

LUNDS TEKNISKA HÖGSKOLA T

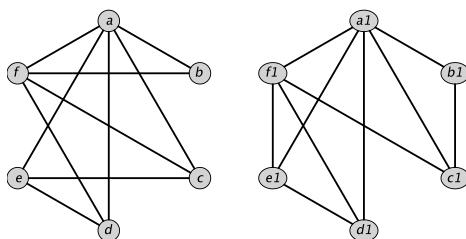
DISKRET MATEMATIK FMAB75/MATB25  
2023-05-30 kl 8-13

No books, notes, computational devices, etc. are allowed. Use clear handwriting and give clear careful motivations. All answers should be fully simplified, but may contain factorials, powers, binomial coefficients and Stirling numbers in instances where those cannot easily be evaluated. Write your personal identifier on each sheet of paper.

1.
    - a) Find the smallest positive integer congruent to  $5^{2023}$  modulo 3.
    - b) Find the smallest positive integer congruent to  $5^{2023}$  modulo 7.
    - c) Find the smallest positive integer congruent to  $5^{2023}$  modulo 21.
  2. Assume we have a bag containing marbles in the six colours red, green, blue, yellow, black and white. All marbles of the same colour are identical. In how many ways can we pick 12
    - a) if there are 15 of each colour in the bag?
    - b) if the bag contains only one red marble but 15 of each of the other colours?
    - c) if there should be an odd number of green marbles and the bag contains only one red marble but 15 of each of the other colours?

*Note: We do not care about the order in which we draw the marbles from the bag.*

- How many strings can be made using the letters in **SUMMERTIME** that do *not* contain any of the strings **TIME**, **SUMMER**, **SUM** or **MERIT**?
  - Consider the following two graphs:



- a) Are the two graphs isomorphic?
  - b) For each graph determine if it contains a Hamilton cycle.
  - c) For each graph determine if it contains an Euler trail.

5. Consider the linear code  $C$  over  $\mathbb{Z}_5$  with generator matrix

$$H = \begin{pmatrix} 1 & 2 & 0 & 1 & 1 & 4 \\ 0 & 1 & 0 & 0 & 4 & 2 \\ 0 & 3 & 1 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 & 4 & 4 \end{pmatrix}$$

- a) Find a control matrix of  $C$ .
  - b) What is the separation of  $C$ ?
  - c) Can the word  $w = (1\ 3\ 1\ 2\ 0\ 4)$  be corrected? If so find the corrected word.
  - d) Is  $C$  a perfect code?
6. Let  $M_0 = (0, 0)$ ,  $M_1 = (a, 0)$  and  $M_2 = (b, c)$  be three points in the plane. Then let  $M_k$  be the midpoint of the line segment between  $M_{k-2}$  and  $M_{k-3}$  for  $k \geq 3$ . Find the coordinates  $(x_k, y_k)$  of  $M_k$  as a function of  $k$ ,  $a$ ,  $b$  and  $c$ . Does  $M_k$  converge to a point in the plane as  $k$  tends to infinity? If so determine the coordinates of that point.

