Topology: Exercises 1

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Problem 1

Proof. ($\Rightarrow$) Assume there exists a bijection $h : A \to B$. Since A is countable, there exists a set C and a bijection $f : A \to C$, where $C = \{1, \dots, n\}$ if A is finite and $C = \mathbb{N}$ if A is countably infinite. Define $g := f \circ h^{-1} : B \to C$. Since compositions of bijections are bijections, g is a bijection. Hence there exisrs a bijection from both A and B to the same set C, so they have the same cardinality.

($\Leftarrow$) Assume A and B have the same cardinality. Then there exists a set $C \subseteq \mathbb{N}$ and bijections $f : A \to C$ and $g : B \to C$. Define $h := g^{-1} \circ f : A \to B$. Since f and g are bijections, h is a bijection. $\square$