Abstract Algebra – Stockholm University (2025)

1. Topic 1: Basic Group Theory

Main concepts: Groups, cyclic groups, permutation groups, dihedral groups, matrix groups; homomorphisms, isomorphisms, kernels, normal subgroups, quotient groups; the four isomorphism theorems.

Reading: Dummit and Foote

- Chapter 0
- Sections 1.1–1.7, 2.1–2.4, 3.1–3.3, 3.5

Exercises:

- 1.1: 6, 9 1.2: 9, 10 1.3: 1, 11, 12
- 1.6: 3, 4, 6, 14 (important), 18 1.7: 1, 4, 10, 15
- 2.1: 1, 2, 6 2.2: 5, 10 2.3: 9, 11, 25
- 2.4: 6, 14, 16 3.1: 1, 6, 7, 24, 30, 43
- 3.2: 6, 8, 11

Videos:

- Wushi Goldring: Lectures 1–4, 10
- Gregory Arone: Lectures 1–5 + slides

2. Topic 2: Group Actions

Main concepts: Group actions, stabilizers, orbits, conjugation, class equation, Cayley's theorem.

Reading: Dummit and Foote, Sections 4.1–4.4

Exercises:

- 4.1: 2, 4, 9, 10 4.2: 2
- 4.3: 3, 6, 17, 19–21, 26, 27, 30

Videos:

- Wushi Goldring: Lectures 5–9
- Gregory Arone: Lectures 6–7 + slides

3. Topic 3: Structure of Groups – Sylow Theorems

Main concepts: p-groups, Sylow theorems, classification of finite groups, structure of abelian groups.

Reading: Dummit and Foote, Sections 4.5–4.6, 5.2

Exercises:

- 4.5: 4, 9, 10, 16, 37 5.2: 1, 4, 7, 8, 9

Videos:

- Wushi Goldring: Lectures 11–12
- Gregory Arone: Lectures 8–9 + slides

4. Topic 4: Introduction to Rings and Fields

Main concepts: Rings, fields, ideals, ring homomorphisms, quotient rings, Chinese Remainder Theorem, polynomial rings.

Reading: Dummit and Foote, Chapter 7

Exercises:

- 7.1: 5, 6, 7, 9, 15, 21
- 7.2: 2, 5, 7
- 7.3: 1, 2, 15, 16, 24, 26, 28, 33, 34
- 7.4: 9, 11, 14, 15, 19, 20, 30, 31, 32, 33

Videos:

- Wushi Goldring: Lectures 13–14
- Gregory Arone: Lectures 10–11 + slides

5. Topic 5: Special Classes of Rings

Main concepts: Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains, polynomial factorization, Gauss's Lemma.

Reading: Dummit and Foote, Chapter 8, Sections 9.1–9.3

Exercises:

- 8.1: 7, 10, 11 8.2: 1, 2, 3, 8
- 8.3: 1, 5, 6, 7, 8 9.2: 1, 2, 3, 4, 5

Videos:

- Wushi Goldring: Lectures 15–17
- Gregory Arone: Lectures 12–14 + slides

6. Topic 6: A Taste of Field Theory

Main concepts: Fields, algebraic extensions, finite fields, field of fractions.
Reading: Dummit and Foote, Sections 13.1–13.2
Exercises:

- 13.1: All exercises
- 13.2: 2, 4, 8, 9, 16

Videos:

- *Not covered in video lectures*