Lecture 04/09/2025

$\underline{X}$ random variable

$(\underline{X}_1, \dots, \underline{X}_k)$ random vector

$\{\underline{X}(t), t \in I\}$ random function

$P(\underline{X} \in A)$

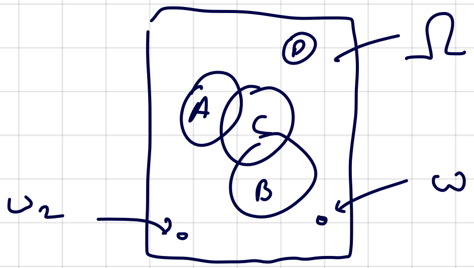
$\uparrow$ probability $\quad \uparrow$ set

probability

Set Theory

Ω - universal set

A, B - subsets



$(\omega_1, \omega_2, \dots)$ - elementary elements

$A \subseteq B \iff \omega \in A \Rightarrow \omega \in B$

$A \equiv B \iff A \subseteq B$ and $B \subseteq A$

$A \cap B = \{\omega : \omega \in A$ and $\omega \in B\}$ intersection

$A \cup B = \{\omega : \omega \in A$ or $\omega \in B\}$ union

Ω - universal set $\leftarrow$ evrything

$\emptyset$ - empty set

A, B are disjoined if $A \cap B = \emptyset$

Mathematical model for randomness

Ω - universal set (contain all possible outcomes)

$\uparrow$ outcome space

Ex: Model outcome when rolling a dice

$\Omega = \{1, 2, 3, 4, 5, 6\}$

A = even outcome $\{2, 4, 6\}$

B = odd outcome $\{1, 3, 5\}$

C = $\{3$ or less$\}$ $\{1, 2, 3\}$

$A \cap B = \varnothing$

$A \cap C = \{2\}$

$A \subseteq \Omega$ - event

ω. (for ex. 2) is elementary event

$\varnothing$ - imposible event

Def (sometime works, sometime does not work)

A probability is a function on the set of ALL subsets of an outcome space

$P: \{A : A \subseteq \Omega\} \ni A \to P(A) \in [0, 1]$

such that

(i) $P(\Omega) = 1$

(ii) $A_1, A_2 \dots$ - countable collection of subsets of Ω disjointed $(A_i \cap A_j = \varnothing, \forall i \neq j)$

that $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

If Ω is uncountable $\Rightarrow$ def doesn't work

Ex: $\Omega = (0; 10]$

$P: \{A : A \in \Omega\} \to [0, 1]$

s.t. $P(A) = \frac{|A|}{10}$ — NO

and def holds

$\Downarrow$

Labesgue integral