 8.28 minMaxEnclosingRectangle [d47db9]

```

const double INF = 1e18, qi = acos(-1) / 2 * 3;
pdd solve(vector<pll> &dots) {
#define diff(u, v) (dots[u] - dots[v])
#define vec(v) (dots[v] - dots[i])
    hull(dots);
    double Max = 0, Min = INF, deg;

```

```

    int n = SZ(dots);
    dots.pb(dots[0]);
    for (int i = 0, u = 1, r = 1, l = 1; i < n; ++i) {
        pll nw = vec(i + 1);
        while (cross(nw, vec(u + 1)) > cross(nw, vec(u)))
            u = (u + 1) % n;
        while (dot(nw, vec(r + 1)) > dot(nw, vec(r)))
            r = (r + 1) % n;
        if (!i) l = (r + 1) % n;
        while (dot(nw, vec(l + 1)) < dot(nw, vec(l)))
            l = (l + 1) % n;
        Min = min(Min, (double)(dot(nw, vec(r)) - dot
            (nw, vec(l))) * cross(nw, vec(u)) / abs2(nw));
        deg = acos(dot(diff(r, l), vec(u)) / abs(diff(r, l)) / abs(vec(u)));
        deg = (qi - deg) / 2;
        Max = max(Max, abs(diff
            (r, l)) * abs(vec(u)) * sin(deg) * sin(deg));
    }
    return pdd(Min, Max);
}

```

## 8.29 Half plane intersection [c3e180]

```

pll area_pair(Line a, Line b)
{ return pll(cross(a.Y
    - a.X, b.X - a.X), cross(a.Y - a.X, b.Y - a.X)); }
bool isin(Line l0, Line l1, Line l2) {
// Check inter(l1, l2) strictly in l0
    auto [a02X, a02Y] = area_pair(l0, l2);
    auto [a12X, a12Y] = area_pair(l1, l2);
    if (a12X - a12Y < 0) a12X *= -1, a12Y *= -1;
    return (__int128
        ) a02Y * a12X - (__int128) a02X * a12Y > 0;
}
/* Having solution, check size > 2 */
/* --^-- Line.X --^-- Line.Y --^-- */
vector<Line> halfPlaneInter(vector<Line> arr) {
    sort(ALL(arr), [&](Line a, Line b) -> int {
        if (cmp(a.Y - a.X, b.Y - b.X, 0) != -1)
            return cmp(a.Y - a.X, b.Y - b.X, 0);
        return ori(a.X, a.Y, b.Y) < 0;
    });
    deque<Line> dq(1, arr[0]);
    auto pop_back = [&](int t, Line p) {
        while (SZ(dq)
            ) >= t && !isin(p, dq[SZ(dq) - 2], dq.back())
            dq.pop_back();
    };
    auto pop_front = [&](int t, Line p) {
        while (SZ(dq) >= t && !isin(p, dq[0], dq[1]))
            dq.pop_front();
    };
    for (auto p : arr)
        if (cmp(
            dq.back().Y - dq.back().X, p.Y - p.X, 0) != -1)
            pop_back(2, p), pop_front(2, p), dq.pb(p);
    pop_back(3, dq[0]), pop_front(3, dq.back());
    return vector<Line>(ALL(dq));
}

```

## 8.30 Vector in poly [4c9a2f]

```

// ori(a
    , b, c) >= 0, valid: "strict" angle from a-b to a-c
bool btwangle(pll a, pll b, pll c, pll p, int strict) {
    return
        ori(a, b, p) >= strict && ori(a, p, c) >= strict;
}
// whether vector
    {cur, p} in counter-clockwise order prv, cur, nxt
bool inside
    (pll prv, pll cur, pll nxt, pll p, int strict) {
    if (ori(cur, nxt, prv) >= 0)
        return btwangle(cur, nxt, prv, p, strict);
    return !btwangle(cur, prv, nxt, p, !strict);
}

```

## 8.31 DelaunayTriangulation dq [e6fa02]

```

/* Delaunay Triangulation:
    Given a sets of points on 2D plane, find a
    triangulation such that no points will strictly
    inside circumcircle of any triangle. */
struct Edge {
    int id; // oidx[id]
    list<Edge>::iterator twin;

```

```

Edge(int _id = 0):id(_id) {}
};
struct Delaunay { // 0-base
    int n, oidx[N];
    list<Edge> head[N]; // result udir. graph
    pll p[N];
    void init(int _n, pll _p[]) {
        n = _n, iota(oidx, oidx + n, 0);
        for (int i = 0; i < n; ++i) head[i].clear();
        sort(oidx, oidx + n, [&](int a, int b)
            { return _p[a] < _p[b]; });
        for (int i = 0; i < n; ++i) p[i] = _p[oidx[i]];
        divide(0, n - 1);
    }
    void addEdge(int u, int v) {
        head[u].push_front(Edge(v));
        head[v].push_front(Edge(u));
        head[u].begin()->twin = head[v].begin();
        head[v].begin()->twin = head[u].begin();
    }
    void divide(int l, int r) {
        if (l == r) return;
        if (l + 1 == r) return addEdge(l, l + 1);
        int mid = (l + r) >> 1, nw[2] = {l, r};
        divide(l, mid), divide(mid + 1, r);
        auto gao = [&](int t) {
            pll pt[2] = {p[nw[0]], p[nw[1]]};
            for (auto it : head[nw[t]]) {
                int v = ori(pt[1], pt[0], p[it.id]);
                if (v > 0 || (v == 0 && abs2(pt
                    [t ^ 1] - p[it.id]) < abs2(pt[1] - pt[0])))
                    return nw[t] = it.id, true;
            }
            return false;
        };
        while (gao(0) || gao(1));
        addEdge(nw[0], nw[1]); // add tangent
        while (true) {
            pll pt[2] = {p[nw[0]], p[nw[1]]};
            int ch = -1, sd = 0;
            for (int t = 0; t < 2; ++t)
                for (auto it : head[nw[t]])
                    if (ori(pt[0], pt[1],
                        p[it.id]) > 0 && (ch == -1 || in_cc
                            ({pt[0], pt[1], p[ch]}, p[it.id])))
                        ch = it.id, sd = t;
            if (ch == -1) break; // upper common tangent
            for (auto it = head
                [nw[sd]].begin(); it != head[nw[sd]].end(); )
                if (seg_strict_intersect
                    (pt[sd], p[it->id], pt[sd ^ 1], p[ch]))
                    head[it->id].erase
                        (it->twin), head[nw[sd]].erase(it++);
                else ++it;
            nw[sd] = ch, addEdge(nw[0], nw[1]);
        }
    }
} tool;

```

### 8.32 Minimum Enclosing Circle [5f3cdb]

```

pdd Minimum_Enclosing_Circle
(vector<pdd> dots, double &r) {
    pdd cent;
    random_shuffle(ALL(dots));
    cent = dots[0], r = 0;
    for (int i = 1; i < SZ(dots); ++i)
        if (abs(dots[i] - cent) > r) {
            cent = dots[i], r = 0;
            for (int j = 0; j < i; ++j)
                if (abs(dots[j] - cent) > r) {
                    cent = (dots[i] + dots[j]) / 2;
                    r = abs(dots[i] - cent);
                    for (int k = 0; k < j; ++k)
                        if (abs(dots[k] - cent) > r)
                            cent = excenter
                                (dots[i], dots[j], dots[k], r);
                }
        }
    return cent;
}

```

### 8.33 Convex hull [2a3008]

```

void hull(vector<pll> &dots) { // n=1 => ans = {}
    sort(dots.begin(), dots.end());
    vector<pll> ans(1, dots[0]);

```

```

    for (int ct = 0; ct < 2; ++ct, reverse(ALL(dots)))
        for (int i = 1,
            t = SZ(ans); i < SZ(dots); ans.pb(dots[i++]))
            while (SZ(ans) > t && ori
                (ans[SZ(ans) - 2], ans.back(), dots[i]) <= 0)
                ans.pop_back();
            ans.pop_back(), ans.swap(dots);
}

```

## 9 Else

### 9.1 ManhattanMST [90cf5a]

```

void solve(Point *a, int n) {
    sort(a, a + n, [](const Point &p, const Point &q) {
        return p.x + p.y < q.x + q.y;
    });
    set<Point> st; // greater<Point::x>
    for (int i = 0; i < n; ++i) {
        for (auto it = st.lower_bound(
            a[i]); it != st.end(); it = st.erase(it)) {
            if (it ->
                x - it -> y < a[i].x - a[i].y) break;
            es.push_back
                ({it -> u, a[i].u, dist(*it, a[i])});
        }
        st.insert(a[i]);
    }
}

void MST(Point *a, int n) {
    for (int t = 0; t < 2; ++t) {
        solve(a, n);
        for (int
            i = 0; i < n; ++i) swap(a[i].x, a[i].y);
        solve(a, n);
        for (int i = 0; i < n; ++i) a[i].x = -a[i].x;
    }
}

```

### 9.2 Mos Algorithm With modification [021725]

```

/*
Mo's Algorithm With modification
Block: N^{2/3}, Complexity: N^{5/3}
*/
struct Query {
    int L, R, LBid, RBid, T;
    Query(int l, int r, int t):
        L(l), R(r), LBid(l / blk), RBid(r / blk), T(t) {}
    bool operator<(const Query &q) const {
        if (LBid != q.LBid) return LBid < q.LBid;
        if (RBid != q.RBid) return RBid < q.RBid;
        return T < q.T;
    }
};

void solve(vector<Query> query) {
    sort(ALL(query));
    int L=0, R=0, T=-1;
    for (auto q : query) {
        while (T < q.T) addTime(L, R, ++T); // TODO
        while (T > q.T) subTime(L, R, T--); // TODO
        while (R < q.R) add(arr[++R]); // TODO
        while (L > q.L) add(arr[--L]); // TODO
        while (R > q.R) sub(arr[R--]); // TODO
        while (L < q.L) sub(arr[L++]); // TODO
        // answer query
    }
}

```

### 9.3 BitsetLCS [027ab4]

```

cin >> n >> m;
for (int i = 1, x; i <= n; ++i)
    cin >> x, p[x].set(i);
for (int i = 1, x; i <= m; i++) {
    cin >> x, (g = f) |= p[x];
    f.shiftLeftByOne(), f.set(0);
    ((f = g - f) ^= g) &= g;
}
cout << f.count() << '\n';

```

### 9.4 BinarySearchOnFraction [dec1bd]

```

struct Q {
    ll p, q;
    Q go(Q b, ll d) { return {p + b.p*d, q + b.q*d}; }
};
bool pred(Q);

```



```
// returns smallest p/q in [lo, hi] such that
// pred(p/q) is true, and 0 <= p,q <= N
Q frac_bs(ll N) {
    Q lo{0, 1}, hi{1, 0};
    if (pred(lo)) return lo;
    assert(pred(hi));
    bool dir = 1, L = 1, H = 1;
    for (; L || H; dir = !dir) {
        ll len = 0, step = 1;
        for (int t = 0; t < 2 && (t ? step/=2 : step*=2);)
            if (Q mid = hi.go(lo, len + step);
                mid.p > N || mid.q > N || dir ^ pred(mid))
                t++;
            else len += step;
        swap(lo, hi = hi.go(lo, len));
        (dir ? L : H) = !len;
    }
    return dir ? hi : lo;
}
```

## 9.5 SubsetSum [8fa070]

```
template<size_t S> // sum(a) < S
bitset<S> SubsetSum(const int *a, int n) {
    vector<int> c(S);
    bitset<S> dp; dp[0] = 1;
    for (int i = 0; i < n; ++i) ++c[a[i]];
    for (size_t i = 1; i < S; ++i) {
        while (c[i] > 2) c[i] -= 2, ++c[i * 2];
        while (c[i]--) dp |= dp << i;
    }
    return dp;
}
```

## 9.6 DynamicConvexTrick [477879]

```
// only works for integer coordinates!! maintain max
struct Line {
    mutable ll a, b, p;
    bool operator
        <(const Line &rhs) const { return a < rhs.a; }
    bool operator<(ll x) const { return p < x; }
};
struct DynamicHull : multiset<Line, less<>> {
    static const ll kInf = 1e18;
    ll Div(ll a,
        ll b) { return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = kInf; return 0; }
        if (x
            ->a == y->a) x->p = x->b > y->b ? kInf : -kInf;
        else x->p = Div(y->b - x->b, x->a - y->a);
        return x->p >= y->p;
    }
    void addline(ll a, ll b) {
        auto z = insert({a, b, 0}); y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin
            ()) && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin
            ()) && (--x)->p >= y->p) isect(x, erase(y));
    }
    ll query(ll x) {
        auto l = *lower_bound(x);
        return l.a * x + l.b;
    }
};
```

## 9.7 DynamicMST [9ac74a]

```
int cnt[maxn], cost[maxn], st[maxn], ed[maxn];
pair<int, int> qr[maxn];
// qr[i].first = id of edge to
// be changed, qr[i].second = weight after operation
// cnt[i] = number of operation on edge i
// call solve(0, q - 1, v,
// 0), where v contains edges i such that cnt[i] == 0

void contract(int l, int
    r, vector<int> v, vector<int> &x, vector<int> &y) {
    sort(v.begin(), v.end(), [&](int i, int j) {
        if (cost[i] == cost[j]) return i < j;
        return cost[i] < cost[j];
    });
    djs.save();
    for (int i = l; i <= r;
        ++i) djs.merge(st[qr[i].first], ed[qr[i].first]);
```

```
for (int i = 0; i < (int)v.size(); ++i) {
    if (djs.find(st[v[i]]) != djs.find(ed[v[i]])) {
        x.push_back(v[i]);
        djs.merge(st[v[i]], ed[v[i]]);
    }
}
djs.undo();
djs.save();
for (int i = 0; i < (
    int)x.size(); ++i) djs.merge(st[x[i]], ed[x[i]]);
for (int i = 0; i < (int)v.size(); ++i) {
    if (djs.find(st[v[i]]) != djs.find(ed[v[i]])) {
        y.push_back(v[i]);
        djs.merge(st[v[i]], ed[v[i]]);
    }
}
djs.undo();
}
```

```
void solve(int l, int r, vector<int> v, long long c) {
    if (l == r) {
        cost[qr[l].first] = qr[l].second;
        if (st[qr[l].first] == ed[qr[l].first]) {
            printf("%lld\n", c);
            return;
        }
        int minv = qr[l].second;
        for (int i = 0; i < (int
            )v.size(); ++i) minv = min(minv, cost[v[i]]);
        printf("%lld\n", c + minv);
        return;
    }
    int m = (l + r) >> 1;
    vector<int> lv = v, rv = v;
    vector<int> x, y;
    for (int i = m + 1; i <= r; ++i) {
        cnt[qr[i].first]--;
        if (cnt
            [qr[i].first] == 0) lv.push_back(qr[i].first);
    }
    contract(l, m, lv, x, y);
    long long lc = c, rc = c;
    djs.save();
    for (int i = 0; i < (int)x.size(); ++i) {
        lc += cost[x[i]];
        djs.merge(st[x[i]], ed[x[i]]);
    }
    solve(l, m, y, lc);
    djs.undo();
    x.clear(), y.clear();
    for (int i = m + 1; i <= r; ++i) cnt[qr[i].first]++;
    for (int i = l; i <= m; ++i) {
        cnt[qr[i].first]--;
        if (cnt
            [qr[i].first] == 0) rv.push_back(qr[i].first);
    }
    contract(m + 1, r, rv, x, y);
    djs.save();
    for (int i = 0; i < (int)x.size(); ++i) {
        rc += cost[x[i]];
        djs.merge(st[x[i]], ed[x[i]]);
    }
    solve(m + 1, r, y, rc);
    djs.undo();
    for (int i = l; i <= m; ++i) cnt[qr[i].first]++;
}
```

## 9.8 Matroid

Start from  $S = \emptyset$ . In each iteration, let

- $Y_1 = \{x \notin S \mid S \cup \{x\} \in I_1\}$
- $Y_2 = \{x \notin S \mid S \cup \{x\} \in I_2\}$

If there exists  $x \in Y_1 \cap Y_2$ , insert  $x$  into  $S$ . Otherwise for each  $x \in S, y \notin S$ , create edges

- $x \rightarrow y$  if  $S - \{x\} \cup \{y\} \in I_1$ .
- $y \rightarrow x$  if  $S - \{x\} \cup \{y\} \in I_2$ .

Find a *shortest* path (with BFS) starting from a vertex in  $Y_1$  and ending at a vertex in  $Y_2$  which doesn't pass through any other vertices in  $Y_2$ , and alternate the path. The size of  $S$  will be incremented by 1 in each iteration. For the weighted case, assign weight  $w(x)$  to vertex  $x$  if  $x \in S$  and  $-w(x)$  if  $x \notin S$ . Find the path with the minimum number of edges among all minimum length paths and alternate it.

## 9.9 cyclicLCS [9b01d1]

```
#define L 0
#define LU 1
#define U 2
```

```

const int mov[3][2] = {0, -1, -1, -1, -1, 0};
int al, bl;
char a[MAXL * 2], b[MAXL * 2]; // 0-indexed
int dp[MAXL * 2][MAXL];
char pred[MAXL * 2][MAXL];
inline int lcs_length(int r) {
    int i = r + al, j = bl, l = 0;
    while (i > r) {
        char dir = pred[i][j];
        if (dir == LU) l++;
        i += mov[dir][0];
        j += mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i = r, j = 1;
    while (j <= bl && pred[i][j] != LU) j++;
    if (j > bl) return;
    pred[i][j] = L;
    while (i < 2 * al && j <= bl) {
        if (pred[i + 1][j] == U) {
            i++;
            pred[i][j] = L;
        } else if (j < bl && pred[i + 1][j + 1] == LU) {
            i++;
            j++;
            pred[i][j] = L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if (al > bl) {
        swap(al, bl);
        strcpy(tmp, a);
        strcpy(a, b);
        strcpy(b, tmp);
    }
    strcpy(tmp, a);
    strcat(a, tmp);
    // basic lcs
    for (int i = 0; i <= 2 * al; i++) {
        dp[i][0] = 0;
        pred[i][0] = U;
    }
    for (int j = 0; j <= bl; j++) {
        dp[0][j] = 0;
        pred[0][j] = L;
    }
    for (int i = 1; i <= 2 * al; i++) {
        for (int j = 1; j <= bl; j++) {
            if (a[i - 1] == b[j - 1])
                dp[i][j] = dp[i - 1][j - 1] + 1;
            else dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
            if (dp[i][j - 1] == dp[i][j]) pred[i][j] = L;
            else if (a[i - 1] == b[j - 1]) pred[i][j] = LU;
            else pred[i][j] = U;
        }
    }
    // do cyclic lcs
    int clcs = 0;
    for (int i = 0; i < al; i++) {
        clcs = max(clcs, lcs_length(i));
        reroot(i + 1);
    }
    // recover a
    a[al] = '\0';
    return clcs;
}

```

## 9.10 HilbertCurve [bc6dec]

```

ll hilbert(int n, int x, int y) {
    ll res = 0;
    for (int s = n / 2; s; s >>= 1) {
        int rx = (x & s) > 0;
        int ry = (y & s) > 0;
        res += s * 1ll * s * ((3 * rx) ^ ry);
        if (ry == 0) {
            if (rx == 1) x = s - 1 - x, y = s - 1 - y;
            swap(x, y);
        }
    }
}

```

```

    }
}
return res;
} // n = 2^k

```

## 9.11 Mos Algorithm On Tree [90ac22]

```

/*
Mo's Algorithm On Tree
Preprocess:
1) LCA
2) dfs with in[u] = dft++, out[u] = dft++
3) ord[in[u]] = ord[out[u]] = u
4) bitset<MAXN> inset
*/
struct Query {
    int L, R, LBid, lca;
    Query(int u, int v) {
        int c = LCA(u, v);
        if (c == u || c == v)
            q.lca = -1, q.L = out[c ^ u ^ v], q.R = out[c];
        else if (out[u] < in[v])
            q.lca = c, q.L = out[u], q.R = in[v];
        else
            q.lca = c, q.L = out[v], q.R = in[u];
        q.Lid = q.L / blk;
    }
    bool operator<(const Query &q) const {
        if (LBid != q.LBid) return LBid < q.LBid;
        return R < q.R;
    }
};
void flip(int x) {
    if (inset[x]) sub(arr[x]); // TODO
    else add(arr[x]); // TODO
    inset[x] = ~inset[x];
}
void solve(vector<Query> query) {
    sort(ALL(query));
    int L = 0, R = 0;
    for (auto q : query) {
        while (R < q.R) flip(ord[++R]);
        while (L > q.L) flip(ord[--L]);
        while (R > q.R) flip(ord[ord[L++]]);
        while (L < q.L) flip(ord[L--]);
        if (~q.lca) add(arr[q.lca]);
        // answer query
        if (~q.lca) sub(arr[q.lca]);
    }
}

```

## 9.12 AdaptiveSimpson [c048eb]

```

template<typename Func, typename d = double>
struct Simpson {
    using pdd = pair<d, d>;
    Func f;
    pdd mix(pdd l, pdd r, optional<d> fm = {}) {
        d h = (r.X - l.X) / 2, v = fm.value_or(f(l.X + h));
        return {v, h / 3 * (l.Y + 4 * v + r.Y)};
    }
    d eval(pdd l, pdd r, d fm, d eps) {
        pdd m((l.X + r.X) / 2, fm);
        d s = mix(l, r, fm).second;
        auto [flm, sl] = mix(l, m);
        auto [fmr, sr] = mix(m, r);
        d delta = sl + sr - s;
        if (abs(delta) <= 15 * eps) return sl + sr + delta / 15;
        return eval(l, m, flm, eps / 2) +
            eval(m, r, fmr, eps / 2);
    }
    d eval(d l, d r, d eps) {
        return eval(
            {{l, f(l)}, {r, f(r)}}, f((l + r) / 2), eps);
    }
    d eval2(d l, d r, d eps, int k = 997) {
        d h = (r - l) / k, s = 0;
        for (int i = 0; i < k; ++i, l += h)
            s += eval(l, l + h, eps / k);
        return s;
    }
};
template<typename Func>
Simpson<Func> make_simpson(Func f) { return {f}; }

```

### 9.13 min plus convolution [b08fbf]

```
// a is convex a[i+1]-a[i] <= a[i+2]-a[i+1]
vector<int> min_plus_convolution
(vector<int> &a, vector<int> &b) {
    int n = SZ(a), m = SZ(b);
    vector<int> c(n + m - 1, INF);
    auto dc = [&](auto Y, int l, int r, int jl, int jr) {
        if (l > r) return;
        int mid = (l + r) / 2, from = -1, &best = c[mid];
        for (int j = jl; j <= jr; ++j)
            if (int i = mid - j; i >= 0 && i < n)
                if (best > a[i] + b[j])
                    best = a[i] + b[j], from = j;
        Y(Y, l, mid - 1, jl, from), Y(Y, mid + 1, r, from, jr);
    };
    return dc(dc, 0, n - 1 + m - 1, 0, m - 1), c;
}
```

### 9.14 cyc tsearch [3dac64]

```
/* bool pred(int a, int b);
f(0) ~ f(n - 1) is a cyclic-shift U-function
return idx s.t. pred(x, idx) is false forall x*/
int cyc_tsearch(int n, auto pred) {
    if (n == 1) return 0;
    int l = 0, r = n; bool rv = pred(1, 0);
    while (r - l > 1) {
        int m = (l + r) / 2;
        if (pred(0, m) ? rv : pred(m, (m + 1) % n)) r = m;
        else l = m;
    }
    return pred(l, r % n) ? l : r % n;
}
```

### 9.15 All LCS [5548b0]

```
void all_lcs(string s, string t) { // 0-base
    vector<int> h(SZ(t));
    iota(ALL(h), 0);
    for (int a = 0; a < SZ(s); ++a) {
        int v = -1;
        for (int c = 0; c < SZ(t); ++c)
            if (s[a] == t[c] || h[c] < v)
                swap(h[c], v);
        // LCS(s[0, a], t[b, c]) =
        // c - b + 1 - sum([h[i] >= b] | i <= c)
        // h[i] might become -1 !!
    }
}
```

### 9.16 NQueens [68bc5d]

```
void solve
(vector<int> &ret, int n) { // no sol when n=2,3
    if (n % 6 == 2) {
        for (int i = 2; i <= n; i += 2) ret.pb(i);
        ret.pb(3); ret.pb(1);
        for (int i = 7; i <= n; i += 2) ret.pb(i);
        ret.pb(5);
    } else if (n % 6 == 3) {
        for (int i = 4; i <= n; i += 2) ret.pb(i);
        ret.pb(2);
        for (int i = 5; i <= n; i += 2) ret.pb(i);
        ret.pb(1); ret.pb(3);
    } else {
        for (int i = 2; i <= n; i += 2) ret.pb(i);
        for (int i = 1; i <= n; i += 2) ret.pb(i);
    }
}
```

### 9.17 Mos Algorithm

- Mo's Algorithm With Addition Only
  - Sort queries same as the normal Mo's algorithm.
  - For each query  $[l, r]$ :
    - If  $l/blk = r/blk$ , brute-force.
    - If  $l/blk \neq r/blk$ , initialize  $curL := (l/blk + 1) \cdot blk$ ,  $curR := curL - 1$ .
    - If  $r > curR$ , increase  $curR$ .
    - decrease  $curL$  to fit  $l$ , and then undo after answering
- Mo's Algorithm With Offline Second Time
  - Require: Changing answer  $\equiv$  adding  $f([l, r], r+1)$ .
  - Require:  $f([l, r], r+1) = f([l, r], r+1) - f([l, l], r+1)$ .
  - Part1: Answer all  $f([l, r], r+1)$  first.
  - Part2: Store  $curR \rightarrow R$  for  $curL$  (reduce the space to  $O(N)$ ), and then answer them by the second offline algorithm.
  - Note: You must do the above symmetrically for the left boundaries.

### 9.18 simulated annealing [d19317]

```
double factor = 100000;
const int base = 1e9; // remember to run ~ 10 times
for (int it = 1; it <= 1000000; ++it) {
    // ans:
    // answer, nw: current value, rnd(): mt19937 rnd()
    if (exp(-(nw - ans) / factor) >= (double)(rnd() % base) / base)
        ans = nw;
    factor *= 0.99995;
}
```

### 9.19 DLX [5d57fa]

```
#define TRAV(i, link, start)
    for (int i = link[start]; i != start; i = link[i])
template<
    bool E> // E: Exact, NN: num of 1s, RR: num of rows
struct DLX {
    int lt[NN], rg[NN], up[NN], dn[NN]
    ], rw[NN], cl[NN], bt[NN], s[NN], head, sz, ans;
    int rows, columns;
    bool vis[NN];
    bitset<RR> sol, cur; // not sure
    void remove(int c) {
        if (E) lt[rg[c]] = lt[c], rg[lt[c]] = rg[c];
        TRAV(i, dn, c) {
            if (E) {
                TRAV(j, rg, i)
                    up[dn[j]]
                        = up[j], dn[up[j]] = dn[j], --s[cl[j]];
            } else {
                lt[rg[i]] = lt[i], rg[lt[i]] = rg[i];
            }
        }
    }
    void restore(int c) {
        TRAV(i, up, c) {
            if (E) {
                TRAV(j, lt, i)
                    ++s[cl[j]], up[dn[j]] = j, dn[up[j]] = j;
            } else {
                lt[rg[i]] = rg[lt[i]] = i;
            }
        }
        if (E) lt[rg[c]] = c, rg[lt[c]] = c;
    }
    void init(int c) {
        rows = 0, columns = c;
        for (int i = 0; i < c; ++i) {
            up[i] = dn[i] = bt[i] = i;
            lt[i] = i == 0 ? c : i - 1;
            rg[i] = i == c - 1 ? c : i + 1;
            s[i] = 0;
        }
        rg[c] = 0, lt[c] = c - 1;
        up[c] = dn[c] = -1;
        head = c, sz = c + 1;
    }
    void insert(const vector<int> &col) {
        if (col.empty()) return;
        int f = sz;
        for (int i = 0; i < (int)col.size(); ++i) {
            int c = col[i], v = sz++;
            dn[bt[c]] = v;
            up[v] = bt[c], bt[c] = v;
            rg[v] = (i + 1 == (int)col.size() ? f : v + 1);
            rw[v] = rows, cl[v] = c;
            ++s[c];
            if (i > 0) lt[v] = v - 1;
        }
        ++rows, lt[f] = sz - 1;
    }
    int h() {
        int ret = 0;
        fill_n(vis, sz, false);
        TRAV(x, rg, head) {
            if (vis[x]) continue;
            vis[x] = true, ++ret;
            TRAV(i, dn, x) TRAV(j, rg, i) vis[cl[j]] = true;
        }
        return ret;
    }
    void dfs(int dep) {
        if (dep + (E ? 0 : h()) >= ans) return;
        if (rg[head]
            ] == head) return sol = cur, ans = dep, void();
    }
}
```

```

if (dn[rg[head]] == rg[head]) return;
int w = rg[head];
TRAV(x, rg, head) if (s[x] < s[w]) w = x;
if (E) remove(w);
TRAV(i, dn, w) {
    if (!E) remove(i);
    TRAV(j, rg, i) remove(E ? cl[j] : j);
    cur.set(rw[i]), dfs(dep + 1), cur.reset(rw[i]);
    TRAV(j, lt, i) restore(E ? cl[j] : j);
    if (!E) restore(i);
}
if (E) restore(w);
}
int solve() {
    for (int i = 0; i < columns; ++i)
        dn[bt[i]] = i, up[i] = bt[i];
    ans = 1e9, sol.reset(), dfs(0);
    return ans;
}
};

```

## 9.20 tree hash [95e839]

```

ull seed;
ull shift(ull x) {
    x ^= x << 13;
    x ^= x >> 7;
    x ^= x << 17;
    return x;
}
ull dfs(int u, int f) {
    ull sum = seed;
    for (int i : G[u])
        if (i != f)
            sum += shift(dfs(i, u));
    return sum;
}

```

## 9.21 DynamicConvexTrick bb [85e4f7]

```

// only works for integer coordinates!!
bool Flag; // 0: insert Line, 1: lower_bound x
template<class val = ll,
        class compare = less<val>> // sort lines with comp
struct DynamicConvexTrick{
    static const ll minx = 0, maxx = ll(1e9) + 5;
    static compare comp;
    struct Line{
        val a, b, l, r; // line ax + b in [l, r]
        Line(val _a, val _b, val _l = minx,
              val _r = maxx):a(_a), b(_b), l(_l), r(_r){}
        val operator () (val x) const {
            return a * x + b;
        }
    };
};
struct cmp{
    bool operator () (const Line a, const Line b){
        if(Flag == 0) return comp(a.a, b.a);
        return a.r < b.l;
    }
};
inline val idiv(val a, val b){
    return a / b - (a % b && a < 0 ^ b < 0);
}
set<Line, cmp> st;
void ins(val a, val b){
    Flag = 0;
    Line L(a, b);
    auto it = st.lower_bound(L);
    if(it != st.begin() && it != st.end()){
        if(!comp((*prev(it))(it->l - 1), L(
            it->l - 1)) && !comp((*it)(it->l), L(it->l)))
            return;
        while(it != st.end()){
            if(it->a == L.a && !comp(it->b, L.b)) return;
            if(comp(
                ((*it)(it->r), L(it->r))) it = st.erase(it);
            else{
                Line M = *it;
                st.erase(it);
                L.r = max(idiv(L.b - M.b, M.a - L.a), minx);
                M.l = L.r + 1;
                it = st.insert(M).X;
                break;
            }
        }
    }
}

```

```

while(it != st.begin()){
    auto pit = prev(it);
    if(comp((*pit)(pit->l), L(pit->l))) st.erase(pit);
    else{
        Line M = *pit;
        st.erase(pit);
        M.r =
            min(idiv(L.b - M.b, M.a - L.a), maxx - 1);
        L.l = M.r + 1;
        st.insert(M);
        break;
    }
}
st.insert(L);
}
val operator () (val x){
    Flag = 1;
    auto it = st.lower_bound({0, 0, x, x});
    return (*it)(x);
}
};

```

DynamicConvexTrick<> DCT;

# 10 JAVA

## 10.1 Big number [05dd09]

```

import java.util.Scanner;
import java.math.BigInteger;

public class JAVA{
    public static void main(String[] args){
        Scanner cin = new Scanner(System.in);
        String a, b, c;
        while(cin.hasNext()){
            a = cin.next();
            b = cin.next();
            c = cin.next();
            BigInteger ia = new BigInteger(a);
            BigInteger ic = new BigInteger(c);
            if(b.charAt(0) == '+')
                System.out.printf("%s\n", ia.add(ic));
            if(b.charAt(0) == '-')
                System.out.printf("%s\n", ia.subtract(ic));
            if(b.charAt(0) == '*')
                System.out.printf("%s\n", ia.multiply(ic));
            if(b.charAt(0) == '/')
                System.out.printf("%s\n", ia.divide(ic));
        }
    }
}

```

# 11 Python

## 11.1 misc

```

from decimal import *
setcontext(Context(prec
    =MAX_PREC, Emax=MAX_EMAX, rounding=ROUND_FLOOR))
print(Decimal(input()) * Decimal(input()))
from fractions import Fraction
Fraction(
    ('3.14159').limit_denominator(10).numerator # 22

```