Solving Simultaneous-play Games

COMP30024 Artificial Intelligence

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Tuesday 13th April, 2021

Administrative updates

- Marks and feedback for project part A will be released this week.
- Looking forward to seeing your project plans (deadline Friday).
- Today's lecture will help you approach project part B, and maybe teach you a little *game theory*.

Review: Zero-sum turn-based games

We've already seen some *game theory*:

Zero-sum game: The **utility** for the opponent is the negative of the utility for the player.

Minimax algorthm: Recursively compute the utility of each action in a turn-based zero-sum game, using the *minimax principle*.

Optimal*, but slow.

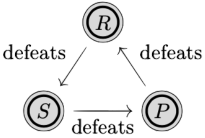
- **Cut-off:** Depth-limited minimax + evaluation function.
- **Pruning:** Alpha-beta pruning + sensible move ordering.

Challenge: Zero-sum *simultaneous-play* games

Can we use minimax in simultaneous-play games? Might not make sense:

- **Indivisible turns**: The game rules only allow for two actions at once.
- **Who moves first?**: Even if possible, dividing turns introduces information asymmetry.

Insight: In simultaneous play games, *information is key*, and *predictability is dangerous*.



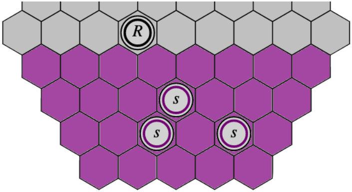


Figure 1: Simultaneous-play challenges

Approaches

Today we will consider two approaches:

1. Go ahead and use minimax anyway: *Artificially divided turns* and the *paranoid reduction*.
2. Address information asymmetry with game theory: *Feedback games* and *equilibrium strategies*

(There are other approaches, and you can feel free to explore or get creative for the project.)

Use minimax anyway

Address the challenges:

- **Artificially divide turns:** Delay the full update until both players have a turn.
- **Player goes first:** Imagine revealing your move to opponent. Find a move to which your opponent has *no* good response (so-called **'paranoid' reduction**)

Optimal*, but *too robust?*

- You might miss *truly good* moves by assuming opponent knows.
- In the worst case, *all* moves *look* bad, you might pick a *truly bad* one.
- Might be able to weaken the opponent's ability to respond (restricted actions) or explicitly accounting for paranoia in evaluation function.

Use *game theory*: Single-stage games

Game theory can help us handle 'single stage' simultaneous-play zero-sum games (like Rock-Paper-Scissors itself):

- **Payoff matrix**: Evaluate *pairs of actions*, in a grid.

$$
\begin{array}{lcc}
\text{opponent:} & & \begin{array}{ccc} \;\; r \;\; & \;\; p \;\; & \;\; s \;\; \end{array} \\
 & \begin{array}{c} R \\ P \\ S \end{array} & \begin{pmatrix} 0 & -1 & +1 \\ +1 & 0 & -1 \\ -1 & +1 & 0 \end{pmatrix} \\
\end{array}
$$

$$
\text{player: } \begin{array}{c} R \\ P \\ S \end{array}
$$

- **Mixed strategy**: Allow *random solutions*, distributions over actions.
- **Minimax principle**: Choose the strategy that minimises the maximum harm the opponent can cause.
- **'Nash equilibrium'**: For a zero-sum game, neither player benefits by individually deviating from such a strategy.

Use *game theory*: Finding the equilibrium strategy

Finding the equilibrium/optimal strategy is a **Linear Programming** problem. A sample implementation using NumPy and SciPy will be provided on the LMS.

```python
def solve_game(V):
    """
    Given a utility matrix V for a zero-sum game, compute
    equilibrium strategy s and value v for row maximiser.
    """
```

$$\texttt{solve\_game}\left(\begin{array}{cc} & \begin{array}{ccc} r & p & s \end{array} \\ \begin{array}{c} R \\ P \\ S \end{array} & \begin{pmatrix} 0 & -1 & +1 \\ +1 & 0 & -1 \\ -1 & +1 & 0 \end{pmatrix} \end{array}\right) \quad \rightarrow \quad \texttt{s} = \begin{array}{c} R \\ P \\ S \end{array} \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}, \quad \texttt{v} = 0$$

Use *game theory*: Multi-stage games

Value of a game: Given an equilibrium solution, a game has a fixed minimum expected utility *value*.

Backward induction algorithm: Recursively solve multi-stage simultaneous-play game by first solving sub-games, then forming a new game at the higher level out of the solution *values*.

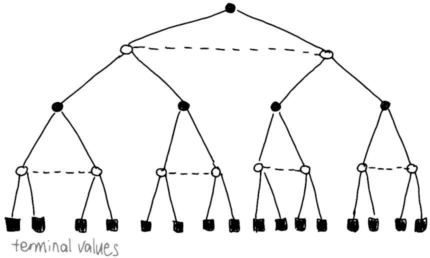


Figure 2: The whole game tree

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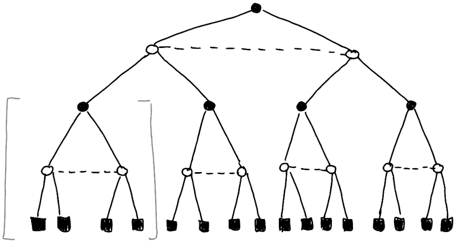

Figure 3: Contains single-stage sub-games

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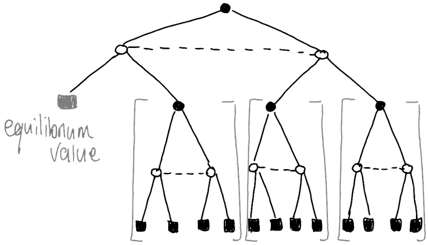

Figure 4: Recursively solve future stages

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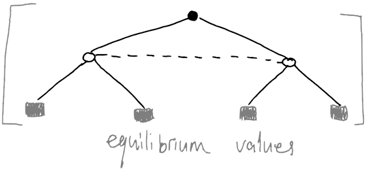

Figure 5: Solve the first stage

Use *game theory*: Cut-off and Pruning

Cut-off: Just like for minimax, rather than traverse the whole backward induction search tree to the bottom, we can search a few layers and then switch to an **evaluation function**.

Pruning: More complex than for minimax, but it is possible to prune the backward induction search tree:

- Can **prune the payoff matrix** (remove clearly suboptimal action rows/columns) without sacrificing Optimality*.
- May get away with **more aggressive pruning** (heuristically remove action rows/columns that are possibly optimal) at some cost to Optimality*.

Further reading

- Two short papers on two different optimal pruning methods for backward induction:

  Saffidine *et al.* (2012). Alpha-Beta Pruning for Games with Simultaneous Moves.

  Bošanský *et al.* (2013). Using Double-Oracle Method and Serialized Alpha-Beta Search for Pruning in Simultaneous Move Games.

- Longer review paper collecting several approaches for solving simultaneous-play games:

  Bošanský *et al.* (2016). Algorithms for computing strategies in two-player simultaneous move games.