LUNDS UNIVERSITET

Matematikcentrum
Matematik NF

Topology, Spring 2026 **MATM36**

Exercises for Seminar 1

1. Let A, B be countable sets. Show: There exists a bijective map $\phi : A \to B$, if and only if A and B have the same cardinality.

2. Let $k \in \mathbb{N}$. Show that the k-fold Cartesian product $\mathbb{N}^k = \mathbb{N} \times \cdots \times \mathbb{N}$ is countable.
Hint. Choose k distinct prime numbers $p_1, \ldots, p_k$ and consider the map

$$f : \mathbb{N}^k \to \mathbb{N}, \quad (n_1, \ldots, n_k) \mapsto p_1^{n_1} \cdot \cdots \cdot p_k^{n_k}.$$

3. Show that $\mathbb{Q}$ is countably infinite.

4. Let A be set. We write $\mathcal{P}(A) = \{B \subseteq A\}$. $\mathcal{P}(A)$ is called the power set of A.

- **a)** Let A be finite and let $n = \operatorname{card} A$. Find a bijective map $\{0,1\}^n \to \mathcal{P}(A)$.
- **b)** Conclude that for any finite set A, $\operatorname{card} \mathcal{P}(A) = 2^{\operatorname{card} A}$. What about the case $A = \emptyset$?
- **c)** Find a bijective map $\{0,1\}^{\mathbb{N}} \to \mathcal{P}(\mathbb{N})$.
- **d)** Show that there exists no bijective map between $\mathbb{N}$ and $\mathcal{P}(\mathbb{N})$, hence $\mathcal{P}(\mathbb{N})$ is not countable.