EXERCISE SHEET 1

To be solved prior, and discussed in the seminar

1.1. Which of the following sets are subspaces of $\mathbb{R}^2$? Justify your answers.
(a) $\{\mathbf{x} \in \mathbb{R}^2 : x_1 = 2x_2\}$,
(b) $\{\mathbf{x} \in \mathbb{R}^2 : (x_1, x_2) = t(1,2) + (1,1),\ t \in \mathbb{R}\}$,
(c) $\{\mathbf{x} \in \mathbb{R}^2 : x_1 = x_2^2\}$,
(d) $\{\mathbf{x} \in \mathbb{R}^2 : (x_1, x_2) = t(1,2),\ t \in \mathbb{R}\}$.

1.2. Show that the set of symmetric $n \times n$ matrices is a subspace of $M_{n\times n}$.

1.3. Let $A \in \mathrm{M}_{m\times n}(\mathbb{R})$. Show that $\ker A$ and $\operatorname{im} A$ are subspaces of $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively.

1.4. Express the plane through the origin and the two points $(1,1,0)$ and $(2,0,1)$ as the image of a matrix and as the kernel of a matrix.

1.5. Consider the following vectors in $\mathbb{R}^4$.

$$\mathbf{u}_1 = (1,1,1,2), \quad \mathbf{u}_2 = (1,2,3,4), \quad \mathbf{u}_3 = (2,1,2,3), \quad \mathbf{u}_4 = (5,1,3,5).$$

Is $\mathbf{u}_1$ a linear combination of $\mathbf{u}_2$, $\mathbf{u}_3$ and $\mathbf{u}_4$? Are $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ and $\mathbf{u}_4$ linearly dependent?

1.6. Which of the following sets of vectors are linearly dependent?
(a) $(1,2,3)$, $(2,3,3)$, $(2,5,7)$ in $\mathbb{R}^3$,
(b) $(1,2,3,1)$, $(2,3,2,3)$, $(1,1,-1,2)$ in $\mathbb{R}^4$,
(c) $(1,2,3,1,2)$, $(2,3,2,3,1)$, $(1,1,-1,2,3)$ in $\mathbb{R}^5$.

1.7. Show that the vectors u_1, u_2 and u_3 in the vector space $C(\mathbb{R})$, defined by

$$u_1(x) = \sin(x), \qquad u_2(x) = \sin(2x), \qquad u_3(x) = \sin(3x)$$

are linearly independent.